Mispricing and Uncertainty in International Markets

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Abstract

We develop Residual MisPricing (RMP), an index capturing mispricing relative to a linear benchmark asset pricing model from the absence of arbitrage. RMP is fully conditional, and depends only on the returns of basic assets. We show that conditional linear asset pricing models perform well on average, and during normal times, while they imply a larger (residual) mispricing during crisis periods. Return data for several economies reveal that RMP is countercyclical and related to financial uncertainty. RMP further shows a strong positive relation to conditional international equity and currency risk premia, even after accounting for linear factors, as well as a close link to market-wide funding liquidity shocks. We find that RMP predicts future market dislocations, including covered interest rate parity (CIP) deviations.

Keywords: stochastic discount factor, residual mispricing, financial uncertainty, exchange rates, machine learning.

JEL Classification: G11, G12, G15

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1 Introduction

Recent literature in financial economics identifies pronounced structure in the pricing residuals generated by linear asset pricing models in the cross section of asset prices (e.g., Ang, Hodrick, Xing, and Zhang, 2006b; Frazzini and Pedersen, 2014), disregarding that linear models may violate the no-arbitrage condition. We reconsider the modeling approach underlying these investigations, and focus on the smallest adjustment of linear pricing models that guarantees the absence of arbitrage, or in other words, the existence of a stochastic discount factor (SDF). This adjustment is easy to compute with minimal data requirements, using only return data on basic assets such as equities and bonds. As the difference between the underlying linear return model and the minimal variance no-arbitrage SDF, it represents a parsimonious and conservative index of mispricing. Empirically, since linear asset pricing models with a small number of factors driving risk premia are widely used in currency markets, we derive the residual mispricing (RMP) index for a cross section of currency pairs. We find a highly significant relation to indicators of financial uncertainty, financial distress measures, conditional risk premia, as well as the cross section of currency returns.

Our results are based on several methodological and empirical contributions. First, we are agnostic about the factor model driving the cross-section of international returns, and recognize that it must lie within the space of linear combinations of the returns available to investors. We proceed thus by proposing a resolution of the dichotomy of the Hansen and Jagannathan (1991) minimum-variance pricing framework, that one needs to choose between the ubiquitous linear pricing model admitting arbitrage, or an arbitrage-free model that is linear in option-type functions of returns, that is difficult to estimate, and thus rarely used in practice. In our framework based on Almeida and Schneider (2021), the model identified through the pricing equations is the sum of a linear part, that is identical to the one from Hansen and Jagannathan (1991), and a nonlinear part, featuring minimal volatility and no price impact. Taken together, they are positive in every state of the world, and represent a valid stochastic discount factor. In economic terms, this nonlinear part can be interpreted as the minimal-cost nonlinear trading strategy (insurance policy) that a representative agent optimizing her Sharpe ratio portfolio would need to enter, in order to price all assets in question and to ensure the absence of arbitrage.

To work with RMP empirically, we estimate conditional expectations from realized returns and conditioning information through novel machine learning techniques. While we can infer RMP also unconditionally, conditional RMP enables us to relate to market events and conditional risk premia. Importantly, in our empirical exercise, we show how crucial time-varying weights are when constructing the optimal Sharpe
ratio portfolio, in particular during market downturns, when we uncover a clear flight-to-safety pattern. Moreover, we show that while RMP is usually downward-sloping in the returns, its shape is inverted during market turmoil, and its overall level is higher and positive, suggesting that insuring against mispricing becomes more expensive. Intuitively, we substantiate the finding that while linear asset pricing models perform well on average and during normal times, they imply a larger (residual) mispricing during crisis periods, when the marginal utility of investors is higher.

We exploit the weak data requirements of RMP and investigate the cross section of aggregate equity and short-term bonds in multiple economies. The motivation is twofold. First, Verdelhan (2018) documents that the strong correlation structure of bilateral exchange rates can be attributed to essentially two risk factors, namely, carry and dollar, accounting for a large portion of the share of systematic variation. Second, Koijen and Yogo (2020) postulate that when determining their optimal demand, global investors hold exchange rate exposure not only through bonds, but also equity. Hence, with U.S. as our base currency, we compute RMP for United Kingdom, Switzerland, Australia, Canada, Japan, New Zealand, and the Euro area, and additionally examine currency returns with respect to the U.S. dollar. Moreover, we account for the interest rate differential and realized volatility as conditioning variables, conforming with Lustig, Roussanov, and Verdelhan (2011) and Menkhoff, Sarno, Schmeling, and Schrimpf (2012), among others.

We summarize our empirical findings as follows. First, we document a strong link between RMP and conditional expected returns. Specifically, we find that the cross section of domestic and international aggregate equity returns, as well as currency returns, are explained by our RMP, even after controlling for linear factors. Hence, RMP is a nonlinear priced risk factor, theoretically and economically motivated, since it is extracted from no-arbitrage without assuming particular preferences for investors, and with the corresponding risk premium on mispricing in equity and currency markets being positive. Importantly, we show that risk premia are positive for conditional expected returns, whereas for unconditional (realized) returns, we obtain the typical negative relation between risk and return documented previously in the literature (see, e.g., Ang, Hodrick, Xing, and Zhang (2006b), among others). We conclude that studying conditional expected returns is thus instrumental for inference and estimation.

Second, we show that RMP is related to extant measures of uncertainty, such as financial uncertainty of Jurado, Ludvigson, and Ng (2015), and the VIX. RMP and financial uncertainty are highly correlated in both contemporaneous and predictive exercises with no ex-ante lead lag relation. To alleviate potential statistical concerns,
we further establish that our results hold in particular also out-of-sample.\(^1\)

Third, we explore the potential origins of residual mispricing in terms of investors’ (in)ability or willingness to trade. To this end, following He, Kelly, and Manela (2017), we investigate the link with financial intermediaries since they are in the unique position of trading most asset classes, especially the more complex ones, involving nonlinear payoffs that trade over-the-counter (OTC). We find no significant relation between RMP and the intermediary capital ratio, defined as the aggregate value of market equity divided by aggregate market equity plus aggregate book debt of primary dealers. In contrast, there is a positive and statistically significant correlation between RMP and the intermediary leverage squared (the squared reciprocal of the capital ratio of the intermediary sector), providing further support that RMP can be interpreted as a state variable capturing the degree of financial distress. This finding is in line with predictions of dynamic intermediary asset pricing models featuring a nonlinear, or state-dependent, relation between the asset risk premium, and the degree of financial sector distress. Still, in regression analysis, the explanatory power varies from 1% to 9% across international markets, implying that investors may choose not to trade during periods of high market uncertainty (see, e.g., Rigotti and Shannon (2005), among others). Finally, we show that our market-implied RMP index exhibits a close link to market-wide funding liquidity shocks, as measured by the TED spread and capturing a commonality in the financial fragility affecting speculators’ capital and margin requirements. Overall, we document that residual mispricing is high during periods of uncertainty and during episodes of low (funding) liquidity, or financial distress. Our results suggest thus that nonlinearities in asset pricing models ensuring absence of arbitrage opportunities are particularly relevant during financial turmoil.

**Literature.** Our framework has implications for, and builds on, several strands in the literature. Powerful empirical results on asset pricing anomalies relative to linear asset pricing models, have been previously documented. Early studies providing such evidence and attempting to explain the empirical failure of the CAPM include Brennan (1971), Black (1972), Black, Jensen, and Scholes (1972), and Haugen and Heins (1975). Also more recent research confirms these patterns. Ang, Hodrick, Xing, and Zhang (2006b, 2009) show that (idiosyncratic) volatility negatively predicts equity returns and that stocks with high sensitivities to aggregate volatility risk earn low returns. Related, Stambaugh, Yu, and Yuan (2015) argue that the sign of the relation between idiosyncratic risk and returns depends on whether stocks are over- or underpriced. Schneider, Wagner, and Zechner (2020) suggest that asset pricing

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\(^1\)Establishing a causal relation between residual mispricing and uncertainty is beyond the scope of this paper. Rather, we document a strong correlation between them in the data.
anomalies stem from coskewness that is not accommodated by SDF models that are linear in returns, but they do not attempt to model this nonlinear SDF component. In this paper, we propose, as a conservative choice, the smallest possible nonlinear SDF component guaranteeing no arbitrage. This explicit model allows us to investigate the systematic error induced from linear pricing models with relatively small cross sections. Importantly, our goal is to document the missing part, or the residual, of linear (factor) models, rather than which of the existing factors drive risk premia. By construction, since the SDF is a nonlinear combination of the traded assets, it encompasses the relevant risk factors driving those assets’ returns. We find no significant relation between well-established currency risk factors and our RMP, suggesting we uncover a new nonlinear priced factor, that guarantees absence of arbitrage opportunities, stemming from unspanned risk and capturing exposure to financial distress or crash risk.

Our approach is also related to the literature examining unspanned risks, that is, factors that affect risk premia, but do not appear in prices. Following the studies of Collin-Dufresne and Goldstein (2002) and Duffee (2011), these unspanned risks have been suggested to be of importance to reconcile prices with risk premia. Further works continuing this line of thought include Gabaix (2012), Joslin, Priebusch, and Singleton (2014), and Filipović, Larsson, and Trolle (2017). The advantage of our approach stems from the formulation of a minimum-variance SDF directly in terms of a linear market model (widely used in industry and academia), as well as a part that is not observed in prices, which we show to be related to conditional risk premia. Since our RMP index extracted exclusively from asset prices is related to uncertainty, in the sense that it is risk not observable in prices, we add to the studies of Drechsler (2013) and Bali, Brown, and Tang (2017), documenting the link between equity premia and uncertainty.

While we perform standard minimum divergence stochastic discount factor (SDF) selection (Almeida and Garcia, 2012, 2017), we do so in a particular polynomial space proposed by Almeida and Schneider (2021) that allows us to control precisely the nonlinearity needed to ensure positivity of the SDF (similar techniques are used in Lasserre, 2010; Renner and Schmedders, 2015). Differently from the positive Hansen and Jagannathan (1991) SDF which is linear in option-payoff-type functions, our model is formulated in terms of a part that is linear in returns, and another part of minimum divergence that is at least quadratic. Sandulescu, Trojani, and Vedolin (2020) employ unconditional minimum dispersion SDFs in an international setting and study the relationships between market incompleteness, financial market structures and international finance puzzles. We significantly depart from this framework, by constructing conditional minimum-variance SDFs that ensure absence of arbitrage.
in every state of the world. Moreover, we document the time series properties of conditional risk premia and the importance of time-varying weights in optimal portfolios, while focusing on the residual mispricing relative to linear factor models.

Lastly, to obtain conditional risk premia, we employ novel machine learning techniques based on distribution embedding in reproducing kernel Hilbert spaces (RKHS), first introduced in Song, Huang, Smola, and Fukumizu (2009). Using reproducing kernel Hilbert spaces (RKHS), we are able to incorporate the information from multiple conditioning variables simultaneously for conditional expectations of a multitude of functions, allowing us to discern between the role of the information set, and the information induced by no-arbitrage.² In contrast with linear regressions, such as ordinary least squares, we do not have to make assumptions regarding the functional form, as the optimal function itself is found to accurately represent the data. In particular, we can accommodate nonlinear interactions between the returns and the conditioning variables, whereas if the true relation between them was linear, RKHS would correctly approximate it. Relative to local polynomial regressions Nagel and Singleton (2011) employ, RKHS distribution embedding is a global machine learning technique that performs well in small samples. Furthermore, as opposed to neural nets, it always operates at globally optimal estimates, with an exactly known functional form of the resulting conditional expectations. Lastly, we estimate conditional expectations, while rolling windows are just noisy estimates of the unconditional expectation. We apply this technique in particular to obtain out-of-sample inference. To the best of our knowledge, RKHS have to this point been used in financial economics to estimate coefficients of a linear pricing kernel model in Kozak (2020), and to estimate discrete probabilities on low-dimensional scenario trees in Schneider (2021).

The rest of the paper is organized as follows. Section 3 describes the framework employed to assess residual mispricing. Section 4 presents the data and our empirical findings. Section 5 contains three robustness checks to our main empirical analysis. In Section 5.1 we study RMP when nonlinear claims are added to the linear return model. Section 5.2 confronts RMP with the mispricing factors from Stambaugh and Yuan (2017). In Section 5.3, we compute RMP under a long-run-risk model, and investigate its behavior under two extreme conditioning events. Section 6 concludes. Appendices A and B contain details of the construction of RMP and the estimation of conditional expectations, respectively.

²RKHS are particularly relevant for statistical learning, or finding a predictive function based on data. They can be thought of as a generalization of linear regressions, in that optimal functions are found that represent the data, rather than optimal coefficients, with little computational overhead. Differently from other machine learning techniques such as neural nets, RKHS are a white, rather than a black box, in that all steps and expressions are deterministic and interpretable.
2 Introductory example

To develop intuition for the arguments to come, imagine a simple two-asset economy accommodating excess returns \( r_X \) (the market) and \( r_Y \), with standard deviations \( \sigma_X \) and \( \sigma_Y \), respectively. Starting from a linear CAPM-type model for the SDF \( M^* := a + br_X \) then yields the well-established relation between the two excess returns

\[
\mathbb{E}[r_Y] = \frac{\text{Cov}(M^*, r_Y)}{\text{Cov}(M^*, r_X)} \mathbb{E}[r_X] \\
= \frac{\text{Cov}(a + br_X, r_Y)}{\text{Cov}(a + br_X, r_X)} \mathbb{E}[r_X] \\
= \frac{\text{Cov}(r_X, r_Y)}{\sigma_Y^2} \mathbb{E}[r_X].
\]  

(1)

The popularity of the linear pricing approach likely stems from the observation that the coefficient \( b = \frac{\text{Cov}(r_X, r_Y)}{\sigma_X^2} \) is the same that would arise from an OLS regression of \( r_Y \) on \( r_X \), with the additional prediction that the intercept is zero.

