Information Disclosure Frequency: Implications for Welfare and Cost of Capital

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Abstract

We study how the frequency of information disclosure affects the cost of capital and traders' welfare under different market conditions. In competitive markets, gradual disclosure reduces uncertainty over time, enhances risk-sharing, and improves welfare for all traders. In non-competitive markets, where informed traders possess market power, increasing disclosure frequency reduces the initial cost of capital, as informed traders spread their trading over time. While more frequent disclosure continues to benefit uninformed traders, it may negatively impact informed traders, who trade based on both private information and liquidity shocks, as higher disclosure frequency increases the cumulative costs of hedging. Our findings suggest that firms may opt for less frequent disclosure when their decisions are influenced by institutional traders with monopoly power.

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Keywords: Frequency of information disclosures, Risk sharing, Welfare, Imperfect competition, Market Liquidity, Information asymmetry

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1 Introduction

Financial markets rely on information disclosure to reduce information asymmetry, facilitate trade, and promote risk sharing. Since the 1930s, US regulators have expanded mandatory disclosure rules to increase both the quantity and frequency of firms' information disclosure. At the same time, there has been a significant increase in firms' voluntary disclosure. There exists extensive research on the effects of information quantity conducted by both regulators and academics.[1] However, research on the effects of the frequency of information disclosure by firms—a crucial aspect influencing market dynamics and participants' welfare—is very limited.[2] In this paper, we examine how the frequency of information disclosure affects cost of capital and the welfare of market participants. Specifically, we aim to address the main question: Are financial market participants better off with more frequent information disclosure? For example, are investors better off if firms release information on sales and other key metrics on a quarterly, monthly, weekly, or even more frequent basis?

We find that the implications of disclosure frequency critically depend on the composition of investors, traders' trading incentives (whether driven by information or hedging), and the degree of market competitiveness. In competitive markets, future public disclosure does not affect the initial cost of capital. Both informed and uninformed traders benefit from a progressive disclosure of information. This approach reduces uncertainty over time, enhances risk-sharing among traders, and thus increases market participants' welfare. In contrast, in non-competitive markets, where informed traders generate price impact through their trading, increasing disclosure frequency reduces the initial cost of capital, as informed traders spread their trading over time. While more frequent disclosure continues to benefit uninformed traders, it may negatively impact informed traders, who trade based on both private information and liquidity shocks, as higher disclosure frequency increases the cumulative costs of hedging.

[1] See, for example, White (2013); Higgins (2014), Dugast and Foucault (2018); Chapman, Reiter, White, and Williams (2019); Goldstein and Yang (2019), among others.

[2] Existing research on disclosure frequency primarily focuses on the empirical effects on the cost of capital and price informativeness. See, for example, Gigler and Hemmer (1998); Jo and Kim (2007); McMullin, Miller, and Twedt (2019).

Our paper suggests that firms with a competitive and diverse institutional investor base might benefit from a gradual approach to information disclosure, progressively releasing information as it becomes available. In contrast, firms with highly concentrated institutional ownership—characterized by a few investors holding a large share of the equity—might opt for less frequent disclosures due to the influence of institutional traders with monopoly power.

To elucidate the underlying mechanisms of the main results, it is essential to consider how risk sharing among traders is measured. Risk sharing is influenced by two primary factors: the price factor, or the risk premium—which is the expected excess return an investor demands for holding a risky asset—and the size factor, represented by the trading volume. A higher trading volume indicates increased trading activity between informed and uninformed traders, signaling a wider scope of risk sharing.

In a competitive market, the release of public information typically leads to an increase in trading volume and a decrease in risk premium. The rise in trading volume occurs as traders react to the new information, adjusting their positions accordingly. At the same time, the risk premium is reduced, due to reduced market uncertainty. Since both trading volume and risk premium affect welfare, the welfare response to increased information quantity is not monotonic. Investors' welfare initially increases, reaching a peak as more information is released, but then declines when there is an overload of information, aligning with Hirshleifer effect (Hirshleifer (1978)), which posits that excessive information can diminish the scope of risk sharing. When the same amount of information is divided and released gradually, it fosters higher overall trading volume, with trades occurring at prices that reflect a greater risk premium compared to a single, comprehensive disclosure. This gradual information release reduces the risk premium more slowly while increasing trading volume, thereby improving risk-sharing among traders. Consequently, the incremental release of information benefits both informed and uninformed traders in a competitive market.

In contrast to competitive markets, where welfare predominantly hinges on risk-sharing, welfare in imperfectly competitive markets, where informed traders generate price impact through their trading, is influenced by both risk-sharing and price impact costs. In these markets, trades by informed traders typically move prices against them but benefit uninformed traders on the opposite side. Consequently, the price impact acts as a form of compensation from informed traders to uninformed traders, facilitating trade. This dynamic introduces additional costs for informed traders, diminishing their welfare while enhancing that of uninformed traders. As a result, the effects of the quantity and frequency of information disclosure on traders' welfare diverge significantly from those observed in competitive markets.

Increased quantity of information disclosure initially leads to improved risk-sharing. However, beyond a certain threshold, similar to the scenario in competitive markets, the level of risk-sharing starts to decline. Similarly, the total cost of the price impact initially increases with more information being disclosed due to heightened trading activity but begins to diminish as even more information is released. Therefore, for uninformed traders who benefit from both risk-sharing and price impact costs, their welfare improves with more public information disclosure up to a point, after which it starts to decrease. The impact of information disclosure on the welfare of informed traders is intricate, influenced by the interplay between risk-sharing and price impact costs. Public information disclosure increases trading volume, enhancing risk-sharing but also raising total price impact costs. For informed traders primarily motivated by hedging, the increased price impact costs outweighs the benefits of increased risk sharing, negatively affecting their welfare. Conversely, for informed traders driven primarily by private information, the benefits of expanded risk-sharing due to information disclosure can outweigh the increased overall price impact costs, thereby enhancing their welfare.