The elegance of the unification of pricing and OLS regression comes with a drawback, however, since \( M^* \) is not necessarily an SDF. As a linear function of \( r_Y \), that as an excess return takes both positive and negative values, it becomes negative with positive probability. To remedy this problem, consider an additive modification

\[
M := M^* + M^0.
\]

To be consistent with pricing in this economy, the additive part \( M^0 \) must satisfy a number of restrictions. First, it ought to be mean-zero, \( \mathbb{E}[M^0] = 0 \), so that the price of a risk-free bond is unaltered, and orthogonal to \( r_X \), i.e. \( \mathbb{E}[r_X M^0] = 0 \), for an interpretation as a projection error. Second, to not change the pricing relation (1), it needs to satisfy \( \mathbb{E}[r_Y M^0] = 0 \). Third, and most importantly for the context of this paper, it needs to be such that \( M^* + M^0 \geq 0 \) in every state of the world.

To sustain the linear structure of \( M^* \), the most natural way of satisfying all these constraints is the specification

\[
M^0 = w_0 + w_1 r_X + w_2 r_X^2.
\]  

(2)

From the aforementioned constraints, the weights \( w_0, w_1, w_2 \) must solve

\[
\mathbb{E}[w_0 + w_1 r_X + w_2 r_X^2] = 0, \\
\mathbb{E}[r_X(w_0 + w_1 r_X + w_2 r_X^2)] = 0, \text{ and} \\
\mathbb{E}[r_Y(w_0 + w_1 r_X + w_2 r_X^2)] = 0.
\]
Concerning the positivity constraint, if the discriminant \((b + w_1)^2 - 4c_2(a + w_0)\) of the quadratic polynomial in \(r_X\): \(M = M^* + M^\circ = (a + w_0) + (b + w_1)r_X + w_2r_X^2\) is non-positive, then \(M \geq 0\) in every state of the world, and thus represents a valid SDF. If the discriminant is positive, a higher even order polynomial in (2) can be chosen to satisfy the linear constraints and to ensure positivity. In any case, a unique and conservative set of coefficients can be chosen by minimizing the second moment of \(M^\circ\) subject to the constraints above.

To see the contradictions and arbitrage opportunities that can arise by taking \(M^*\) as a SDF, suppose that \(b\) is negative, \(a\) is positive, and consider a call option on \(r_X\) with strike price \(K = a/|b|\), which is precisely the cutoff from which \(M^*\) becomes negative.\(^3\) The payoff from the call option is \((r_X - K)^+\), with \((z)^+\) denoting the positive part of \(z\), and the price using \(M^*\) as an SDF is

\[
\mathbb{E}\left[M^*(r_X - K)^+\right] = \mathbb{E}\left[(a + br_X)\left(r_X - \frac{a}{|b|}\right)^+\right] \leq 0,
\]

since the payoff is positive precisely when \(M^*\) is negative, \(r_X - \frac{a}{|b|} > 0 \iff a + br_X < 0\). Taking together the non-negative payoff with the non-positive price computed with \(M^*\) constitutes a (weak) arbitrage portfolio.

On the other hand, the price computed with \(M = M^* + M^\circ\) reads

\[
\mathbb{E}\left[M \left(r_X - \frac{a}{|b|}\right)^+\right] = \mathbb{E}\left[(M^* + M^\circ) \left(r_X - \frac{a}{|b|}\right)^+\right] \geq 0,
\]

since \(M\) is non-negative in every state of the world, and so is \(r_X - \frac{a}{|b|}^+\), excluding arbitrage.

The minimum-variance modification \(M^\circ\) is the most conservative (since it has minimal variance) residuum that makes \(M = M^* + M^\circ\) a valid SDF without changing the linear pricing equations. As such, it is interpreted as residual mispricing, in the sense that it measures the distance between the linear pricing model \(M^*\) and the closest no-arbitrage model \(M\).

### 3 Conditional residual mispricing

In this section, we describe our modeling assumptions to assess the degree of residual mispricing implied by different asset markets. Following our introductory example

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\(^3\)This can be seen from \(a + br_X \leq 0 \iff r_X \geq a/|b|\) for \(a \geq 0, b < 0\). Different signs of \(a\) and \(b\) would lead to different arbitrage portfolios. For instance, with \(b > 0\), we could work with a put option.
in the previous section, we build our approach upon a unifying framework between standard linear asset pricing models, relative to which asset pricing anomalies are commonly measured, and the notion of no-arbitrage. No-arbitrage implies the existence of a strictly positive stochastic discount factor (SDF), but SDF models that are linear in excess returns can not be positive in every state of the world. Below, we develop the discrepancy between the linear SDF and the closest positive SDF as our RMP index, and motivate why it measures mispricing.

To this end, we reformulate the Hansen and Jagannathan (1991) asset pricing framework, such that linear SDF models are merely projections onto the linear asset span and thus naturally accommodate linear asset pricing models, while being positive in all states of the world. We achieve this task by working with a family of SDFs that are non-negative polynomials in the underlying asset excess returns. We are not the first to consider polynomial pricing kernels, as starting from Kraus and Litzenberger (1976), a large literature is built on power series expansions (Harvey and Siddique, 2000, and many other papers). However, our approach is the first to yield positive polynomial SDFs, to the best of our knowledge. This is not an arbitrary choice, as we are interested in modifying linear models for positivity, and this is naturally done with polynomials. Moreover, this specification allows for an additive decomposition: the linear factor model plus the nonlinear part. Importantly, this decomposition is appealing because the nonlinear part is orthogonal to the factor model, allowing us to isolate the differential effect. As the polynomial degree becomes large, the specification converges to the true, unknown SDF. While they are positive by construction, exponentially affine models share none of these advantages.

In incomplete arbitrage-free markets, there may be many candidate SDFs. Following the literature, we therefore choose a representative of this set that exhibits minimal variance. It is well known that such a minimum-variance SDF is also associated with the sharpest upper bound for the Sharpe ratio of all trading strategies in this economy (see Hansen and Jagannathan, 1991). For assets with excess returns \( \mathbf{R}_{i,t+1} := (R^i_{1,t+1} - R^i_{0,t+1}, \ldots, R^i_{m_i,t+1} - R^i_{0,t+1})^\top \) from time \( t \) to time \( t+1 \) to be priced, with \( R^i_{0,t+1} \) the one-period riskless gross bond return in economy \( i \) known at time \( t \).

\footnote{From an econometric point of view, under mild technical conditions, polynomials are a basis of the space of square-integrable random variables under the real-world probability \( P \) and thus well-suited for approximation.}
we select the SDF according to the following procedure.

\[
\begin{align*}
\minimize_{M_{t+1} \in P_{n,m_i}} & \mathbb{E}^P \left[ M_{t+1}^2 \mid \mathcal{F}_t \right], \\
& \text{subject to } \mathbb{E}^P \left[ M_{t+1} R_{i,t+1} \mid \mathcal{F}_t \right] = 0_{m_i}, \mathbb{E}^P \left[ M_{t+1} \mid \mathcal{F}_t \right] = \frac{1}{R_{0,t+1}} \tag{3} \\
& M_{t+1} \overset{p.a.s.}{\geq} 0,
\end{align*}
\]

where \(0_{m_i}\) denotes a zero vector of length \(m_i\), \(\mathcal{F}_t\) is the information set available to investors at time \(t\), and \(P_{n,m_i}\) denotes the space of polynomials in \(R_{i,t+1}\) with fixed even degree \(n\). We denote the optimal solution in Equation (3) by \(M_{i,t+1}\), with \(i \in \{d,f\}\). It is linear in powers of \(R_{i,t+1}\), and with \(\mathcal{B}_{i,n,t+1} := (1, R_{1,t+1}^i, \ldots, R_{m_i,t+1}^i, (R_{1,t+1}^i)^2, R_{1,t+1}^i R_{2,t+1}^i, \ldots, (R_{m_i,t+1}^i)^n)^\top\). we can write it as

\[
M_{i,t+1} = w_t^\top \mathcal{B}_{i,n,t+1}, \tag{4}
\]

where the weights \(w_t\) are known at time \(t\).

The original construction in Hansen and Jagannathan (1991) performs the optimization in \(L_2^P\), the space of square-integrable random variables under the real-world probability \(P\). We perform the optimization in formulation (3) on a smaller set than in the original formulation.\(^6\) The reason for this simplification is twofold. Firstly, we can accommodate linear models in this way. Linear models, such as the one implied by the Capital Asset Pricing Model or the Fama French factor models are widely used in academia and industry. Secondly, we can easily control the condition that the SDFs are positive, through a simple sum of squares condition that is available for positive polynomials. Positive Hansen and Jagannathan (1991) SDFs are linear in option-type functions of the returns, hard to estimate conditionally, and not easy to reconcile with practitioners’ models.\(^7\) Moreover, we can uniquely decompose the solution \(M_i\) as\(^8\)

\[
M_{i,t+1} = M_{i,t+1}^\star + M_{i,t+1}^\circ = \underbrace{w_t^\top \mathcal{B}_{i,n,t+1}}_{\text{HJ linear part}} + \underbrace{w_t^\circ \mathcal{B}_{i,n,t+1}}_{\text{RMP}},
\]

\(^5\)This optimization problem is considered in Almeida and Schneider (2021) to engineer positive polynomial likelihood ratios. Since problem (3) is convex, it can be solved rapidly to a unique optimal solution. We describe the corresponding finite-dimensional program in detail in Appendix A.

\(^6\)Specifically, we consider \(P_{n,m_i}\) with fixed, finite and even maximal degree \(n \geq 2\), using the expectation inner product associated with \(L_2^P\). When \(L_2^P\) is separable, a mild assumption, the space of polynomials of fixed degree is a subspace and a strict subset of \(L_2^P\).

\(^7\)Hansen and Jagannathan (1991) show that the positive random variable with the smallest variance pricing \(R_{i,t+1}\) is the positive part of the linear specification, \((\delta^\top R_{i,t+1})^+\), such that \(\mathbb{E}^P \left[ M_{t+1}^2 \mid \mathcal{F}_t \right] \geq \mathbb{E}^P \left[ (\delta^\top R_{i,t+1})^+ \mid \mathcal{F}_t \right]\) and the variance of the polynomial kernel is an upper bound for the HJ distance. The polynomial form makes our conditional study possible, that would be computationally intractable with the positive part formulation.

\(^8\)This follows directly from the properties of a Hilbert space.
where $M^*_{i,t+1}$ is linear in $R_{i,t+1}$, identical to linear Hansen-Jagannathan SDFs with no positivity constraint. The second component $M^\circ_{i,t+1}$ is the random variable with smallest variance, such that $M_{i,t+1}$ is positive. Furthermore, it is orthogonal to $M^*_{i,t+1}$ and all its components. As a second benefit of orthogonality, the second moment of $M_{i,t+1}$, an upper bound for the conditional Hansen-Jagannathan bound, can be decomposed into

$$
\mathbb{E}^P \left[ M_{i,t+1}^2 \mid \mathcal{F}_t \right] = \mathbb{E}^P \left[ M^*_{i,t+1}^2 \mid \mathcal{F}_t \right] + \mathbb{E}^P \left[ M^\circ_{i,t+1}^2 \mid \mathcal{F}_t \right],
$$

where $\mathbb{E}^P \left[ M^\circ_{i,t+1}^2 \mid \mathcal{F}_t \right]$ can be interpreted as the marginal Sharpe ratio increase arising from nonlinear claims necessary for hedging arbitrage portfolios.

We identify the discrepancy $M^\circ_{i,t+1}$ between the linear SDF model $M^*_{i,t+1}$, and the closest positive SDF $M_{i,t+1} = M^*_{i,t+1} + M^\circ_{i,t+1}$, as the most conservative measure of mispricing relative to the linear SDF model. It is conservative, since $M^\circ_{i,t+1}$ is the modification to the linear SDF $M^*_{i,t+1}$ with the smallest variance ensuring no-arbitrage. It is a measure of mispricing, since it reflects the fraction of the positive SDF that is missed by the linear model to guarantee the absence of arbitrage. In what follows, we use the terms $M^\circ_{i,t+1}$ and RMP interchangeably.

The presence of conditional expectations in formulation (3) necessitates a model. Rather than working with a parametric formulation, or nonparametric local linear or local constant regressions, we learn the conditional expectation function in reproducing kernel Hilbert spaces (RKHS). RKHS are a powerful tool used in machine learning to transform potentially infinite-dimensional features of an object to be studied through data, such as a conditional expectation, to simple finite-dimensional linear algebra operations through the so-called kernel trick. In financial economics they are to this date rarely used (a notable exception is Kozak, 2020, who uses RKHS to learn the coefficients of a linear SDF). Learning the conditional expectation function requires additional steps over the standard RKHS machinery. Importantly, one must specify which variables make up investors’ information set $\mathcal{F}_t$. Contrary to local constant and local linear nonparametric regressions, RKHS conditional distribution embedding allows us to use multidimensional conditioning information with comparably short time series. In our empirical application, we are able to use a bivariate information set, despite a relatively small number of data points ($\approx 540$).\footnote{We describe the procedure along with its in-sample and out-of-sample cross validation in Appendix B.}
4 RMP in international markets

In this section, we describe the data employed in the empirical analysis and study the time series properties of international RMP indices. We then document a strong positive premium for RMP in the cross-section of international stocks, as well as for currencies. Subsequently, we link the extracted RMP to measures of financial uncertainty, and explore potential origins of mispricing. Finally, we analyze the link between RMP and exchange rates.

4.1 Data

We obtain price data for MSCI country indices, exchange rates and one-month deposit rates from Datastream, for the period spanning January 1975 to June 2020. Due to the minimal data requirements of our index, we are able to examine eight developed economies, where we consider the U.S. as domestic market, and we define as foreign markets: the United Kingdom, Switzerland, the Euro area\textsuperscript{10}, Japan, Canada, Australia and New Zealand. Lastly, we derive aggregate equity returns at the country level and risk-free rates from monthly deposit rates, and focus on bilateral trading.

With international data in mind, we follow Lustig, Roussanov, and Verdelhan (2011) and Menkhoff, Sarno, Schmeling, and Schrmpf (2012) and use the interest rate differential, as well as exchange rate volatility as conditioning variables. We calculate two sets of RMP indices, one using the information of the entire sample (in-sample), and the other one using only information available prior to each data point (out-of-sample).\textsuperscript{11}

4.2 Estimation procedure and market structure

In our estimation, we assume that investors can trade both equities and short-term bonds internationally, through the exchange rate $X$. Markets are thus integrated internationally. We follow extant literature and study linear factor models in bilateral exchange rates (see, e.g. Backus, Foresi, and Telmer (2001), Lustig, Roussanov, and Verdelhan (2014) and Verdelhan (2018), among others). The results that we derive are hence based on bilateral trading, assuming that U.S. is the domestic economy. Consequently, the conditional SDFs that we derive correctly price domestic and international equities and short-term bonds. Specifically, for domestic U.S. investors, the vector of tradable returns reads $\mathbf{R}_{d,t+1} := (R_{1,t+1}^d - R_{0,t+1}^d, R_{2,t+1}^d - R_{0,t+1}^d, R_{3,t+1}^d - R_{0,t+1}^d)^\top$, as well as the one-period domestic riskless gross bond return $R_{0,t+1}^d$, where

\textsuperscript{10}We use Germany as a proxy for the Euro area prior to the introduction of the euro currency.

\textsuperscript{11}The time series of the in-sample RMP index is highly correlated with the one computed from unconditional expectations (sample averages).
is the domestic market gross return, and \( R_{d,t+1}^d = R_{0,t+1}^d X_{t+1} \) is the foreign risk-free rate in domestic units, and \( R_{d,t+1}^d = R_{0,t+1}^d X_{t+1} \) is the foreign market gross return in domestic currency. Similarly, for foreign investors, the vector of tradable returns is \( R_{f,t+1} : = (R_{1,t+1}^f - R_{0,t+1}^f, R_{2,t+1}^f - R_{0,t+1}^f, R_{3,t+1}^f - R_{0,t+1}^f)\), as well as the one-period foreign riskless gross bond return \( R_{0,t+1}^f \), where \( R_{1,t+1}^f \) is the foreign market gross return, and \( R_{2,t+1}^f = R_{0,t+1}^d / X_{t+1} \) is the domestic risk-free rate in foreign units, and \( R_{3,t+1}^f = R_{1,t+1}^f / X_{t+1} \) is the domestic market gross return in foreign currency.

In order to estimate conditional expectations, we employ RKHS embedding of conditional distributions. Estimating functions such as conditional expectations in RKHS is a machine learning technique that is particularly tractable, consistent in an econometric sense, as well as generally favorable to asymptotics (Boudabsa and Filipović, 2020). Through the so-called kernel trick, infinitely-dimensional features can be processed through the evaluation of a carefully chosen inner product, leading to simple solutions in terms of numerical linear algebra operations. Analogously to ordinary least squares, that recovers optimal coefficients that represent the data in a linear model, RKHS help recover optimal approximating functions that best describe the data. With various constructions of conditional expectations in RKHS in the literature (Song et al., 2009; Grünewälder et al., 2012; Park and Muandet, 2020), their computation necessitates a parameter for the kernel of the so-called input space, the conditioning variables, as well as a regularization parameter. The kernel parameter controls the smoothness of the family of functions represented, the regularization parameter controls the trade-off between smoothness and the error of the conditional expectation fitted to the realizations. In our estimation, we implement a Gaussian kernel featuring one (volatility) parameter.\(^{12}\) Similarly to local linear and local constant nonparametric regressions, we tune the parameters in a cross validation procedure as described in Appendix B. Importantly, the two aforementioned parameters are used for conditional expectations of any function that is an element of the RKHS, making the procedure very parsimonious with respect to the number of parameters. Since the RKHS representation of conditional expectations follows a sequence of simple linear algebra operations, it is also a very fast procedure. For the in-sample RMP, we perform time series cross-validation over the entire sample, while we repeat the out-of-sample cross validation for each data point (using only information prior to the data point). We also consider our program (3) in terms of unconditional expectations estimated through sample averages, and refer to the corresponding residual mispricing index as the unconditional RMP.

With (un)conditional expectations at hand, we solve the minimization problem

---

\(^{12}\)This choice of kernel is standard in the literature. Our results are not driven by a particular kernel function.
for each data point as a conic program for a unique optimal polynomial $M^*_{i,t+1} + M^o_{i,t+1}$.

The optimization program is detailed in Appendix A. $M^*_{i,t+1}$ is obtained as the classical Hansen and Jagannathan (1991) solution. The time series of $M^o_{i,t+1}$, or the RMP index, is then obtained by evaluating the polynomial $M^o_{i,t+1}$ at the return data $R_{i,t+1}$ realized at time $t+1$, and weights estimated using the information set at time $t$. In Section 4.3, we describe how the RMP index evolves over time, and how it differs between the conditional and the unconditional formulation.

Owing to our international sample comprised of seven currencies GBP, CHF, JPY, EUR, AUD, CAD, and NZD, we distinguish between domestic (U.S.), and international RMP. Since our estimation is carried out for bilateral trading, we take domestic (U.S.) RMP as the average

$$M^o_{dom,t+1} = \frac{1}{7} \left( M^o_{USDGBP,t+1} + \cdots + M^o_{USDNZD,t+1} \right),$$

and international RMP computed as

$$M^o_{int,t+1} = \frac{1}{7} \left( M^o_{GBPUSD,t+1} + \cdots + M^o_{NZDUSD,t+1} \right),$$

in what follows. We also consider foreign RMP as the constituents of international RMP in (7).

### 4.3 Time series properties of RMP

In this section, we describe the statistical properties of the RMP index. Figure 1, top panel, shows the time series of realized U.S. domestic $M^o_{dom}$. As constructed, RMP has an (un)conditional mean of zero, with deviations from zero reflecting the under- or overestimation relative to the linear reference model $M^*_{dom}$. This under- or overestimation refers to the addition or subtraction of a random variable with smallest variance, complementing $M^*_{dom}$ such that absence of arbitrage is ensured in all states of the world. On average, and whenever RMP is close to zero, the linear model provides a good approximation of the return data. It is noteworthy that the RMP realizations with the largest magnitudes occur during crisis times and are uniformly positive. The largest realization occurred during the 1987 crisis, while the Black Wednesday, the Asian crisis and LTCM feature comparable values. RMP during the 2008 financial crisis is as large as during the recent COVID-19 crisis, suggesting that we are experiencing new record highs since the strikingly low idiosyncratic risk over

---

13 This optimization is performed in a fraction of a second with a conic solver such as MOSEK.

14 Transforming first the foreign RMP in U.S. dollars and then taking the average yields virtually identical results. The two indices feature a perfect positive correlation.
the past decade. We plot in the bottom panel of Figure 1 the international RMP (7), computed as the average across all foreign countries (excluding the U.S.), and document patterns similar to those of the U.S. RMP. Importantly, this high similarity in the time series of the two indices suggests that U.S. residual mispricing provides a good approximation of the aggregate international residual mispricing (and vice versa). Indeed, the co-movement of RMP indices between domestic and individual foreign markets is high as well (see, e.g. Table C1).

Figure 1: Time series of international residual mispricing

![Figure 1: Time series of international residual mispricing](image)

This figure plots the time series of the domestic U.S. RMP (top panel) and the international RMP, computed as the average across all foreign countries excluding U.S. (bottom panel). Gray bars denote NBER recessions. The sample period is January 1985 to June 2020.

4.4 Determinants of RMP

In this section, we explore potential determinants of residual mispricing, both domestically and internationally. Guided by the countercyclical patterns documented for RMP, we start by studying its relation with financial uncertainty proxies in international markets. Uncertainty data containing monthly proxies for the macro economy and the financial sector are obtained from Jurado, Ludvigson, and Ng (2015)

\[15\] On the other hand, RMP is mildly persistent, and a question arises whether this is due to the conditional expectations used in the computation of the RMP. We rule out this possibility, as Figure C1 shows in a scatter plot that the realizations of the unconditional vs. the conditional SDFs do not deviate systematically.
and Ludvigson, Ma, and Ng (2015). Specifically, they extract an economic uncertainty index as the conditional volatility of the unpredictable component of a large number of economic indicators. To further examine the link between RMP and uncertainty in international markets, we use the VIX, a volatility index provided by CBOE, as an alternative measure of uncertainty.\footnote{For the sample period between January 1985 and December 1989 we use the monthly data extended for S&amp;P 500 implied volatility from Berger, Dew-Becker, and Giglio (2020).}

Figure 2 (top panel) plots the time-series of U.S. RMP and proxies of financial uncertainty, illustrating a strong positive relation, especially during periods of financial distress. To explore their link more systematically, we regress RMP on financial uncertainty proxies. Specifically, our empirical specification reads\footnote{For ease of interpretation and comparability of coefficients, we standardize all variables to have mean zero and unit variance.}

\[ M_{i,t}^{o} = \alpha + \beta \mathcal{X}_{t} + \epsilon_{t}, \]  

where \( \mathcal{X} \) is a proxy for financial uncertainty, and \( M_{i,t}^{o} = RMP_{i} \) is the smallest no-arbitrage correction to linear pricing models that can be interpreted as a measure of residual mispricing in market \( i \in \{\text{dom, int}\} \).

Table 1 reports the estimation results. All betas are positive and statistically significant at the 1% level, suggesting that periods when the underlying financial uncertainty is high are accompanied by larger residual mispricing. The explanatory power is similar for the U.S. RMP and the international RMP (which is the average across international markets excluding the U.S.), with \( R^{2} \)'s of 19.3% and 19.6%, respectively. Similarly, we find a strong positive relationship between RMP and the VIX. On average, VIX accounts for 24.9% (25.4%) of the variation of RMP in domestic (international) economies. Both, the financial uncertainty index, and VIX, remain significant when running multivariate regressions.\footnote{Since financial uncertainty index and VIX feature a correlation of 80% during the sample period considered, we do not simultaneously include them in the multivariate regressions in Table 1.} Overall, we document a close link between residual mispricing as captured by the RMP index and financial uncertainty in international markets. Thus, RMP can be interpreted not only as an ex-ante measure consistent with the absence of arbitrage opportunities, but also as a financial distress measure. Since existing proxies only capture about 20-25% of the variation in RMP, the latter is a non-redundant measure, economically grounded and theoretically motivated by the no-arbitrage condition. Moreover, since our RMP index uses only return data on international equities and bonds, it is readily available for several economies, without having to rely on options data for instance, or having to aggregate a considerable number of economic indicators to derive a measure of uncertainty.

We proceed by exploring additional potential sources of residual mispricing, in
Figure 2: Determinants of RMP

This figure plots the time series of the U.S. RMP index (dashed-line) against (i) the financial uncertainty index of Jurado, Ludvigson, and Ng (2015) (Top left Panel); (ii) VIX (Top right Panel); (iii) the intermediary leverage ratio squared, or the squared reciprocal of the capital ratio of the intermediary sector from He, Kelly, and Manela (2017) (Bottom left Panel); (iv) TED spread (Bottom right Panel). Gray bars denote NBER recessions. The sample period is January 1985 to June 2020 (January 1986 to June 2020 for TED spread).

Table 1: Determinants of RMP

This table reports the estimated coefficients from regressing the RMP index on various determinants, in the $i = \text{domestic (6), and international market (7): $ M_{\text{RMP}}^{i,t} = \alpha_i + \beta_i X_t^i + \epsilon_{i,t}$. Proxies for financial uncertainty are the financial uncertainty index of Jurado, Ludvigson, and Ng (2015) and the VIX; intermediary leverage ratio squared is from He, Kelly, and Manela (2017), whereas TED spread is from FRED. The domestic market is the U.S., whereas international is the average across foreign markets excluding the U.S. All variables are standardized. Data are monthly and span January 1985 to June 2020 (January 1986 to June 2020 for the TED spread). Newey-West standard errors are reported in brackets. ***, ** and * indicate significant at the 1%, 5% and 10% level, respectively.