Given the total quantity of disclosed information, uninformed traders consistently benefit from more frequent releases of information. This positive impact results from incremental disclosure that enhances risk-sharing and increases the overall price impact costs borne by informed traders. This suggests that gradual information disclosure always benefits uninformed market participants. However, the effect of disclosure frequency on the welfare of informed traders is more complicated and largely influenced by their trading motivations. If an informed trader's trading is primarily driven by hedging demands, his welfare tends to decrease with more frequent disclosures. In such cases, the rise in overall price impact costs due to gradual disclosure outweighs the benefits of enhanced risk sharing. Conversely, when an informed trader's trading is primarily driven by private information, his welfare improves with increased frequency of disclosure. In this scenario, the benefits from enhanced risk sharing due to gradual disclosure outweigh the increased total price impact costs because the price impact per unit is reduced by a larger magnitude due to decreased adverse selection compared to the case with hedging-driven trading. For informed traders with more balanced trading involving both hedging and private information, there is an initial increase in welfare with increasing disclosure frequency, followed by a decrease. This indicates the presence of an optimal frequency of disclosure for such traders.

Moreover, our paper shows that in an imperfectly competitive market, the risk premium begins to decline even before public information is released—unlike in competitive markets, where future disclosures do not affect the current risk premium. Anticipating future public announcements, informed traders strategically shift part of their trading—both for private information and hedging—before the release and trade the rest afterward to minimize overall trading costs. This behavior helps smooth market liquidity over time and reduces the price impact. As a result, the cost of capital prior to public disclosure declines. When total precision is held constant, increasing the frequency of disclosure leads informed traders to spread their trading more evenly across periods, further lowering the price impact. Consequently, higher disclosure frequency results in a monotonic decline in the initial cost of capital.

Our paper sheds light on how the ownership structure of institutional investors influences a firm's information disclosure approach. It suggests that firms with a high concentration of institutional ownership, where a small number of investors hold substantial equity, may adopt less frequent information disclosure. This can be attributed to the monopolistic influence of these major investors. In contrast, firms with a more diversified and competitive institutional investor base may disclose information gradually, as it becomes available, rather than in infrequent batches. Moreover, if a firm receives information in a lumpy fashion, smoothing public disclosures can enhance risk-sharing. Therefore, our paper highlights the importance of considering the investor composition, their trading motivations, and market competitiveness when analyzing the impact of a firm's disclosure practices. These insights are useful for regulators aiming to promote effective risk-sharing and improve the welfare of market participants.

Our paper contributes to two streams of literature. First, it contributes to the understanding of the impacts of information disclosure frequency. One contemporaneous and complementary theoretical paper by Jiang, Wei, Yang, and Zhang (2023) examines the impact of frequent public disclosure on price informativeness and cost of capital in competitive markets. They find that more frequent disclosure unequivocally increases price informativeness and subtly affects the ex-ante cost of capital. In contrast, our paper highlights that, in imperfective competitive markets, the effects of frequency of information disclosure on traders' welfare diverge significantly from those observed in competitive markets. Our model reveals that, with imperfective competition, information disclosure frequency can reduce the ex-ante cost of capital prior to disclosure, offering alternative empirical and regulatory insights. The other theoretical work by Rostek and Weretka (2015) develops a dynamic equilibrium model focused on asset pricing, trade, and consumption in markets characterized by large institutional investors who are aware of their impact on prices. Their paper emphasizes the timing effects of information disclosure on market welfare and liquidity. It concludes that while postponing information disclosure can improve consumption and welfare by enhancing diversification before risk resolution, it also results in periods of reduced market liquidity. Brennan and Cao (1996) analyze the value of improved trading opportunities in a competitive market, either through more frequent trading in the underlying asset (driven by more frequent information disclosure) or by trading in a derivative asset. They demonstrate that the welfare gain for an individual investor from increased trading frequency is positively related to that investor's trading volume. Furthermore, they show that as trading approaches continuity, Pareto efficiency is achieved. Additionally, they illustrate that trading in an appropriate derivative security allows Pareto efficiency to be attained in just a single trading round. Different from both papers, our paper highlights that the welfare implications of information disclosure frequency critically depend on the composition of investors, traders' trading incentives, and the degree of market competitiveness.

Second, our paper contributes to the literature on the relationship between institutional investors and the information environment of firms (see, for example, Armstrong, Balakrishnan, and Cohen (2012); Boone and White (2015); Bird and Karolyi (2016)). Empirical evidence, as highlighted by Bushee and Noe (2000), generally indicates a positive association between institutional ownership and the quality of a firm's information environment. However, recent studies (Chen and Jung (2016); Baik, Kim, Kim, and Patro (2020)) present a more nuanced view, especially regarding hedge fund ownership. These works suggest that, unlike other institutional investors, hedge fund ownership may adversely affect voluntary disclosure practices in portfolio firms. Our model proposes a theoretical framework to reconcile these seemingly contradictory findings. We provide an argument that hedge fund investors, in contrast to other types of institutional investors, possess considerable informational market power and may face frequent re-balancing pressures. Consequently, firms with significant hedge fund ownership are likely to engage in less frequent information disclosure practices. In contrast, ownership by competitive non-hedge fund institutions may correlate positively with the extent of a firm's information disclosure. Moreover, consistent with the implications of our model, Ge, Bilinski, and Kraft (2021) document that concentrated institutional ownership reduces firms' voluntary disclosure, as measured by the propensity to issue management forecasts, the comprehensiveness of guidance, the likelihood of engaging in conference calls, and the number of 8-K filings.

The paper is structured as follows: Section 2 presents the model. Section 3 analyzes the competitive equilibrium and examines the effects of the quantity and frequency of public information disclosure on cost of capital and market participants' welfare. Section 4 presents the imperfectly competitive equilibrium and explores the implications of information disclosure frequency on cost of capital and welfare. Section 5 concludes. All proofs are in the Appendix.

2 The Model

We consider a multi-period model of trading in a market with two groups of traders: a mass of N_I identical informed traders, and a mass of N_U identical uninformed traders, denoted by superscripts I and U, respectively. Traders can trade one risk-free asset and one risky security at dates $1, \ldots, T$. The risk-free asset has a zero supply and serves as the numeraire and thus the risk-free interest rate is normalized to 0. The total supply of the risky asset is Θ, and each share pays a liquidation value of $V \sim \mathcal{N}(\bar{V}, \tau_V^{-1})$ at the final date $T+1$, where $\bar{V}$ is a constant, $\tau_V^{-1} > 0$, and $\mathcal{N}$ denotes the normal distribution. No investor is endowed with any risk-free asset. Each investor of type $i \in \{I, U\}$ is initially endowed with θ_{-1}^i shares of the risky asset, making the total supply of the risky security equal to the total endowment, i.e., $\Theta = N_I \theta_{-1}^I + N_U \theta_{-1}^U$.