<table>
<thead>
<tr>
<th>Panel A: U.S. Residual Mispricing</th>
<th>Panel B: International Residual Mispricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial uncertainty</td>
<td>0.441*** [0.098]</td>
</tr>
<tr>
<td>VIX</td>
<td>0.501*** [0.112]</td>
</tr>
<tr>
<td>Intermediary leverage</td>
<td>0.215*** [0.073]</td>
</tr>
<tr>
<td>TED spread</td>
<td>0.295** [0.127]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>19.3%</td>
</tr>
</tbody>
</table>
order to have a deeper understanding of whether it stems from investors not being able to trade, or being unwilling to trade. To document the ability to trade assets with nonlinear payoffs, that are consistent with our RMP index, we investigate the link with financial intermediaries, and in particular periods when their ability to provide liquidity is hindered. The reason we focus on financial intermediaries is twofold. First, they represent specialized investors, who are in the unique position of trading most asset classes, especially the more complex ones, that trade over-the-counter (OTC). Second, we can only measure the propensity to trade of intermediaries, and periods when they experience financial constraints such that their risk bearing capacity is impaired, are likely to coincide with periods of heightened degrees of unpriced risk, potentially leading to limits to arbitrage and subsequent dislocations in financial markets. Intuitively, an absent relation between residual mispricing and intermediaries or dealers suggests that the latter are not able to provide the average investors the nonlinear payoffs necessary to hedge. If instead there is a strong relation in the data between intermediaries and RMP, but uncertainty is not reduced, it might suggest the unwillingness of investors to trade during high levels of financial uncertainty.\footnote{Importantly, our extracted SDFs are strongly negatively correlated with the intermediary capital risk factor, and significantly positively correlated with the leverage factor of Adrian, Etula, and Muir (2014).}

We use the proxies for financial intermediaries constraints of He, Kelly, and Manela (2017): (i) the intermediary capital ratio, which is the aggregate value of market equity divided by aggregate market equity plus aggregate book debt of primary dealers, (ii) the intermediary capital risk factor, or the shock to the capital ratio converted to a growth rate, and (iii) the intermediary leverage ratio squared, or the squared reciprocal of the capital ratio of the intermediary sector. We find a negative, but insignificant correlation between the degree of residual mispricing and intermediary capital ratio, both domestically and internationally. In contrast, there is a positive and statistically significant correlation between RMP and the intermediary leverage ratio squared (Figure 2 bottom left panel). This result is not surprising, as the RMP itself represents a (nonlinear) correction to standard (linear) asset pricing models that ensures absence of arbitrage opportunities in every state of the world. Moreover, following the predictions of the dynamic intermediary asset pricing model of He and Krishnamurthy (2013) featuring a nonlinear, or state-dependent, association between asset risk premium and the degree of financial sector turmoil, this relation provides further support that RMP can be interpreted as a measure of (international) financial distress.

To examine whether the intermediary leveraged squared is a potential driver of
RMP, we run regressions similar in spirit to (8),

\[ M_{i,t} = \alpha_i + \beta_i \times \frac{1}{\xi^2_{it}} + \varepsilon_{it}, \]  

(9)

with \( i \) denoting the domestic (6), or international market (7), and \( \xi^2 \) the squared intermediary capital ratio. Results are reported in Table 1. The coefficient estimates are positive and significant, suggesting that high leverage (low intermediary capital ratio) states correspond to higher degrees of residual mispricing. Still, the mild explanatory power of 5% implies that investors may choose not to trade during periods of high market uncertainty.

Having established the link with financial intermediaries capital constraints, we explore next the relation with funding liquidity constraints, as measured by the TED spread, which is the difference between three-month interbank (LIBOR) rate and U.S. Treasury bills. The spread widens precisely during times of uncertainty, as banks then generally increase interest rates on unsecured loans. We document a positive correlation of 25% of the TED spread with domestic U.S. RMP. Figure 2 (bottom right panel) plots the time-series of U.S. RMP and the TED spread: the positive relation is apparent, suggesting that the price for insuring against bad states is naturally higher during high uncertainty periods, when the TED spread becomes larger. In summary, we show that our RMP index exhibits a close link with market-wide funding liquidity shocks, as measured by the TED spread and capturing a commonality in the financial fragility affecting speculators’ capital and margin requirements. Overall, residual mispricing is high during periods of uncertainty and during episodes of low (funding) liquidity.

A natural question arises whether already the linear part of the SDF, i.e. \( M^* \), is driven by financial uncertainty and funding constraints proxies, rather than the RMP index, since \( M^* \) itself is likely to increase during periods of financial distress, when the marginal utility of investors is high. To address this possibility, we provide empirical results from regressing the linear part of the SDF on various determinants in Table 2. The coefficient estimates and the \( R^2 \) are significantly lower than the ones in Table 1. In multivariate regressions, most of the explanatory power comes from VIX, but also the financial uncertainty index and TED spread, albeit weaker. We conclude therefore that nonlinear corrections, ensuring arbitrage-free international markets, represent a non-redundant measure of distress, highlighting the importance of residual mispricing for financial markets and the macro-economy.

To examine the robustness of our empirical findings, we additionally perform an out-of-sample exercise. We train our ML model from January 1975 to December 1984, with monthly sequential increments, such that all prior information is accounted
This table reports the estimated coefficients from regressing the linear component of SDF $M^*$ in the $i = \text{domestic}$ (6) and international markets (7) on various determinants: $M^*_{i,t} = \alpha_i + \beta_i X_t + \epsilon_{i,t}$. Proxies for financial uncertainty are the financial uncertainty index of Jurado, Ludvigson, and Ng (2015) and the VIX; intermediary leverage ratio squared is from He, Kelly, and Manela (2017), whereas TED spread is from FRED. All variables are standardized. The domestic market is the U.S., whereas international is the average across international markets excluding the U.S. Data are monthly and span January 1985 to June 2020 (January 1986 to June 2020 for TED spread). Newey-West standard errors are reported in brackets. 

<table>
<thead>
<tr>
<th></th>
<th>Panel A: U.S. linear model</th>
<th>Panel B: International linear model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Uncertainty Index</td>
<td>0.199***</td>
<td>0.136***</td>
</tr>
<tr>
<td></td>
<td>[0.080]</td>
<td>[0.069]</td>
</tr>
<tr>
<td>VIX</td>
<td>0.397***</td>
<td>0.394***</td>
</tr>
<tr>
<td></td>
<td>[0.066]</td>
<td>[0.068]</td>
</tr>
<tr>
<td>Intermediary leverage squared</td>
<td>0.141**</td>
<td>-0.046</td>
</tr>
<tr>
<td></td>
<td>[0.065]</td>
<td>[0.064]</td>
</tr>
<tr>
<td>TED Spread</td>
<td>0.197**</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>[0.080]</td>
<td>[0.067]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>3.7%</td>
<td>15.6%</td>
</tr>
<tr>
<td></td>
<td>15.6%</td>
<td>1.8%</td>
</tr>
<tr>
<td></td>
<td>3.6%</td>
<td>5.7%</td>
</tr>
<tr>
<td></td>
<td>15.7%</td>
<td>15.7%</td>
</tr>
<tr>
<td></td>
<td>5.1%</td>
<td>18.6%</td>
</tr>
<tr>
<td></td>
<td>2.2%</td>
<td>3.5%</td>
</tr>
<tr>
<td></td>
<td>6.8%</td>
<td>18.2%</td>
</tr>
</tbody>
</table>

for, but free of look-ahead bias. We consider the out-of-sample period starting from January 1985 and ending in June 2020. To study the link between residual mispricing and potential determinants out-of-sample, we run regressions in the spirit of Equation (8) and provide the results in Table 3. Although the $R^2$s vary between 5 to 10% in univariate regressions, and are as high as 16% in multivariate specifications, only the financial uncertainty index is able to explain RMP out-of-sample. Hence, our extracted RMP indices are an appropriate proxy to measure financial uncertainty, as the evidence provided is not likely to be spurious or driven by overfitting in sample. Moreover, the advantage of our RMP index is that it can be easily computed and is readily available directly from international asset returns.

We end this section by investigating the relationship between RMP indices in domestic and international markets and the macro economy, by studying the economic policy uncertainty (EPU) index and the equity market volatility (EMV) index of Baker, Bloom, and Davis (2016) and Baker, Bloom, Davis, and Kost (2019), as well as the Chicago Fed National Activity Index (CFNAI). EPU is constructed to account for (i) newspaper coverage of policy-related economic uncertainty, (ii) the number of federal tax code provisions set to expire in future years and (iii) disagreement among economic forecasters as a proxy for uncertainty. EMV is similar to a newspaper-based Equity Market Volatility tracker that moves with the CBOE Volatility Index (VIX) and with the realized volatility of returns on the S&P 500. EMV is further decomposed into policy-related EMV trackers and a suite of trackers that quantify the importance of each category in the level of U.S. stock market volatility and its movements over time. The CFNAI index is designed to gauge overall economic activity and related inflationary pressure. The results from regressing RMP on different measures of
This table reports the estimated coefficients from regressing the out-of-sample RMP index on various determinants, in the \( i = \text{domestic (6), and international market (7)} \): \( M^i_{t+1} = \alpha_i + \beta_i X_t + \epsilon_{i,t} \). Proxies for financial uncertainty are the financial uncertainty index of Jurado, Ludvigson, and Ng (2015) and the VIX; intermediary leverage ratio squared is from He, Kelly, and Manela (2017), whereas TED spread is from FRED. All variables are standardized. The domestic market is the U.S., whereas international is the average across international markets excluding the U.S. The training period is from January 1975 to December 1984, with monthly sequentially increments, such that all prior information is accounted for. Out-of-sample period is from January 1985 to June 2020 (January 1986 to June 2020 for TED spread). Newey-West standard errors are reported in brackets. ** and * indicate significant at the 5% and 10% level, respectively.

<table>
<thead>
<tr>
<th>Financial Uncertainty Index</th>
<th>0.236</th>
<th>0.163*</th>
<th>0.225*</th>
<th>0.169**</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>0.169</td>
<td>0.099</td>
<td>0.136</td>
<td>0.085</td>
</tr>
<tr>
<td>Intermediary leverage squared</td>
<td>0.337</td>
<td>0.289</td>
<td>0.317</td>
<td>0.290</td>
</tr>
<tr>
<td>TED Spread</td>
<td>0.255</td>
<td>0.206</td>
<td>0.221</td>
<td>0.196</td>
</tr>
<tr>
<td>R²</td>
<td>0.133</td>
<td>0.000</td>
<td>0.111</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Panel A: U.S. Residual Mispricing
Panel B: International Residual Mispricing

economic uncertainty and activity are reported in Table 4. We document a strong link between RMP and EPU, as the coefficient is positive and significant at the 1% level. The coefficient for CFNAI, it is not only statistically significant, but also economically meaningful: a lower output in economic activity, as measured by CFNAI, is accompanied by a larger RMP. We additionally uncover a tight link between RMP and EMV, as well as its components. Consequently, periods characterized by higher RMP coincide with periods of heightened uncertainty in macro news and outlook. Policy-related EMV trackers of volatility sources for the financial markets, in particular for financial regulation, and monetary and fiscal policies also display a nontrivial explanatory power for RMP. Lastly, we show that RMP is relevant for other asset classes, such as commodity markets. Similar results are obtained in Panel B, for international RMP. The largest explanatory power (17%) comes from equity market volatility (EMV), also in the multivariate regression reported in the last column of Table 4. Overall, our empirical findings display a tight link between RMP and proxies of policy and market uncertainty, suggesting that RMP is a relevant, non-redundant index for financial markets’ ability to correctly price traded assets.

4.5 RMP and arbitrage violations in the data

We investigate whether the RMP index is related to observed arbitrage violations in the data, or potential sources of mispricing, such as liquidity, information, sentiment, or noise. To this end, we provide a comparison with the market dislocation index of Pasquariello (2014), which captures arbitrage violations in international markets.
Table 4: RMP, economic policy uncertainty and equity market volatility

This table reports the estimated coefficients from regressing RMP on measures of economic policy uncertainty and equity market volatility, in the $i =$ domestic (6) market (Panel A), and international market (7) (Panel B): $M_{it}^c = \alpha + \beta X_t + \epsilon_t$. The explanatory variables in $X$ are from Baker, Bloom, and Davis (2016) and Baker, Bloom, Davis, and Kost (2019). EPU is the economic policy uncertainty, CFNAI is the Chicago Fed National Activity Index measuring overall economic activity, and EMV is the equity market volatility. All variables are standardized. The domestic market is the U.S., whereas international is the average across foreign markets excluding the U.S. The data are monthly and span January 1985 to June 2020. Newey-West standard errors are reported in brackets. *** and ** indicate significant at the 1% and 5% level, respectively.