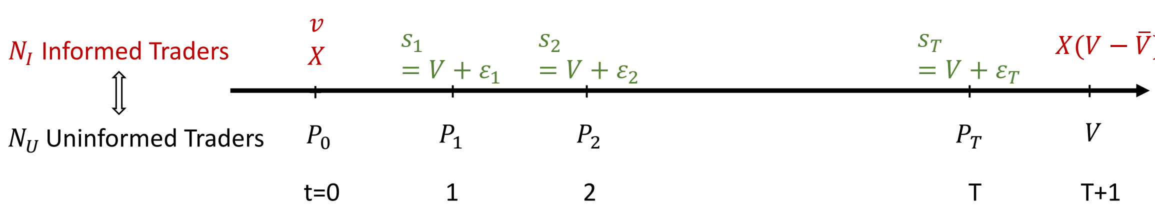


Figure 1. Timeline of the model.

Figure 1 illustrates the timeline of the model. At the start of trading ($t = 0$), informed traders may receive a private signal

$$v = V + \eta, \tag{1}$$

about the liquidation value V, where η is an independent noise following a normal distribution $\eta \sim \mathcal{N}(0, \tau_\eta^{-1})$.[3] To prevent informed investors' private information from being fully revealed

[3]To make the intuition of our main result as clear as possible, the precision of the private information at date 0 is given in our model. Our model can be extended to include date -1, before trading, where the informed traders can acquire a costly signal with a precision of τ_η at a cost of $c(\tau_\eta) := k\tau_\eta^2$, where k is a positive constant. Thus, τ_η is a decreasing function of k. The main results remain qualitatively similar with endogenous information acquisition. Increasing the frequency of information disclosure may still negatively impact informed traders in a non-competitive market. For example, if it is too costly for informed traders to acquire private information with high precision and their trading is primarily driven by hedging, increasing the frequency of information disclosure makes them worse off in a non-competitive market.

in equilibrium, we assume that each informed trader is also endowed with X shares of a nontradable risky asset, where $X \sim \mathcal{N}(0, \tau_X^{-1})$.[4] The quantity of this nontradable asset, independent from V and η, is only observable to informed traders and can only be liquidated at $T+1$. Without loss of generality, we assume that each unit of the nontradable asset pays a liquidation value of $V - \bar{V}$ at $t = T+1$.[5] In this setting, informed investors, characterized by dual trading motives, initially demand liquidity in the market, whereas the uninformed traders initially provide liquidity.

At each of the dates $t = 1, ..., T$, public information regarding the liquidation value of the risky asset may be disclosed:

$$s_t = V + \varepsilon_t, \tag{2}$$

where ε_t is an independent noise term following a normal distribution $\mathcal{N}(0, \tau_{\varepsilon,t}^{-1})$. By altering the precision $\tau_{\varepsilon,t}$ of the public information for each period, we can examine the effects of both the quantity and frequency of information disclosure. We define

$$\tau_t = \sum_{s=1}^{t} \tau_{\varepsilon,s}, \quad \forall\, t \in [1, T], \tag{3}$$

as the cumulative quantity of information disclosed up to time t. Therefore, τ_T represents the total amount of public information disclosed by the final trading period T.

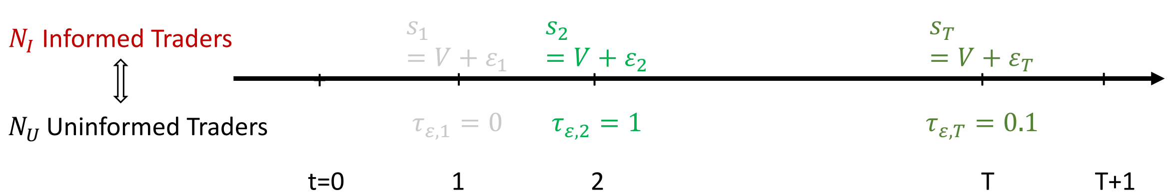


Figure 2. Adjustable quantity and frequency of information disclosure.

[4]Similar to Vayanos and Wang (2012) and Goldstein, Li, and Yang (2014), to keep information from being fully revealed in equilibrium, we assume that informed investors have two trading motives. They have liquidity shocks in addition to private information. One can interpret this assumption to mean that there are some pure liquidity traders who trade in the same direction as the informed.

[5]In general, we can assume that the nontradable asset has a payoff correlated with V. The correlation between the nontradable asset and the security results in a liquidity demand for the risky asset to hedge the nontradable asset payoff. This generalization would not yield qualitatively different results.

In Figure 2, we provide an illustrative example where $\tau_{\varepsilon,2} = 1$ and $\tau_{\varepsilon,T} = 0.1$, implying that more information is released at $t = 2$ than at $t = T$. Additionally, setting $\tau_{\varepsilon,1} = 0$ indicates that no information is released at $t = 1$. By adjusting $\tau_{\varepsilon,t}$ to various values, including zero, we can effectively control the timing and frequency of information disclosure in our model.

Define $\underline{Z}_t := (Z_0, Z_1, ..., Z_t)$ for any stochastic process $\{Z_t\}$, representing the history of Z_t up to t. On each date t, each informed investor chooses a demand schedule $\Theta_t^I(v, X, \underline{s}_t; \cdot)$. Each uninformed trader chooses a demand schedule $\Theta_t^U(\cdot)$. The schedules Θ_t^I and Θ_t^U are traders' strategies. Given prices $\underline{P}_t$, the quantities demanded by informed and uninformed investors can be written as $\theta_t^I = \Theta_t^I(v, X, \underline{s}_t, \underline{P}_t)$ and $\theta_t^U = \Theta_t^U(\underline{s}_t, \underline{P}_t)$.

Let $\mathscr{F}_t^i$ denote a type i investor's information set at period t for $i \in \{I, U\}$. Investors' information sets can be written as

$$\mathscr{F}_t^I = \{v, X, \underline{s}_t, \underline{P}_t\}, \;\; \mathscr{F}_t^U = \{\underline{s}_t, \underline{P}_t\}. \tag{4}$$

Given prices $\underline{P}_t$, for type $i \in \{I, U\}$, a type i investor's problem is to solve

$$\max_{\theta_t^i} \mathrm{E}[-e^{-A^i W_{T+1}^i} \mid \mathscr{F}_t^i], \tag{5}$$

where W_{T+1}^i and A^i represent the final wealth and risk aversion coefficient of type i traders, respectively, and

$$\begin{aligned} W_{T+1}^I &= W_{-1}^I - \sum_{t=0}^{T} (\theta_t^I - \theta_{t-1}^I) P_t + \theta_T^I V + X(V - \bar{V}), \\ W_{T+1}^U &= W_{-1}^U - \sum_{t=0}^{T} (\theta_t^U - \theta_{t-1}^U) P_t + \theta_T^U V, \end{aligned} \tag{6}$$

where $W_{-1}^i = \theta_{-1}^i \bar{V}$ is the initial wealth of a type i investor.

3 A Competitive Market

To examine the effects of varying information disclosure frequencies under different market conditions, we first solve for the equilibrium in a competitive market, where both informed and uninformed traders are price takers. The definition of a Bayesian Nash equilibrium is given as follows.

Definition 1 (Competitive Equilibrium). *A competitive equilibrium* $\left(\Theta_t^I(P_t), \Theta_t^U(P_t), P_t\right)$ *is such that*

1. *given any* P_t, $\Theta_t^i(P_t)$ *solves a type* i *investor's Problem* (5) *for* $i \in \{I, U\}$, *where the information set of the informed and the uninformed traders is as given in equation* (4)*;*
2. P_t *clears the risky security market,* $\Theta = N_I \Theta_t^I(P_t) + N_U \Theta_t^U(P_t)$, *and the risk-free asset market;*
3. *for every realization of the signals* $\{\underline{s}_t\}$, v, *and* X, *the beliefs of all investors are consistent with the joint conditional probability distribution in equilibrium.*

3.1 The Equilibrium

We focus on a linear equilibrium where the equilibrium stock price can be expressed as a linear function of the state variables in the model.

At $t = 0$, both informed investors' private signal v and liquidity shock X affect informed investors' trading and thus the equilibrium price. From market price P_0, other investors can only infer the value of the composite signal

$$s_0 := v - hX = V + \varepsilon_0, \tag{7}$$

where

$$\varepsilon_0 = \eta - hX \sim \mathcal{N}(0, \tau_0^{-1}), \quad h = \frac{A^I}{\tau_\eta}, \quad \tau_0^{-1} = \tau_\eta^{-1} + h^2 \tau_X^{-1}. \tag{8}$$

Therefore, P_0 is informationally equivalent to the composite signal s_0, which is a linear combination of informed investors' private signal and the amount of the nontradable risky asset. Therefore, the information set at period t for uninformed investors $\mathscr{F}_t^U = \{s_0, \underline{s}_t\}$.

We obtain the following lemma on the dynamic Kalman filtering formulas.

Lemma 1. *(Kalman filtering). At t, for $i \in \{I, U\}$, trader i's estimate of the stock's liquidation value $\hat{V}_t^i := \mathrm{E}[V \mid \mathscr{F}_t^i]$ and trader U's estimate of the liquidity shocks $\hat{X}_t^U := \mathrm{E}[X_t \mid \mathscr{F}_t^U]$ are*

$$\hat{V}_t^I = \hat{V}_{t-1}^I + o_{V,t}^I \tau_{\varepsilon,t} e_t^I, \quad \hat{V}_t^U = \hat{V}_{t-1}^U + o_{V,t}^U \tau_t e_t^U, \quad \hat{X}_t^U = \hat{X}_{t-1}^U + K_{X,t}^U e_t^U, \tag{9}$$

where

$$e_t^i = s_t - \hat{V}_{t-1}^i, \;\; \text{for } i \in \{I, U\} \text{ and } t \geq 1, \text{ and } e_0^I = v - \bar{V}, \; e_0^U = s_0 - \bar{V}, \tag{10}$$

$\hat{X}_{-1}^U = 0$, $\hat{V}_{-1}^I = \hat{V}_{-1}^U = \bar{V}$, the coefficient $K_{X,t}^U$ is defined in (A-13) in the Appendix, and the conditional variances of V for informed and uninformed traders at t are,

$$o_{V,t}^I := \mathrm{Var}[V \mid \mathscr{F}_t^I] = \big(\tau_V + \sum_{s=0}^{t} \tau_{\varepsilon,s}\big)^{-1}, \quad o_{V,t}^U := \mathrm{Var}[V \mid \mathscr{F}_t^U] = \big(\tau_V + \sum_{s=0}^{t} \tau_s\big)^{-1},$$

where $\tau_{\varepsilon,0} = \tau_\eta$, $\tau_0 := (\tau_\eta^{-1} + h^2 \tau_X^{-1})^{-1}$, and $\tau_s = \tau_{\varepsilon,s}$ for $s \geq 1$.

Proof. See Appendix A.1. □

Lemma 1 shows that informed traders' estimate $\hat{V}_t^I$ can be written as a recursive equation of $\hat{V}_{t-1}^I$ with an innovation term $s_t - \hat{V}_{t-1}^I$ and uninformed traders' estimate $(\hat{V}_t^U, \hat{X}_t^U)$ can be expressed as a recursive equation of $(\hat{V}_{t-1}^U, \hat{X}_{t-1}^U)$ with an innovation term $s_t - \hat{V}_{t-1}^U$. The conditional variance and mean of the security's payoff for traders have intuitive expressions. Specifically, the conditional variance of the payoff corresponds to the inverse of the total precision of traders' signals, while the conditional mean of the payoff (or the estimate of V) is determined by the precision-weighted average of signal innovations.

The sufficient statistic $\hat{V}_t^I - \mu_t X$ is a composite signal of informed traders, capturing their dual trading motives: speculating based on private information and hedging against risks from liquidity shocks. The noise-to-signal ratios μ_t is

$$\mu_t = A^I o_{V,t}^I = A^I \big(\tau_V + \sum_{s=0}^{t} \tau_{\varepsilon,s}\big)^{-1}. \tag{11}$$

Since the sufficient statistic $\hat{V}_t^I - \mu_t X$ can be inferred by uninformed traders from prices, it follows that $\hat{V}_t^I - \mu_t X = \hat{V}_t^U - \mu_t \hat{X}_t^U$. Equation (11) implies that the noise-to-signal ratio for uninformed traders μ_t decreases over time, as prices progressively convey more information about informed traders' estimation of the stock value.