<table>
<thead>
<tr>
<th>Panel A: U.S. Residual Mispricing</th>
<th></th>
<th></th>
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<tbody>
<tr>
<td>EPU</td>
<td>0.251***</td>
<td>0.078</td>
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<td></td>
<td>[0.075]</td>
<td>[0.053]</td>
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<tr>
<td>CFNAI</td>
<td>-0.237***</td>
<td>-0.139***</td>
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<td></td>
<td>[0.050]</td>
<td>[0.053]</td>
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<tr>
<td>EMV</td>
<td>0.418***</td>
<td>0.359***</td>
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<td></td>
<td>[0.115]</td>
<td>[0.125]</td>
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<tr>
<td>Macro News and Outlook</td>
<td>0.412***</td>
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<td></td>
<td>[0.118]</td>
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<tr>
<td>Financial crises</td>
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<tr>
<td>Financial regulation</td>
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<td>Monetary policy</td>
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<td>Fiscal policy</td>
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<tr>
<td>Commodity markets</td>
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<td></td>
<td>[0.124]</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>6.1%</td>
<td>5.4%</td>
</tr>
<tr>
<td></td>
<td>17.3%</td>
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<td>7.3%</td>
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<td>16.3%</td>
</tr>
<tr>
<td></td>
<td>17.0%</td>
<td>19.7%</td>
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</table>

<table>
<thead>
<tr>
<th>Panel B: International Residual Mispricing</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>EPU</td>
<td>0.228***</td>
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</tr>
<tr>
<td></td>
<td>[0.070]</td>
<td>[0.052]</td>
</tr>
<tr>
<td>CFNAI</td>
<td>-0.226***</td>
<td>-0.133**</td>
</tr>
<tr>
<td></td>
<td>[0.041]</td>
<td>[0.065]</td>
</tr>
<tr>
<td>EMV</td>
<td>0.416***</td>
<td>0.367***</td>
</tr>
<tr>
<td></td>
<td>[0.126]</td>
<td>[0.144]</td>
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<tr>
<td>Macro News and Outlook</td>
<td>0.406***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.127]</td>
<td></td>
</tr>
<tr>
<td>Financial crises</td>
<td>0.200**</td>
<td></td>
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<tr>
<td></td>
<td>[0.083]</td>
<td></td>
</tr>
<tr>
<td>Financial regulation</td>
<td>0.346***</td>
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<tr>
<td></td>
<td>[0.123]</td>
<td></td>
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<tr>
<td>Monetary policy</td>
<td>0.281**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.113]</td>
<td></td>
</tr>
<tr>
<td>Fiscal policy</td>
<td>0.400***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.141]</td>
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<tr>
<td>Commodity markets</td>
<td>0.425***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.138]</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>5.0%</td>
<td>4.9%</td>
</tr>
<tr>
<td></td>
<td>17.1%</td>
<td>16.3%</td>
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<td></td>
<td>3.8%</td>
<td>11.8%</td>
</tr>
<tr>
<td></td>
<td>6.6%</td>
<td>15.8%</td>
</tr>
<tr>
<td></td>
<td>17.9%</td>
<td>18.9%</td>
</tr>
</tbody>
</table>
Table 5: RMP and arbitrage violations in the data

This table reports the estimated coefficients from regressing observed arbitrage violations and financial market dislocations on RMP, in the $i =$ domestic (6) market (Panel A), and international market (7) (Panel B): $Y_i = \alpha + \beta M_{i,t}^o + \epsilon_i$. The independent variables are the market dislocation index (MDI) of Pasquariello (2014) and covered interest rate parity (CIP) deviations in the data, computed as the average across developed markets, at different horizons. All variables are standardized. The domestic market is the U.S., whereas international is the average across foreign markets excluding the U.S. The data are monthly and span January 1985 to December 2009 for MDI (January 2000 to June 2020 for CIP). Newey-West standard errors are reported in brackets. *** , ** and * indicate significant at the 1%, 5% and 10% level, respectively.

<table>
<thead>
<tr>
<th>Panel A: U.S. Residual Mispricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDI</td>
</tr>
<tr>
<td>CIP 3M</td>
</tr>
<tr>
<td>CIP 1Y</td>
</tr>
<tr>
<td>CIP 2Y</td>
</tr>
<tr>
<td>CIP 3Y</td>
</tr>
<tr>
<td>CIP 5Y</td>
</tr>
<tr>
<td>CIP 7Y</td>
</tr>
<tr>
<td>CIP 10Y</td>
</tr>
<tr>
<td>$R^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: International Residual Mispricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDI</td>
</tr>
<tr>
<td>CIP 3M</td>
</tr>
<tr>
<td>CIP 1Y</td>
</tr>
<tr>
<td>CIP 2Y</td>
</tr>
<tr>
<td>CIP 3Y</td>
</tr>
<tr>
<td>CIP 5Y</td>
</tr>
<tr>
<td>CIP 7Y</td>
</tr>
<tr>
<td>CIP 10Y</td>
</tr>
<tr>
<td>$R^2$</td>
</tr>
</tbody>
</table>
Table 6: RMP and arbitrage violations in the data

This table reports the estimated coefficients from running predictive regressions of observed arbitrage violations and financial market dislocations on RMP, in the $i = \text{domestic (6)}$ market (Panel A), and international market (7) (Panel B): $Y_t = \alpha + \beta M_{i,t-1} + \epsilon_t$. The independent variables are the market dislocation index (MDI) of Pasquariello (2014) and covered interest rate parity (CIP) deviations in the data, computed as the average across developed markets, at different horizons. All variables are standardized. The domestic market is the U.S., whereas international is the average across foreign markets excluding the U.S. The data are monthly and span January 1985 to December 2009 for MDI (January 2000 to June 2020 for CIP). Newey-West standard errors are reported in brackets. ***, ** and * indicate significant at the 1%, 5% and 10% level, respectively.

Panel A: U.S. Residual Mispricing

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDI</td>
<td>0.301**</td>
<td>0.139</td>
</tr>
<tr>
<td>CIP 3M</td>
<td>0.231</td>
<td>0.173</td>
</tr>
<tr>
<td>CIP 1Y</td>
<td>0.350**</td>
<td>0.152</td>
</tr>
<tr>
<td>CIP 2Y</td>
<td>0.399***</td>
<td>0.161</td>
</tr>
<tr>
<td>CIP 3Y</td>
<td>0.384**</td>
<td>0.152</td>
</tr>
<tr>
<td>CIP 5Y</td>
<td>0.368***</td>
<td>0.129</td>
</tr>
<tr>
<td>CIP 7Y</td>
<td>0.279***</td>
<td>0.098</td>
</tr>
<tr>
<td>CIP 10Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.210***</td>
<td>0.067</td>
</tr>
</tbody>
</table>

Panel B: International Residual Mispricing

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDI</td>
<td>0.275**</td>
<td>0.130</td>
</tr>
<tr>
<td>CIP 3M</td>
<td>0.213</td>
<td>0.172</td>
</tr>
<tr>
<td>CIP 1Y</td>
<td>0.283*</td>
<td>0.161</td>
</tr>
<tr>
<td>CIP 2Y</td>
<td>0.349**</td>
<td>0.167</td>
</tr>
<tr>
<td>CIP 3Y</td>
<td>0.350**</td>
<td>0.156</td>
</tr>
<tr>
<td>CIP 5Y</td>
<td>0.329**</td>
<td>0.131</td>
</tr>
<tr>
<td>CIP 7Y</td>
<td>0.237**</td>
<td>0.099</td>
</tr>
<tr>
<td>CIP 10Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.197***</td>
<td>0.067</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A</td>
<td>8.7% 5.0% 11.9% 15.5% 14.4% 13.2% 7.4% 4.0%</td>
</tr>
<tr>
<td>Panel B</td>
<td>7.8% 6.8% 5.7% 9.7% 9.8% 8.4% 4.5% 2.8%</td>
</tr>
</tbody>
</table>
While our index is related to the dislocation index of Pasquariello (2014) statistically, providing thus a reassuring sanity check, there is an important conceptual difference to our RMP. The dislocation index is based on realized arbitrage opportunities, or mispricing, while the RMP is computed precisely such that there is no arbitrage. While we can not exclude the existence of arbitrage opportunities between certain markets empirically, they may be hard to disentangle from market frictions, trading costs, or limits to arbitrage in general, making data collection difficult. Moreover, riskless arbitrage opportunities will likely never be observed in practice since they are short-lived. As a consequence, the data requirements of our index are substantially reduced in that the RMP can be calculated from a bond and a market return only, with no reference to bid-ask spreads and fees. In addition, we consider covered interest rate parity (CIP) violations observed for the developed markets in our sample.

We investigate the explanatory and predictive power of our domestic and international RMPs for financial market dislocations and CIP violations in Tables 5 and 6. We find a positive and significant relation between RMP and arbitrage violations in the data, both from MDI and CIP at different tenors considered. The $R^2$s are rather high in both contemporaneous and predictive specifications. For the CIP deviations, both the coefficients estimates and the $R^2$s feature a non-monotonic relation, with the highest being encountered for the two year tenor.

To summarize, we conclude that our market-implied residual mispricing index contains additional information not subsumed by proxies of intermediaries constraints or existing deviations from the no-arbitrage condition, such as the covered interest rate parity violation. Constructed as an ex-ante measure consistent with no-arbitrage, RMP explains and predicts financial market dislocations and CIP deviations.

### 4.6 RMP and expected returns

#### 4.6.1 Univariate estimation

In this section, we examine the link between RMP and expected returns. Since our RMP index is by construction orthogonal to asset returns and therefore unspanned by them, we are interested in uncovering whether this unspanned risk is instead related to asset risk premia. A long-standing literature on unspanned risk premia documents the possibility that expected returns accommodate information that is not contained in prices. As a measure of mispricing, the RMP index lends itself to an investigation in this context.

We consider the empirical specification

$$
\mathbb{E}[r_{j,t+1} \mid \mathcal{F}_t] = \alpha_j + \beta_j M_{dom,t}^\circ + \varepsilon_{j,t},
$$

(10)
where $\mathbb{E}[r_{j,t+1} \mid \mathcal{F}_t]$ is the conditional expected return on the test assets $j$ at time $t$ and $M^i_{t,t} = RMP_t$ (either domestically (6) or internationally (7)) is the correction to linear SDFs that ensures positivity and can be interpreted as a measure of residual mispricing in market $i$. The novelty of our approach stems from the fact that we can use conditional expected returns as the dependent variables, as opposed to realized returns previously employed in the literature.

We estimate the model in Equation (10) using the two-stage procedure of Fama and MacBeth (1973). The methodology proceeds in two steps. First, we run time-series regressions to obtain the betas, or loadings, on the corresponding mispricing index. Second, we run cross-sectional regressions on the estimated betas from the first step in order to retrieve the price of risk of the mispricing index, or the $\lambda$s from

$$\mathbb{E}[r_{j,t+1} \mid \mathcal{F}_t] = \lambda_0 + \lambda_\hat{\beta}_j + \eta_j.$$  

(11)

When performing this two-stage regression, we adjust the standard errors to account for errors-in-variables following Shanken (1992), since the betas are estimated in the first step, for heteroskedasticity, as the variance of residuals is not constant, and for potential autocorrelation in error terms.\(^{20}\)

We provide estimates of the model parameters in (11) for different asset classes in Table 7. The test assets considered are: i) domestic equity in each market, (ii) international equity converted to U.S. dollars, (iii) currency returns, i.e. borrowing at foreign risk-free rates and investing in the domestic U.S. risk-free rate, and (iv) domestic and international equities jointly. We find that RMP explains the cross-section of equities, both domestically and internationally, as well as the cross-section of currencies, with estimated prices of residual mispricing risk that are positive and statistically significant at the 1% level, and $R^2$ between 41% and 90%.

Figure 3 substantiate the regression results, by plotting the conditional expected return against the model-implied predicted return. The test assets lie close to the 45 degree line, suggesting that residual mispricing explains the expected returns considered. In particular, since higher yield currencies exhibit more volatile RMPs, they earn a higher expected return on average (Figure 3 bottom left panel).

Overall, it follows that RMP is a priced risk factor and assets that are more exposed to this risk require higher conditional expected returns. Consequently, the RMP fits the cross-section of international returns well, with small pricing errors.

To emphasize the relevance of conditional expectations, we perform the same exercise on realized returns, using simple averages. We show in Table 8 that the

\(^{20}\)Specifically, the standard errors are going to be generalized method of moments (GMM) errors, that are adjusted for serial correlation using Newey and West (1987).
Table 7: RMP index and asset returns

This table reports the prices of risk $\lambda$, Fama and MacBeth t-stats in parentheses, root mean squared errors (RMSE), and the cross-sectional $R^2$ for the residual mispricing index. Each column contains the following test assets: (i) the domestic equity in each market, (ii) the international equity converted in U.S. dollars, (iii) the currency returns, i.e. borrowing at foreign risk-free rates and investing in the domestic U.S. risk-free rate, and (iv) the domestic and international equities jointly. The first and last columns have eight test assets (individual economies), whereas the second and fourth columns have seven test assets (corresponding to the bilateral pairs having U.S. as domestic economy). The sample period is January 1985 to June 2020. The standard errors in the Fama and MacBeth approach account for errors-in-variables, heteroskedasticity and autocorrelation in error terms. *** indicates significant at the 1% level.

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>International equity</th>
<th>Currency</th>
<th>Domestic &amp; international equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.155***</td>
<td>0.042***</td>
<td>0.040***</td>
<td>0.035***</td>
</tr>
<tr>
<td></td>
<td>(19.91)</td>
<td>(6.108)</td>
<td>(5.078)</td>
<td>(5.235)</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0008</td>
<td>0.0007</td>
<td>0.0003</td>
<td>0.0008</td>
</tr>
<tr>
<td>$R^2$</td>
<td>48.07%</td>
<td>52.20%</td>
<td>90.09%</td>
<td>40.82%</td>
</tr>
</tbody>
</table>

Figure 3: RMP and expected returns

This figure plots mean conditional expected returns versus the model-implied predicted mean returns (in percent) for the domestic residual mispricing index using the Fama MacBeth approach. The top left (right) panel depicts the domestic (international) aggregate equity, whereas the bottom left (right) panel depicts the currency (U.S. and international aggregate equity) returns. The sample period is January 1985 to June 2020.
price of risk is negative, albeit insignificant, when using unconditional expectations of returns. This result is in line with previous evidence documenting the relationship with idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang (2006b)). Moreover, as illustrated in Figure 4, the cross-sectional fit worsens.

Table 8: RMP index and average returns

This table reports the prices of risk $\lambda$, Fama and MacBeth $t$-stats in parentheses, root mean squared errors (RMSE), and the cross-sectional $R^2$ for the residual mispricing index. Each column contains the following test assets: (i) the domestic equity in each market, (ii) the international equity converted in U.S. dollars, (iii) the currency returns, i.e. borrowing at foreign risk-free rates and investing in the domestic U.S. risk-free rate, and (iv) the domestic and international equities jointly. The first and last columns have eight test assets (individual economies), whereas the second and fourth columns have seven test assets (corresponding to the bilateral pairs having U.S. as domestic economy). The sample period is January 1985 to June 2020. The standard errors in the Fama and MacBeth approach account for errors-in-variables, heteroskedasticity and autocorrelation in error terms.

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>International equity</th>
<th>Currency</th>
<th>Domestic &amp; international equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$-0.004$</td>
<td>$-0.002$</td>
<td>$-0.004$</td>
<td>$-0.002$</td>
</tr>
<tr>
<td>($-1.25$)</td>
<td>($-0.77$)</td>
<td>($-1.23$)</td>
<td>($-0.63$)</td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0014</td>
<td>0.0013</td>
<td>0.0008</td>
<td>0.0013</td>
</tr>
<tr>
<td>$R^2$</td>
<td>32.97%</td>
<td>23.42%</td>
<td>46.20%</td>
<td>16.19%</td>
</tr>
</tbody>
</table>

4.6.2 Multivariate estimation

Since international equity and currency premia may be related to other priced risk factors, we investigate this possibility by including in specification (10) the linear component of the SDF, $M^\ast$. Specifically, we consider

$$
\mathbb{E}[r_{j,t+1} | \mathcal{F}_t] = \alpha_j + \beta_{j}^o M_{dom,t}^o + \beta_{j}^\ast M_{dom,t}^\ast \varepsilon_{j,t}, 
$$

(12)

and

$$
\mathbb{E}[r_{j,t+1} | \mathcal{F}_t] = \lambda_0 + \lambda^o \hat{\beta}_{j}^o + \lambda^\ast \hat{\beta}_{j}^\ast + \eta_j.
$$

(13)

We report the estimated prices of risk in Table 9. The price of risk for RMP remains positive and statistically significant, even after accounting for the linear factors embedded in $M_{dom}^\ast$. Notice that, by construction, $M^\ast$ will price the corresponding bilateral assets from which it is derived. Interestingly, when considering the average domestic RMP in (6), and examining its performance across different test assets than the ones from which it is constructed, we find that $M_{dom}^\ast$ is a priced risk factor only in the individual equities and marginally in the cross-section of currencies. On the other hand, domestic RMP is relevant for all the test assets we consider, and in particular for currencies.
This figure plots mean realized expected returns versus the model-implied predicted mean returns (in percent) for the domestic residual mispricing index using the Fama MacBeth approach. The top left (right) panel depicts the domestic (international) aggregate equity, whereas the bottom left (right) panel depicts the currency (U.S. and international aggregate equity) returns. The sample period is January 1985 to June 2020.

Table 9: RMP, linear component of SDF and asset returns

This table reports the prices of risk $\lambda^\circ$ and $\lambda^*$ for the domestic RMP and the linear component of the domestic SDF, Fama and MacBeth t-stats in parentheses, root mean squared errors (RMSE), and the adjusted cross-sectional $R^2$. Each column contains the following test assets: (i) the domestic equity in each market, (ii) the international equity converted in U.S. dollars, (iii) the currency returns, i.e. borrowing at foreign risk-free rates and investing in the domestic U.S. risk-free rate, and (iv) the domestic and international equities jointly. The first and last columns have eight test assets (individual economies), whereas the second and fourth columns have seven test assets (corresponding to the bilateral pairs having U.S. as domestic economy). The sample period is January 1985 to June 2020. The standard errors in the Fama and MacBeth approach account for errors-in-variables, heteroskedasticity and autocorrelation in error terms. ***, ** and * indicate significant at the 1%, 5% and 10% level, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>International equity</th>
<th>Currency</th>
<th>Domestic &amp; international equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda^\circ$</td>
<td>0.174***</td>
<td>0.027**</td>
<td>0.048***</td>
<td>0.025**</td>
</tr>
<tr>
<td></td>
<td>(21.52)</td>
<td>(2.219)</td>
<td>(5.293)</td>
<td>(2.029)</td>
</tr>
<tr>
<td>$\lambda^*$</td>
<td>-0.499***</td>
<td>0.317</td>
<td>-0.241*</td>
<td>0.223</td>
</tr>
<tr>
<td></td>
<td>(-2.903)</td>
<td>(1.441)</td>
<td>(-1.803)</td>
<td>(1.031)</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0007</td>
<td>0.0007</td>
<td>0.0003</td>
<td>0.0008</td>
</tr>
<tr>
<td>Adj.$R^2$</td>
<td>43.25%</td>
<td>45.42%</td>
<td>89.25%</td>
<td>32.24%</td>
</tr>
</tbody>
</table>
This figure plots mean conditional expected returns versus the model-implied predicted mean returns (in percent) for the domestic residual mispricing index and the linear component of the domestic SDF using the Fama MacBeth approach. The top left (right) panel depicts the domestic (international) aggregate equity, whereas the bottom left (right) panel depicts the currency (U.S. and international aggregate equity) returns. The sample period is January 1985 to June 2020.
Overall, our findings suggest that RMP is a priced risk factor, relevant for understanding the cross-section of assets’ conditional mean expected returns. Moreover, RMP is theoretically motivated as an index insuring absence of arbitrage opportunities and is not redundant, as it is not subsumed by linear factors.

4.7 RMP and Exchange Rates

With RMP indices for multiple currencies, it is natural to investigate their exchange rate implications. Canonical models in international finance usually assume that markets are complete and integrated (see, e.g., Colacito and Croce (2011), Colacito and Croce (2013), among others). Whenever these assumptions hold, the change in the exchange rate is equal to the ratio of foreign to domestic SDFs, \( X = \frac{M_f}{M_d} \), i.e. the so-called asset market view holds. In more general settings, in which the completeness assumption is relaxed, deviations from the asset market view can be expressed through a stochastic exchange rate wedge (see, e.g. Backus, Foresi, and Telmer (2001))

\[
X = \frac{M_f}{M_d} \exp (\eta),
\]

where the stochastic wedge \( \eta \) measures the degree of international market incompleteness. It is important to mention that in incomplete markets, each particular choice of SDFs will entail a different wedge.

The RMP indices that we derive are consistent with arbitrage-free local markets, in the sense that each economy has its own SDF, and measure the correction to linear asset pricing models. We study next their role in international settings. To this end, we rewrite Equation (14) as

\[
X = \frac{M_f^{\star}}{M_d^{\star}} \left( \frac{1 + M_f^{\circ}/M_f^{\star}}{1 + M_d^{\circ}/M_d^{\star}} \right) \exp (\eta) = \frac{M_f^{\star}}{M_d^{\star}} \exp (\eta^{\star}),
\]

with \( \eta^{\star} \) the stochastic wedge implied by the ratio of the linear SDFs \( M_f^{\star} \) and \( M_d^{\star} \). Sandulescu, Trojani, and Vedolin (2020) provide an economic interpretation of the stochastic wedge as a measure of unspanned risk, capturing potential nonlinearities in each country. We further decompose this wedge to reflect the no-arbitrage correction, or the projection error on the space of linear returns with the minimal variance, i.e. the term in the middle in the above equation, capturing the relative residual mispricing. Notice that whenever markets are complete, \( \eta \) is naturally equal to zero. Interestingly,

\[\text{We consider in this case a multiplicative wedge, similar to Lustig and Verdelhan (2019). An additive wedge, i.e. } X = \frac{M_f}{M_d} + \lambda, \text{ following Bakshi, Cerrato, and Crosby (2018), delivers qualitatively the same implications. We obtain a correlation of 0.98 between the multiplicative and additive wedge.}\]
if the relative importance of RMP is similar in the two markets, such that the ratio is trivially one, we retrieve the standard stochastic wedge relative to the linear minimum variance SDFs. Moreover, by construction, since RMP is orthogonal to $M^*$ in each market, it will have no effect on the prices of assets. However, it is an empirical question whether the ratio will impact exchange rates in international markets settings. Figure 6 shows that the ratio of relative RMP is close to one, even during spikes in financial distress episodes. It follows that the correction, or relative mispricing in each market is roughly proportional, such that the ratio of RMPs does not contribute to the change in exchange rates, similarly to the wedge $\eta$. This is further depicted in Figure 7, where we report the $R^2$ from regressing the changes in exchange rates on the ratio of SDFs, as well as the $R^2$ from regressing the changes in exchange rates on the ratio of RMPs. Consequently, the latter provides no explanatory power for exchange rates. Hence, when international financial markets are integrated, the risk factors driving premia are similar, such that the residual mispricing is proportional in domestic and foreign markets.

Still, Figure 6 shows relevant spikes around periods of financial distress and recessions, such as during the 1987 crash, the Asian crisis and LTCM default, the financial crisis of 2008 and more importantly, during the recent COVID-19 crisis. Overall, most of the spikes encountered are below one, suggesting that during financial turmoil, on average, the degree of residual mispricing is more relevant in the domestic U.S. market, rather than in the foreign economies studied. Overall, we find a positive cross-market correlation between the relative RMPs across the economies studied.\(^{22}\)

Lastly, Panel B of Figure 6 plots the time series of the stochastic wedge $\eta^*$ defined in Equation (15). Accordingly, the international unspanned risk fluctuates around zero, exhibiting both positive and negative spikes, especially during financial distress periods.

Next, we explore whether the stochastic wedge is related to asset risk premia. We have established in Section 4.6 that RMP is a priced risk factor in the cross section of (international) equities and currencies. We report in Table 10 the results obtained following the same Fama MacBeth approach as in Section 4.6, where we consider the stochastic wedge ($\eta$), and the stochastic wedge ($\eta^*$) in Equation 15, respectively, as the factors. We plot the estimated prices of risk in Table 10. There are two noteworthy observations. First, even if the stochastic wedge appears to be priced in the cross-section of international equities and currencies, the reported $R^2$ are significantly lower than the ones in Table 7 having RMP as a risk factor. Second, it is crucial what SDFs are used to determine the stochastic wedge, as they will give rise to different implications in terms of the associated prices of risk. Using the SDFs that ensure

\(^{22}\)We report the cross-market correlations between the relative RMPs in Table C2.
Figure 6: RMP and Exchange Rates

Panel A: Relative residual mispricing in international markets

Panel B: Stochastic wedge in international markets

This figure plots the time series of the relative RMP term (Panel A) and the time series of the stochastic wedge $\eta^*$, relative to the linear HJ SDFs (Panel B). Gray bars denote NBER recessions. The sample period is January 1985 to June 2020.
absence of arbitrage opportunities (Panel A), the corresponding wedge is not priced for domestic equities, but it commands a positive risk premia for international equities and currencies. By using instead the ratio of the minimum variance SDFs implied by linear models, that do not rule out arbitrage (Panel B), the ensuing wedge gives rise to distinct implications: it commands a negative price of risk for domestic equities and currencies, but a positive one for international equities. Moreover, the $R^2$s are lower than the ones in Panel A of Table 10. Overall, we conclude that using SDFs accounting for residual mispricing yields more robust and stable results than the SDFs derived from linear asset pricing models alone.

5 Robustness

5.1 Hedging in nonlinear instruments and RMP

In this section we investigate how RMP behaves if nonlinear trading strategies become available to investors. Specifically, we are interested in the question how the behaviour of $M_{t+1}^{\diamond}$ is altered when adding nonlinear securities to the linear SDF. In the context of our setting, we approach this problem by adding $r_{t+1}^2$, the excess return squared, as an additional term to $M_{t+1}^\star$. This instrument corresponds to the payoff of a Martin (2017) simple variance swap. The price of this payoff is an option portfolio holding out-of-the-money calls and out-of-the-money puts scaled by the squared forward price. A lower bound for the price of this instrument is the price of an at-the-money straddle scaled by the squared forward price of the underlying. Denote this lower bound at
Table 10: Stochastic wedge and asset returns

This table reports the prices of risk, Fama and MacBeth t-stats in parentheses, root mean squared errors (RMSE), and the cross-sectional $R^2$ for the stochastic wedge. Panel A (B) reports results for $\eta$ ($\eta^*$). Each column contains the following test assets: (i) the domestic equity in each market, (ii) the international equity converted in U.S. dollars, (iii) the currency returns, i.e. borrowing at foreign risk-free rates and investing in the domestic U.S. risk-free rate, and (iv) the domestic and international equities jointly. The first and last columns have eight test assets (individual economies), whereas the middle columns have seven test assets (corresponding to the bilateral pairs having U.S. as domestic economy). The sample period is January 1985 to June 2020. The standard errors in the Fama and MacBeth approach account for errors-in-variables, heteroskedasticity and autocorrelation in error terms. *** indicates significant at the 1% level.