The following proposition provides the equilibrium prices and equilibrium quantities at t ($t = 0, 1, 2, ..., T$) in closed form.

Proposition 1. *1. The equilibrium stock price is*

$$P_t = \omega_t(\hat{V}_t^I - \mu_t X) + (1-\omega_t)\hat{V}_t^U - \frac{\Theta}{\frac{N_I}{A^I o_{V,t}^I} + \frac{N_U}{A^U o_{V,t}^U}}, \quad \text{where} \quad \omega_t = \frac{\frac{N_I}{A^I o_{V,t}^I}}{\frac{N_I}{A^I o_{V,t}^I} + \frac{N_U}{A^U o_{V,t}^U}}. \tag{12}$$

2. The equilibrium quantities demanded at t are

$$N_I\theta_t^I = \Theta - N_I(1-\omega_t)\phi_t, \quad N_U\theta_t^U = N_I(1-\omega_t)\phi_t, \quad \text{where} \quad \phi_t = \hat{X}_t^U + \frac{\Theta}{N_I}. \tag{13}$$

3. The value functions are

$$\begin{aligned} J_t^I &= -\rho_t^I e^{-A^I[W_t^I + X\hat{V}_t^I + \frac{1}{2}m_t^I\phi_t^2 + \frac{1}{2}\mu_t(1-\omega_t)(\hat{X}_t^U)^2]}, \\ J_t^U &= -\rho_t^U e^{-A^U[W_t^U + \frac{1}{2}m_t^U\phi_t^2]}, \end{aligned} \tag{14}$$

where $m_t^I = -\mu_t\omega_t(1-\omega_t)$, $m_t^U = -\frac{N_I}{N_U}m_t^I$, ρ_t^I and ρ_t^U can be computed recursively using equations (A-47) and (A-37).

Proof. See Appendix A.2. □

Proposition 1 implies that the equilibrium price increases with informed traders' composite signal ($\hat{V}_t^I - \mu_t X$) and uninformed estimate of the asset's liquidation value ($\hat{V}_t^U$) and decreases with the total supply of the asset, as shown in equation (12).

The positions of investors are influenced by their updated estimates of payoffs, realized hedging demands, and the risk premium they require. Consequently, the quantities demanded at time t are influenced by the discrepancy between informed investors' composite signal and the uninformed traders' estimate of the asset's liquidation value, as well as the differential in risk premiums. As suggested by equation (13), holdings by informed investors decrease with $\hat{X}_t^U$ whereas those by uninformed investors increase with it. Intuitively, if uninformed traders were to experience identical liquidity shocks and share the same level of risk aversion as informed traders, their quantities demanded would be identical. Therefore, trading activity primarily stems from differences in liquidity shocks, as indicated by the variations in estimates of these shocks and the associated estimation of the risk premium.

3.2 The Effects on Cost of Capital

Proposition 2. *In a competitive market setting, future information does not influence the current price or the quantities of stock holdings and thus it does not affect the current cost of capital.*

Proof. This follows directly from Proposition 1. □

Proposition 2 suggests that, at time t, the coefficients defining both the price and stock holdings depend solely on the precisions of information available up to and including time t. From equation (12), the risk premium at t is

$$\mathrm{E}[V - P_t] = \frac{\Theta}{\frac{N_I}{A^I o^I_{V,t}} + \frac{N_U}{A^U o^U_{V,t}}}. \tag{15}$$

The explicit formula of the expected trading volume, $\mathrm{Vol}_t = \mathrm{E}\left[N_I \mid \theta^I_t - \theta^I_{t-1} \mid\right]$, is computed and detailed in Appendix A.3. Both the risk premium and expected trading volume at time t depend only on the information available up to and including time t. When new information is disclosed at time t, the risk premium tends to decrease as uncertainty is reduced.

In a competitive market, the anticipation of future public information does not affect the current risk premium or expected trading volume. However, the actual disclosure of information can lead to an increase in trading volume and a reduction in the risk premium. Since the risk premium is often used as a measure of a firm's cost of capital (as discussed in Goldstein et al. (2014)), it follows that in a competitive financial market, the anticipation of future public information does not affect the firm's current cost of capital.

3.3 The Effects on Welfare

In this section, we examine the effects of information disclosure frequency on market participants' welfare.

Equation (14) presents an analytical expression for trader i's ($i \in \{I, U\}$) utility function at t, J_t^i. The ex-ante utility and certainty equivalent wealth of trader i are represented as follows:

$$\mathscr{W}^i = \mathrm{E}[J_0^i], \quad \mathrm{CE}^i = -\frac{1}{A^i}\log(-\mathscr{W}^i). \tag{16}$$

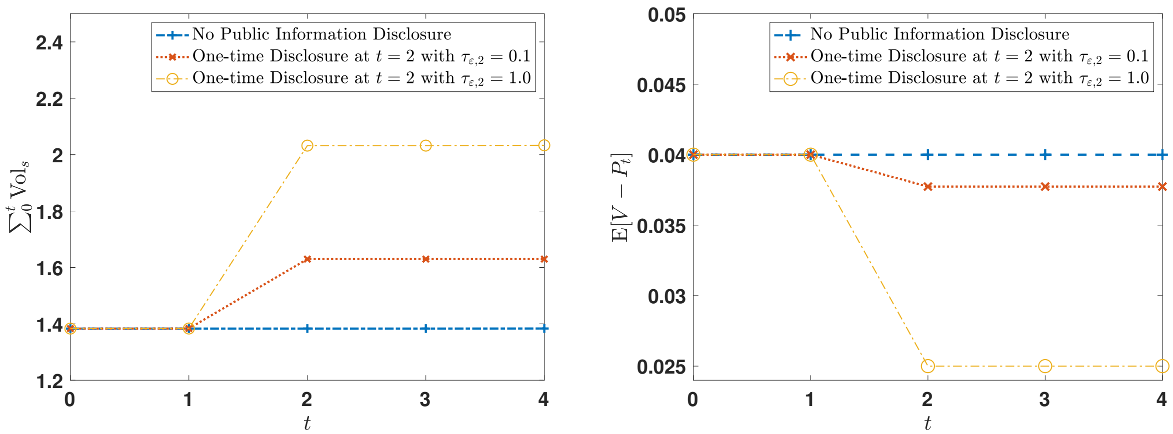
Our goal is to assess how public information disclosure influences traders' ex-ante certainty equivalent wealth. For this purpose, we first calculate the certainty equivalent wealth in the absence of public information disclosure (CE_{np}^i) to serve as a reference point. This allows us to quantify the effect of information disclosure by examining the change in certainty equivalent wealth, denoted as

$$\Delta_{CE}^i := \mathrm{CE}^i - \mathrm{CE}_{np}^i. \tag{17}$$

To understand the welfare implication of information disclosure, it is helpful to discuss how to measure the extent of risk sharing among traders. The extent of risk sharing is influenced by two key factors. The first is the price factor, which refers to the risk premium, essentially the expected excess return demanded by an investor for holding the risky asset. The second factor is the quantity, represented by the trading volume. A larger trading volume indicates heightened trading activity between informed and uninformed traders, suggesting a broader scope of risk sharing.

3.3.1 The Effects of Public Information Quantity

We first examine the effects of information quantity measured by the precision of public information. In the presence of information asymmetry, disclosing public information typically results in increased trading volume and a reduced risk premium. The surge in trading volume occurs as traders react to and incorporate the new information, adjusting their positions accordingly. Simultaneously, the risk premium diminishes, attributed to the enhanced information environment that reduces market uncertainty. Thus, information disclosure yields contrasting effects on two critical factors: the size (trading volume) and the price (risk premium), both of which are important in determining traders' welfare.



(a) Cumulative Trading Volume (b) Risk Premium

Figure 3. Cumulative Trading Volume (left) and Risk Premium (right) with Varying Precision of Public Information. Parameters: $T = 4$, $N_I = 5$, $N_U = 10$, $A^I = A^U = 1$, $\tau_V = 1$, $\tau_\eta = 1$, $\tau_X = 1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$.

To focus on the effects of information quantity, we keep the frequency of information disclosure constant. Consider a scenario where a single piece of public information is released. In our model, when no new information is released, prices—and thus the risk premium—remain unchanged. During these periods, traders transact based on the optimal quantities determined by their existing information and hedging demands. The specific allocation of trading during these periods does not affect traders' welfare, as prices remain unchanged. When new information is disclosed, traders adjust their optimal quantities, and prices are updated accordingly. Suppose that this information release occurs at $t = 2$, the left panel of Figure 3 plots cumulative trading volume from time 0 to time t, comparing three scenarios: high information precision (dot-dashed curve), low precision (dotted curve), and no information release (dashed curve). Figure 3 depicts that higher precision in disclosed information results in greater total trading volume and a corresponding reduction in the risk premium. Since both trading volume and risk premium affect welfare, the welfare response to increased information quantity is not monotonic. Investors' welfare initially increases, reaching a peak as more information is released, but then declines when there is an overload of information.

Proposition 3. *Assume there is only one piece of public information. The optimal welfare for both informed and uninformed traders is achieved when the precision of the public information, denoted as Γ, is*

$$\Gamma = \tau_V + \bar{\tau}, \quad where \quad \bar{\tau} = \frac{\frac{N_I}{A^I}\tau_\eta + \frac{N_U}{A^U}\tau_0}{\frac{N_I}{A^I} + \frac{N_U}{A^U}}, \tag{18}$$

and $\tau_0 = \left(\tau_\eta^{-1} + h^2\tau_X^{-1}\right)^{-1}$ represents the precision of the signal inferred from the initial price P_0 about the liquidation value V.

Proof. See Appendix A.5. □

Releasing some public information, rather than withholding it completely, is beneficial for both informed and uninformed traders. According to Proposition 3, there is an optimal level of precision for public information that maximizes welfare for both groups. Initially, as more information is released, investors' welfare increases, reaching a peak. However, past this point, welfare begins to decline due to an overload of information.

In competitive markets, the welfare of traders is closely tied to the degree of risk sharing. Too much information can lead to a significantly low risk premium, while too little information might result in reduced trading volume. The peak of traders' welfare, is achieved when the precision of public information is moderate. Initially, the positive impact on trading volume outweighs the effect on risk premium, leading to increased welfare. Later, the influence of reduced risk premium becomes more dominant, causing a decline in welfare. Equation (18) shows that the optimal precision is determined by a weighted average of the precision of information available to both informed and uninformed traders.

As illustrated in Figure 4 and shown in Proposition 3, investors' welfare initially increases

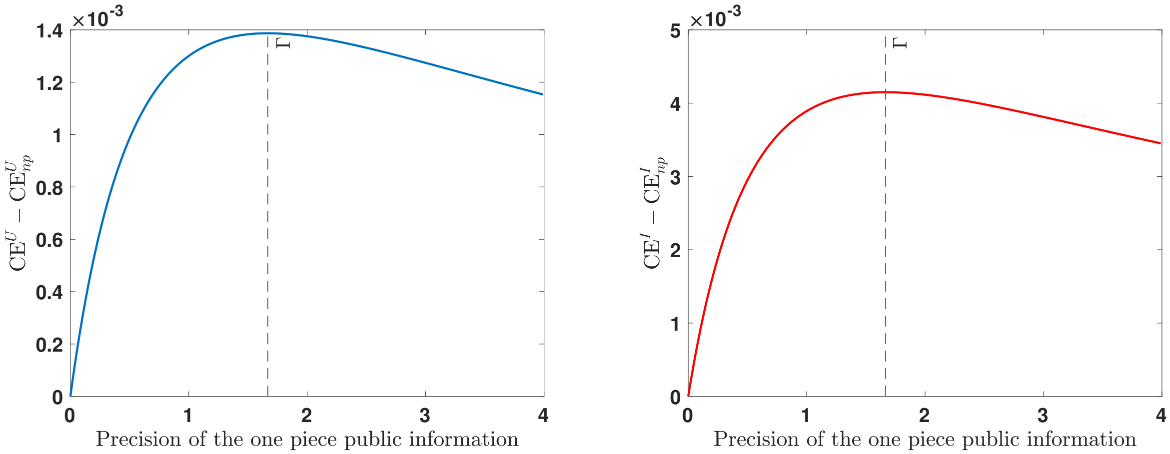


Figure 4. Changes in Certainty Equivalent Welfare for Both Traders (Δ^i_{CE}) Versus the Precision of Public Information. Parameters: $N_I = 5$, $N_U = 10$, $A^I = A^U = 1$, $\tau_V = 1$, $\tau_\eta = 1$, $\tau_X = 1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$.

with the release of more precise public information, reaching a peak. However, beyond this point, it begins to decline.