<table>
<thead>
<tr>
<th>Panel A: Stochastic wedge ($\eta$)</th>
<th>Equity</th>
<th>International equity</th>
<th>Currency</th>
<th>Domestic &amp; international equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>-0.045</td>
<td>0.242***</td>
<td>0.229***</td>
<td>0.214***</td>
</tr>
<tr>
<td></td>
<td>(-0.700)</td>
<td>(5.497)</td>
<td>(3.635)</td>
<td>(4.879)</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0011</td>
<td>0.0008</td>
<td>0.0009</td>
<td>0.0008</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.15%</td>
<td>39.16%</td>
<td>11.22%</td>
<td>32.22%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Stochastic wedge implied by linear models ($\eta^*$)</th>
<th>Equity</th>
<th>International equity</th>
<th>Currency</th>
<th>Domestic &amp; international equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>-0.338***</td>
<td>0.307***</td>
<td>-0.167***</td>
<td>0.281***</td>
</tr>
<tr>
<td></td>
<td>(-4.210)</td>
<td>(5.844)</td>
<td>(-3.083)</td>
<td>(5.271)</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0010</td>
<td>0.0009</td>
<td>0.0010</td>
<td>0.0008</td>
</tr>
<tr>
<td>$R^2$</td>
<td>4.86%</td>
<td>33.80%</td>
<td>4.79%</td>
<td>29.02%</td>
</tr>
</tbody>
</table>
time $t$ by $C_t$. We add to the original problem (3) the constraint that

$$\mathbb{E}^P [M_{t+1} r_{t+1}^2 | \mathcal{F}_t] \geq C_t.$$  \hfill (16)

This lower bound induces $M_{t+1}^* r_{t+1}^2$, meaning that the linear benchmark model $M_{t+1}^*$ accommodates an additional quadratic hedging term. It is expected that the presence of this quadratic hedging term renders the market more complete, and thus reduces the scale of $M_{t+1}^o$.

Empirically, we need to work with observed option prices to implement constraint (16). Since options data for our international sample outlined in Section 4 are not readily available, we restrict ourselves to the S&P 500 index return, along with S&P 500 options data from OptionMetrics for this exercise. Hence, we derive RMP from Equation (3) only for the U.S. market with the additional constraint (16), and plot in Figure 8 the corresponding time series, along with the original RMP extracted from the linear model. The two series are termed RMP, and variance hedged RMP, accordingly.

**Figure 8: Nonlinear hedging**

This figure plots the time series of the U.S. RMP, as well as the variance hedged U.S. RMP (dashed line). Gray bars denote NBER recessions. The data are S&P 500 returns, as well as at-the-forward put and call options. The sample period is January 1996 to December 2019.

The scale of variance hedged RMP is orders of magnitude smaller than the one of the original RMP, suggesting that nonlinear hedging in the options market will indeed reduce residual mispricing. As another striking observation, the variance hedged RMP does not show any extraordinary movement during recessions. This fact may suggest that trading variance contracts is particularly valuable during distress periods. Finally, the few times the variance hedged RMP does spike, it goes in the opposite direction of the original RMP.

To have a better understanding of the links between nonlinear instruments and
mispricing, we examine next the option trading volume, including both puts and calls. We find no significant relation between option trading volume and the original (international) RMP.\textsuperscript{23} However, there is a significantly positive relation between the variance of RMP and option trading volume, implying heightened option trading during periods of high expected uncertainty. It follows that option pricing moves in response to hedging demand, rather than the increasing volume experienced over the past decade. Notice that the second moment of RMP has an economic interpretation as the price of a nonlinear payoff necessary to render the markets arbitrage-free. Indeed, its price shares similar time-series properties as the VIX, which can be regarded as the price of variance, such as both spiking during periods of financial distress. Overall, we find that augmenting the set of assets available to investors to account for nonlinear payoffs decreases the magnitude of the underlying residual mispricing.

5.2 Mispricing factors

In this section, we explore the link between our residual mispricing index and existing factors capturing mispricing in the U.S. equity market. Specifically, Stambaugh and Yuan (2017) construct two mispricing factors by aggregating the information contained in 11 prominent anomalies, i.e. long-short decile portfolios of U.S. equities, sorted on different firm characteristics, related to management and performance. We report in Table 11 the correlation between our RMP indices and the mispricing factors. We find no significant relation, suggesting that we uncover a new priced nonlinear factor, whereas Panel B shows that the mispricing factors are positively correlated with our linear SDFs, both in domestic and international markets.

Table 11: Correlation between RMP and mispricing factors

This table reports the correlation between RMP (linear component of SDF) and the mispricing factors of Stambaugh and Yuan (2017) in Panel A (Panel B). MGMT and PERF are mispricing factors aggregating information from 11 anomalies, related to firm management and performance. The domestic market is the U.S., whereas international is the average across international markets excluding the U.S. Data are monthly and span January 1985 to December 2016. *** indicates significant at the 1% level.

\begin{tabular}{lcccc}
<table>
<thead>
<tr>
<th>Panel A: Residual mispricing</th>
<th>U.S.</th>
<th>International</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGMT</td>
<td>0.051</td>
<td>0.062</td>
</tr>
<tr>
<td>PERF</td>
<td>0.005</td>
<td>0.006</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Linear component of SDF</th>
<th>U.S.</th>
<th>International</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGMT</td>
<td>0.392***</td>
<td>0.386***</td>
</tr>
<tr>
<td>PERF</td>
<td>0.338***</td>
<td>0.337***</td>
</tr>
</tbody>
</table>

\textsuperscript{23}In interest of space, we do not report results in the main body of the paper.
5.3 RMP in Benchmark Models

RMP is computed nonparametrically from moments of input distributions having finite second-order moments. It can be interpreted as the minimal-cost of a nonlinear payoff of a representative agent optimizing her Sharpe ratio portfolio, such that markets are arbitrage free. Since the input is a moment matrix (A.3), it can originate also from parametric models. In the following, we illustrate the behavior of RMP using the well-known benchmark model from Colacito and Croce (2013) using the Gaussian approximation developed in their Appendix C.

The ingredients are dynamics for the log SDF

\[ m^i_{t+1} = \log \delta - \frac{1}{\psi} \Delta c^i_{t+1} + \left( \frac{1}{\psi} - \gamma \right) \log \bar{U}^i_{t+1} \]

\[ - \frac{1}{\psi} - \gamma \log \mathbb{E} \left[ \exp \left( (1 - \gamma) \log \bar{U}^i_{t+1} \right) | \mathcal{F}_t \right], \quad i \in \{d, f\}, \]

where \( \Delta c^i_{t+1}, \log \bar{U}^i_{t+1} \), denoting the log consumption growth and the continuation value of utility, respectively, are assumed to be jointly normal with mean \( \mu = (0, 0, 0, 0)^\top \), and covariance matrix \( \Sigma \).\(^{24}\) For utmost parsimony, and without applying log-linearization, we specify the equity returns as \( \log R^i_{1,t+1} = -m^i_{t+1}, \ i \in \{d, f\} \), so that the Euler conditions are satisfied. Furthermore, following Colacito and Croce (2013), we assume complete markets, so that log exchange rate growth is given by the log SDF difference \( \log X_{t+1} = m^f_{t+1} - m^d_{t+1} \). The corresponding bond prices are accordingly \( R^i_{0,t+1} = \mathbb{E} \left[ \exp m^i_{t+1} | \mathcal{F}_t \right] \). Consistent with our empirical section, we define as state variables the domestic return \( (R^f_{1,t+1} - R^d_{0,t+1}) \), the excess return associated with the foreign equity return in domestic currency \( R^f_{1,t+1} X_{t+1} - R^d_{0,t+1} \), and the carry \( R^f_{0,t+1} X_{t+1} - R^d_{0,t+1} \). We compute the moments of these state variables using the moment-generating function of consumption growth and long-run factors and follow the procedure outlined in Appendix A to derive the coefficients of \( M^*_{t,t+1} \) and \( M^\circ_{t,t+1} \), or the RMP.

We restrict our attention to the U.S. - UK pair, with U.S. being the domestic, and UK the foreign market, similarly to the calibration used in Colacito and Croce (2013). Figure 9 plots the RMP extracted from the benchmark long run risk model and our nonparametric RMP obtained with unconditional estimates, for different levels of the realized carry trade. There are several noteworthy observations. First, RMP in both settings features a U-shape pattern and is high and positive during distress periods, i.e. whenever both domestic and foreign returns are negative. Second, our nonparametric specification for the RMP appears to be more stable when varying the level of the carry

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\(^{24}\) Croce (2021) shows that log \( \bar{U}^i_{t+1} \) are not exactly long-run shocks, and that the error is reflected in a constant. For simplicity, we ignore this effect here.
trade, whereas RMP in long-run risk models display a higher variability. Overall, the unconditional nonparametric RMP is similar to the one extracted from benchmark models incorporating long run risk components.

In the following, we highlight the importance of conditioning information and time-varying estimates, by plotting in Figure 10 our conditional RMP. For illustrative purposes, we focus on two polar cases: one in which markets are relatively calm (July 2005) and another one during the collapse of the Lehman Brothers during the financial crisis (September 2008). While during calm periods, conditional RMP features similar properties with the unconditional RMP, regardless of the level of the carry trade considered (left column in Figure 10), a different picture emerges during distress periods: conditional RMP features an inverted U-shape, such that it is higher on average, positive, and flatter, suggesting that insuring against mispricing becomes more expensive.
The range of in the left column, and unconditionally from our data in the right column, for different scenarios of the carry trade.

Figure 9: RMP in long-run risk models vs. unconditionally

(a) $R^f_0X - R^d_0 = -0.1$

(b) $R^f_0X - R^d_0 = -0.1$

(c) $R^f_0X - R^d_0 = 0$

(d) $R^f_0X - R^d_0 = 0$

(e) $R^f_0X - R^d_0 = 0.1$

(f) $R^f_0X - R^d_0 = 0.1$

This figure shows RMP as a function of $r^i_1 = \log R^i_{1,1.1}, i \in \{d, f\}$ computed from the Colacito and Croce (2013) model in the left column, and unconditionally from our data in the right column, for different scenarios of the carry trade. The range of $r^i_1$ and $r^d_1$ is chosen to be two standard deviations around the mean.
Figure 10: Conditional RMP

(a) $R_{0i}^d X - R_{0i}^d = -0.1$

(b) $R_{0i}^f X - R_{0i}^f = -0.1$

(c) $R_{0i}^l X - R_{0i}^l = 0$

(d) $R_{0i}^l X - R_{0i}^l = 0$

(e) $R_{0i}^l X - R_{0i}^d = 0.1$

(f) $R_{0i}^f X - R_{0i}^d = 0.1$

This figure shows RMP as a function of $r_i^d = \log R_{1,i+1}^{d}$, $i \in \{ d, f \}$ computed from our conditional estimations for two dates: July 2005 (left column) chosen to represent a calm period, and September 2008 (right column), reflecting the collapse of Lehman Brothers during the financial crisis. The range of $r_i^d$ and $r_i^f$ is chosen to be two standard deviations around the mean.
6 Conclusions

We develop a measure of mispricing from basic asset returns. It is constructed as the nonlinear stochastic discount factor component with the smallest variance, that renders a linear asset pricing model arbitrage-free in all states of the world. Our measure is easy to compute with parsimonious data requirements, and fully conditional. In particular, we do not need option prices to identify the nonlinear component. We term our measure Residual MisPricing (RMP). We show that conditional linear asset pricing models perform well on average, and during normal times, while they imply a larger (residual) mispricing during crisis periods.

We exploit the weak data requirements of RMP and produce time series for the United States, United Kingdom, Switzerland, Australia, Canada, Japan, New Zealand, and the Euro area. Empirically, RMP is related to the Jurado et al. (2015) financial uncertainty index, to the VIX, as well as to measures of economic policy uncertainty. Moreover, we show that RMP drives risk premia, even after including linear factors, providing further evidence for unspanned components that cannot be found in prices of basic assets, such as international stocks and bonds. Lastly, our RMP is related to nonlinear proxies of intermediary leverage constraints and exhibits a close link with market-wide funding liquidity shocks, as measured by the TED spread, similar to a financial distress measure. Consequently, we find that RMP predicts future market dislocations, including covered interest rate parity (CIP) deviations. Overall, we document that RMP is high during periods of uncertainty and during episodes of low (funding) liquidity, exhibiting a substantial effect not only across different asset classes, but also on the macro-economy. Our findings thus contribute to the study of the relation between uncertainty and mispricing, while providing a unifying approach between financial markets and the macro-economy.

Our framework is modular and expandable, for example with modifications to introduce trading frictions such as short-selling constraints. We leave this, an extensive study involving larger cross sections, and related topics for future research.
References


A Optimization program

From (4), each pricing kernel \( M_{i,t+1} = w^\top B_{i,n,t+1} \). A sufficient condition for the constraint \( c^\top B_{i,n} \overset{p.a.s.}{\geq} 0 \) is the so-called sum-of-squares (SOS) condition

\[
w^\top B_{i,n,t+1} = B_{i,n/2,t+1}^\top A B_{i,n/2,t+1}
\]

for some symmetric and positive semidefinite matrix \( A \). It is easy to see that the coefficients \( c \) are related to \( A \) through a linear map \( L \). Equipped with this condition, Program (3) thus becomes

\[
\begin{align*}
\text{minimize} & \quad w^\top H_{i,n,t} w \\
\text{subject to} & \quad G_{i,n,t} w = 0_{m_i}, \quad w^\top \mu_{i,n,t} = \frac{1}{R_{0,t+1}} \\
& \quad w = L(A), \quad A \succeq 0, \quad \text{with}
\end{align*}
\]

\[
H_{i,n,t} := \mathbb{E}^p \left[ B_{i,n,t+1}^\top B_{i,n,t+1} \mid \mathcal{F}_t \right].
\]

The vector \( \mu_{i,n,t} \) denotes the first row of, and \( G_{i,n,t} \) denotes rows 2 through \( m_i + 1 \) of \( H_{i,n,t} \). Since \( R_{0,t+1} \) is \( \mathcal{F}_t \)-measurable, so are the coefficients \( w \). This program is a conic (convex) optimization problem for which efficient solution algorithms exist that yield an optimal and unique solution (provided the constraints are feasible). For our empirics we use the software package MOSEK. Solutions obtain in fractions of a second. In practice we substitute the strict constraint in Program (3) with the non-strict one in (A.1) allowing for weak arbitrages. If needed, condition (A.1) could be made strict by a translation with some \( \epsilon > 0 \) through a change of variables.