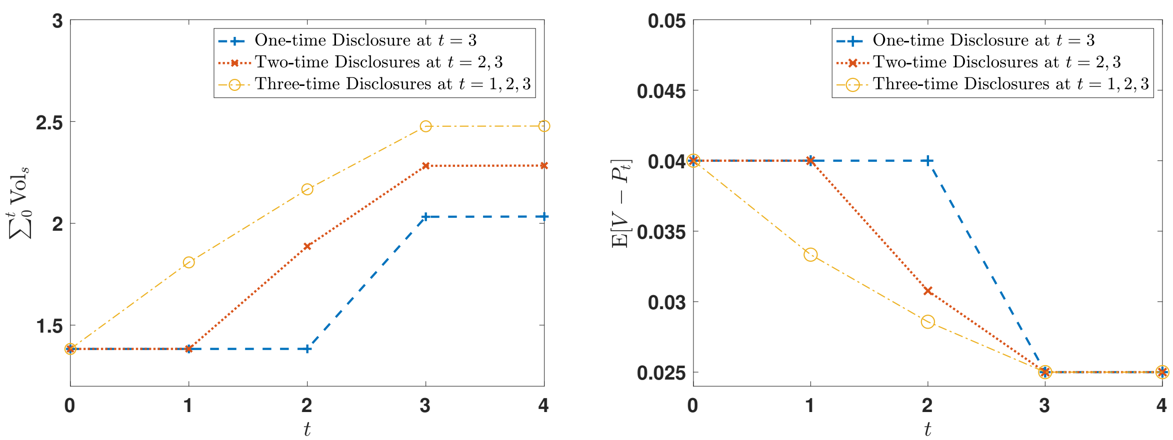
3.3.2 The Effects of Information Disclosure Frequency

We next examine the impact of the frequency of information disclosure. For instance, consider a scenario where the total precision of public information, denoted as τ_T, remains constant. Does the method of releasing all information at once versus distributing it in two sequential parts make a difference? This question is addressed in the following proposition.

Proposition 4. *(1) The welfare of both informed and uninformed traders improves when a fixed amount of information is divided into two parts (i.e., $\tau_T = \tau_{\varepsilon,1} + \tau_{\varepsilon,2}$) and released sequentially to the market. (2) When a fixed amount of information is divided into two parts, the welfare of both informed and uninformed traders is maximized when: $\tau_{\varepsilon,1} = \frac{\tau_T}{2+\frac{\tau_T}{\tau_V+\bar{\tau}}} < \frac{\tau_T}{2}$. This implies that the optimal release scheme for both groups involves gradually disclosing an increasing amount of information to the market over time.*

Proof. See Appendix A.6. □

Proposition 4 demonstrates that gradually dividing and releasing a fixed amount of information to the market enhances the welfare of all traders. The left panel of Figure 5 illustrates the cumulative trading volume when the information, with fixed total precision, is divided into two pieces (dotted curve) or three pieces (dot-dashed curve), compared to releasing it all at once (dashed curve). The gradual release increases total trading volume. Meanwhile, the right panel of Figure 5 shows that gradual information release reduces the risk premium more slowly while increasing trading volume. Consequently, trades occur at prices that reflect a higher risk premium compared to a single, comprehensive release. This enhances risk-sharing among traders.



(a) Cumulative Trading Volume (b) Risk Premium

Figure 5. Cumulative Trading Volume (left) and Risk Premium (right) with Varying Information Disclosure Frequency. Parameters: $T = 4$, $\tau_T = 1$, $N_I = 5$, $N_U = 10$, $A^I = A^U = 1$, $\tau_V = 1$, $\tau_\eta = 1$, $\tau_X = 1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$. Dashed curve represents scenarios with a single disclosure at $t = 3$, with $\tau_{\varepsilon,3} = \tau_T$; dotted curve represents two disclosures at $t = 2, 3$, with $\tau_{\varepsilon,2} = \tau_{\varepsilon,3} = \tau_T/2$; dot-dashed curve represents three information disclosures with $\tau_{\varepsilon,1} = \tau_{\varepsilon,2} = \tau_{\varepsilon,3} = \tau_T/3$.

Therefore, as Figure 6 demonstrates, the incremental release of a fixed quantity of information is beneficial for both informed and uninformed traders in a competitive market. Nonetheless, the increase in welfare is upper bounded, as illustrated in Figure 6 and shown in Proposition 5.[6]

[6]For expositional simplicity, in Proposition 5 and Figure 6, we focus on the case where the total precision is equally split for each release. When the majority of the information is released in the earlier periods, the scenario resembles one with less frequent disclosures, which is suboptimal.