B RKHS conditional moments

From the previous section, to solve program (A.2), we need to estimate conditional moments. In particular, we need to estimate matrix \( H_{i,n,t} \) that is positive semidefinite (PSD) for population moments but not necessarily so for estimated moments. To ensure the PSD property of \( H_{i,n,t} \) that is also necessary to maintain convexity of program (A.2), we resort to RKHS embedding of conditional distributions with subsequent nearest PSD regularization.

RKHS embedding of conditional distributions is a technique from machine learning to estimate the conditional expectation function (Song et al., 2009). A complete self-contained introduction of this topic is beyond the scope of this paper, so we introduce very briefly the bare minimum of the necessary concepts. A RKHS \( H \) on \( \mathcal{X} \) with kernel
\( k_H \) is a Hilbert space of functions \( f : \mathcal{X} \to \mathbb{R} \). Its defining properties are written in terms of its inner product
\[
\langle f, k(x, \cdot) \rangle_H = f(x), \\
\langle k(x, \cdot), k(x', \cdot) \rangle_H = k(x, x').
\]
The kernel hence is a linear evaluation functional that through the inner product maps function objects to their evaluation at any \( x \in \mathcal{X} \). The marvellous power of RKHS arises through the fact that the kernel function \( k(x, \cdot) \) itself may be infinite-dimensional, while when used inside the inner product it maps elements of \( H \) to their evaluation at a point. This mapping from a potentially infinite-dimensional feature space to simple evaluations is called the kernel trick. Learning about objects \( H \) that best represent given observed data, say \( x_1, \ldots, x_N \), usually leads to a linear algebra operation involving the kernel matrix
\[
K_H := \begin{pmatrix}
k(x_1, x_1) & \cdots & k(x_1, x_N) \\
\vdots & \ddots & \vdots \\
k(x_N, x_1) & \cdots & k(x_N, x_N)
\end{pmatrix},
\]
through a so-called representer theorem.

### B.1 RKHS embedding of conditional distributions

Turning to the problem of the present paper, RKHS embedding of distributions is performed in the tensor product space \( F \otimes G \) of two RKHS \( F \) and \( G \), where it has to be assumed that for all \( g \in G \) the conditional expectation \( \mathbb{E} [g(R) \mid z] \in F \). With this in place we may define \( \mu_{R \mid z} := \mathbb{E} [k_G(R, \cdot) \mid z] \) so that
\[
\mathbb{E} [g(R) \mid z] = \langle g, \mu_{R \mid z} \rangle_G. \tag{B.4}
\]
From Song et al. (2009) and Park and Muandet (2020), given conditioning data \( z_1, \ldots, z_N \) and return realizations \( R_{i,1}, \ldots, R_{i,N} \), the desired object \( \mu_{R \mid z} \) can be estimated with an additional regularization parameter \( \lambda \) via
\[
\widehat{\mu}_{R \mid z} = \left( k_G(R_{i,1}, \cdot) \quad \cdots \quad k_G(R_{i,N}, \cdot) \right) (K_F + N\lambda I_N)^{-1} \begin{pmatrix} k_F(z_1, \cdot) \\ \vdots \\ k_F(z_N, \cdot) \end{pmatrix}, \tag{B.5}
\]
where \( I_N \) is the identity matrix of dimension \( N \). With this set-up in place, to estimate the first conditional moment of \( R \) given some value \( z \), it suffices to perform a linear-
For our empirical study we use Gaussian kernels

\[ k_F(z, z') := e^{-\|z - z'\|^2/\sigma}. \]  

(B.7)

The literature on RKHS has found the particular choice of kernel not important for the results. We perform the above operation (B.6) for all the moments making up the matrix \( H_{i,n} \) in the optimization program A. Subsequently, to obtain a PSD matrix, we find the closest (in Frobenius norm) positive definite matrix and use this matrix in our optimization program (A.2).

### B.2 In-sample and out-of-sample cross validation

We describe here out-of-sample cross validation of the parameters \( \sigma > 0 \) and \( \lambda > 0 \), with the former parameterizing the Gaussian kernel (B.7), and the latter controlling the regularization in (B.5). As with traditional local linear and local constant nonparametric regression, also RKHS exhibit a bias-smoothness tradeoff. This is clear from Eq. (B.5) that would yield the observations as conditional expectations with \( \lambda = 0 \).

To perform cross validation in our context, suppose that we would like to estimate conditional expectations using information up to and not including some index \( N \leq N \).

From an initial sample ranging from one to \( N := [N/3] \) we compute the moment predictions

\[
(\mathbb{E}[R_i^\alpha | z_N], \ldots, \mathbb{E}[R_i^\alpha | z_{\min(N+m,N)}]) = \begin{pmatrix} k_F(z_1, z_N) & \cdots & k_F(z_1, z_{\min(N+m,N)}) \\ \vdots & \ddots & \vdots \\ k_F(z_N, z_N) & \cdots & k_F(z_N, z_{\min(N+m,N)}) \end{pmatrix}^{-1} \begin{pmatrix} k_F(z_1, z_N) \\ \vdots \\ k_F(z_N, z_N) \end{pmatrix}.
\]

(B.8)

for all \( \alpha \) necessary to fill the matrix \( B_{i,n}^\top B_{i,n} \) giving \( H_{i,n,N}, \ldots, H_{i,n,\min(N+m,N)} \) for \( M \) around 10 (for our application we choose \( M = 6 \)). The sum of the squared prediction
errors can be written concisely in Frobenius norm as

\[ C_{N,M} := \left\| B_{i,n,N+1}^T B_{i,n,N+1} - H_{i,n,N} \right\|^2 + \cdots + \left\| B_{i,n,\min(N+M+1,N)}^T B_{i,n,\min(N+M+1,N)} - H_{i,n,\min(N+K,N)} \right\|^2. \]

Denoting by \( \# := \lfloor (N - N) / M \rfloor \) the number of windows used in the cross validation, we

\[
\min_{\sigma, \lambda} C_{N,M} + \cdots + C_{N+\#,M} \tag{B.9}
\]

to find the optimal kernel and regularization parameters \( \sigma \) and \( \lambda \). Since this problem is non-convex, we obtain a solution from solving a large number (20) of optimization problems started from a random initial parameter vector. The in-sample results are obtained from using this procedure with the entire data set. Our out-of-sample estimates are obtained by performing this procedure for each of the data points at indices \( N/3, N/3 + 1, \ldots, N \).

C Appendix Figures and Tables

Figure C1: Correlations Between Conditional and Unconditional SDFs

This figure shows the scatter plot between conditional and unconditional domestic SDFs, for the corresponding bilateral pair. The sample period is January 1985 to June 2020.
Figure C2: Time series of the HJ bound

This figure plots the time series of the HJ bound of the domestic SDF corresponding to each bilateral pair studied. Gray bars denote NBER recessions. The sample period is January 1985 to June 2020.

Table C1: Cross-market correlations of the degree of residual mispricing

This table reports the correlations between the residual mispricing indices across different economies. The international RMP is computed as the average across all foreign countries (excluding the U.S.). Data are monthly and span January 1985 to June 2020. *** indicates significant at the 1% level.

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>UK</th>
<th>CH</th>
<th>JP</th>
<th>EU</th>
<th>AU</th>
<th>CA</th>
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<td>0.727***</td>
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<td>CA</td>
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<td>0.691***</td>
<td>0.607***</td>
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<td>0.869***</td>
<td>0.732***</td>
<td>0.811***</td>
<td>0.724***</td>
<td>0.823***</td>
<td>0.635***</td>
<td></td>
</tr>
<tr>
<td>International</td>
<td>0.920***</td>
<td>0.952***</td>
<td>0.900***</td>
<td>0.854***</td>
<td>0.893***</td>
<td>0.918***</td>
<td>0.778***</td>
<td>0.891***</td>
</tr>
</tbody>
</table>

Table C2: Cross-market correlations of the arbitrage-free correction

This table reports the cross-market correlations between relative RMP (15). Data are monthly and span January 1985 to June 2020.

<table>
<thead>
<tr>
<th></th>
<th>UK</th>
<th>CH</th>
<th>JP</th>
<th>EU</th>
<th>AU</th>
<th>CA</th>
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<tr>
<td>CH</td>
<td>0.288</td>
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<tr>
<td>JP</td>
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<td>0.563</td>
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<tr>
<td>CA</td>
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<td>0.043</td>
<td>-0.054</td>
<td>0.023</td>
<td>0.044</td>
<td></td>
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<tr>
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<td>0.004</td>
<td>0.031</td>
<td>0.042</td>
<td>0.009</td>
</tr>
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</table>
D Conditional risk premia and time-varying weights

We start by investigating the relation between conditional risk premia on equities and currencies, as well as the time-series properties of the weights, or coefficients, associated with the linear model $M^\star$.

Figure D3: Time-varying Weights

Panel A: Domestic (U.S.) investor’s asset weights

Panel B: Foreign (Swiss) investor’s asset weights

This figure plots the time-varying asset weights (the coefficients of $M^\star$) of a domestic (foreign) U.S. (Swiss) investor in Panel A (Panel B). Gray bars denote NBER recessions. The sample period is January 1985 to June 2020.
Figure D3 plots the time-varying weights in the domestic and foreign equities, as well as in the respective currency (left y-axis), and in the risk-free rate (right y-axis).\textsuperscript{25} Specifically, these are the optimal weights of a Sharpe ratio portfolio corresponding to the linear minimum variance SDF $M^\star$. There are several noteworthy observations. First, weights in risky assets exhibit a significant variation across time, suggesting that analyzing only unconditional models might be an oversimplification. Second, the currency weights display large swings, that mirror the behavior of the short-term risk-free bond exposure, and especially during crisis periods, there is a flight to quality, as there is a drop in the currency weight followed by a concomitant rise in the domestic bond. Still, the risk-free bond exposure is much less volatile, with an average coefficient of 1.015. Third, the exposure to domestic equity (Panel A) is similar to the one entered in foreign equity by a foreign investor (Panel B).\textsuperscript{26} The same relation holds for the foreign (domestic) equity of a domestic (foreign) investor. The exposure to the currency has a natural inverse relation from the perspective of a domestic investor, as opposed to the foreign investor.

To provide further support for the importance of time-varying weights, and subsequently, time-varying risk premia, we plot in Figure D4 conditional versus unconditional, or constant, weights associated with the currency return. Importantly, unconditional weights are virtually the same for domestic and foreign investors, of approximately 1.5 during the sample period analyzed, whereas there are clear differences in the conditional weights. For instance, the sudden adjustment in the weight following the unpeg of the Swiss franc from the Euro at the beginning of 2015 is apparent, but completely ignored by an unconditional approach.

Finally, we examine the conditional risk premia of domestic and foreign investors. We document a strikingly close link between the risk premia of international equities and the currency risk premium. This finding is in line, and at the same time strengthens the evidence in Lustig and Verdelhan (2007), Lustig, Roussanov, and Verdelhan (2014) and Lettau, Maggiori, and Weber (2014), among others. Moreover, consistently with Ang, Chen, and Xing (2006a), the correlation between aggregate equity markets and currency markets is the largest during market downturns. It follows that one can estimate the conditional currency risk premium as the difference between foreign and domestic equity conditional risk premia. Put differently, we find the relation

$$E_t[r_{1}^{f,d}] - E_t[r_{1}^{d}] \approx E_t[r_{0}^{f,d}]$$

holds. Indeed, Figure D5 shows that the difference between the conditional expectation

\textsuperscript{25}For illustrative purposes and in interest of space, we report the weights for the U.S.-Swiss bilateral pair. The time-series behavior of the coefficients is similar for the other currency pairs studied.

\textsuperscript{26}Note that from the perspective of a foreign investor, foreign equity is the U.S. equity.
Figure D4: Time-varying currency weights

This figure plots the time-varying currency weights (coefficients of $M^*$) versus the unconditional weight of a domestic (foreign) U.S. (Swiss) investor in the top panel (bottom panel). The red dot during 2015 corresponds to the unpeg of the Swiss franc from the Euro. Gray bars denote NBER recessions. The sample period is January 1985 to June 2020.

of the foreign aggregate equity translated in domestic terms and the conditional expectation of the domestic aggregate equity provides a close approximation of the conditional expectation of the currency.

Having documented the time-series properties of conditional risk-premia, we explore next whether residual mispricing is related to equity and currency risk premia.
Figure D5: Conditional risk premia

Panel A: Domestic risk premia

Panel B: Foreign risk premia

This figure plots the conditional risk premia of a domestic (foreign) U.S. (Swiss) investor in Panel A (Panel B). The dashed dotted lines represent the difference between foreign and domestic equity risk premia. Gray bars denote NBER recessions. The sample period is January 1985 to June 2020.