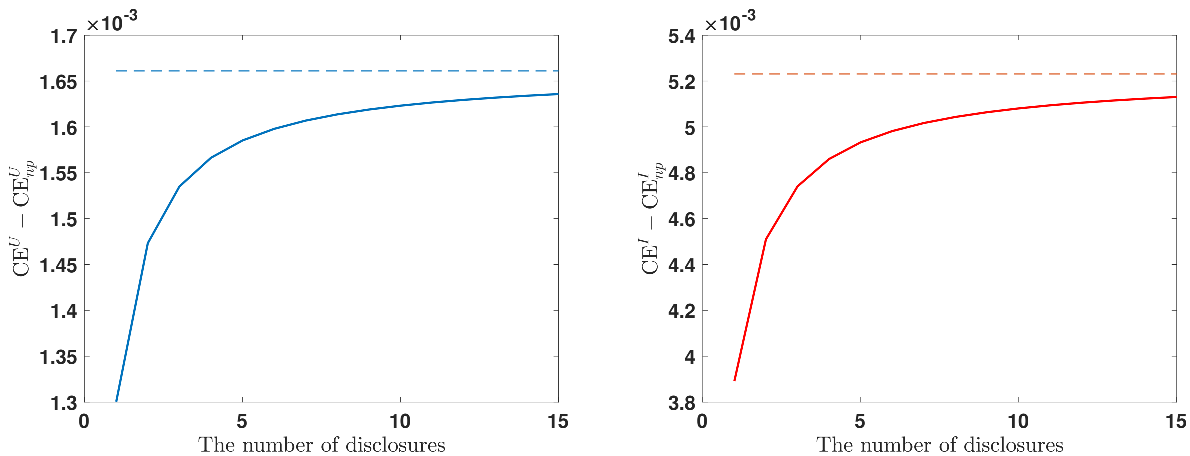


Figure 6. Changes in Certainty Equivalent Welfare for Both Traders (Δ^i_{CE}) Versus Varying Information Disclosure Frequency. Parameters: $N_I = 5$, $N_U = 10$, $A^I = A^U = 1$, $\tau_T = 1$, $\tau_{\varepsilon,t} = \tau_T/T$, $\tau_V = 1$, $\tau_\eta = 1$, $\tau_X = 1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$.

Proposition 5. *With a fixed total precision τ_T, adopting a gradual approach to information disclosure, where $\tau_{\varepsilon,t} = \frac{\tau_T}{T}$, results in an increase in certainty equivalent wealth as*

$$\lim_{T\to\infty} \Delta^I_{CE} = \frac{1}{2}\left(\frac{\frac{N_U}{A^U}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\tau_\eta-\bar{\tau}}\right)^2 \frac{E^I}{A^I}, \quad \lim_{T\to\infty} \Delta^U_{CE} \;= \frac{1}{2}\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\bar{\tau}-\tau_0}\right)^2 \frac{E^U}{A^U},$$

where

$$E^I = \frac{\tau_T(\tau_\eta-\bar{\tau})}{(\tau_V+\bar{\tau})(\tau_T+\tau_V+\bar{\tau})} + \ln\frac{(\tau_V+\bar{\tau})(\tau_T+\tau_V+\tau_\eta)}{(\tau_V+\tau_\eta)(\tau_T+\tau_V+\bar{\tau})}, \quad E^U = \frac{\tau_T(\tau_0-\bar{\tau})}{(\tau_V+\bar{\tau})(\tau_T+\tau_V+\bar{\tau})} + \ln\frac{(\tau_V+\bar{\tau})(\tau_T+\tau_V+\tau_0)}{(\tau_V+\tau_0)(\tau_T+\tau_V+\bar{\tau})}.$$

Thus, for a fixed τ_T, the welfare of both traders is an increasing and concave function of T with a bounded limit. The limit is increasing with τ_T and bounded by

$$\begin{aligned} \lim_{\tau_T\to\infty}\lim_{T\to\infty} \Delta^I_{CE} &= \frac{1}{2A^I}\left(\frac{\frac{N_U}{A^U}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\tau_\eta-\bar{\tau}}\right)^2\left(\frac{\tau_\eta-\bar{\tau}}{\tau_V+\bar{\tau}} + \ln\frac{\tau_V+\bar{\tau}}{\tau_V+\tau_\eta}\right), \\ \lim_{\tau_T\to\infty}\lim_{T\to\infty} \Delta^U_{CE} &= \frac{1}{2A^U}\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\bar{\tau}-\tau_0}\right)^2\left(\frac{\tau_0-\bar{\tau}}{\tau_V+\bar{\tau}} + \ln\frac{\tau_V+\bar{\tau}}{\tau_V+\tau_0}\right). \end{aligned} \tag{19}$$

Proof. See Appendix A.7. □

Proposition 5 indicates that while gradual information disclosure consistently enhances the welfare of all participants in a competitive market, this benefit has a finite upper limit. This limit is determined by the total quantity of information, τ_T, and other model parameters, as detailed in Equation (19).

4 An Imperfectly Competitive Market

We next examine how information disclosure frequency influences cost of capital and the welfare of market participants in an imperfectly competitive market.

For expositional simplicity, let's assume a market with a single monopolistic informed trader ($N_I = 1$) and a large number of uninformed traders, represented by a continuum with mass N_U.[7] The portfolio choice problems for both the informed trader and uninformed traders follow the formulations in equations (5) and (6). The primary distinction in this imperfectly competitive model is the inclusion of price impact in the monopolistic informed trader's strategy. Specifically, the price impact, denoted as λ_t, is defined by:

$$\lambda_t = \frac{\partial P_t}{\partial \theta_t^I}. \tag{20}$$

The definition of an imperfectly competitive equilibrium is given as follows.

Definition 2 (Imperfectly Competitive Equilibrium)**.** *An imperfectly competitive equilibrium* $\left(\Theta_t^I(P_t), \Theta_t^U(P_t), P_t\right)$ *is such that*

1. *given any* P_t, $\Theta_t^U(P_t)$ *solves Problem* (5) *for uninformed investors, where the information set of traders is as given in equation* (4)*;*
2. *The strategy* $\Theta_t^I(P_t)$ *solves the monopolistic informed trader's problem as outlined in equation* (5)*, by incorporating the price impact* λ_t *as defined in equation* (20)*.*
3. P_t *clears the risky security market,* $\Theta = \Theta_t^I(P_t) + \sum_{j=1}^{N_U} \Theta_t^{U,j}(P_t)$*, and the risk-free asset markets;*
4. *for every realization of the signals* $\{\underline{s}_t\}$*,* v *and* X*, the beliefs of all investors are consistent with the joint conditional probability distribution in equilibrium.*

[7] While a closed-form solution is also possible for a market with oligopolistic informed traders ($N_I \geq 2$), this extension does not alter the qualitative nature of our results.

4.1 The Equilibrium

As in the competitive market case, we focus on a linear equilibrium where the equilibrium stock price can be expressed as a linear function of the state variables in the model. The following proposition provides the equilibrium prices and quantities in closed form.

Proposition 6. *1. The equilibrium stock price is*

$$P_t = \omega_t P_t^I + (1-\omega_t) P_t^U - f_t \Theta, \tag{21}$$

where

$$P_t^I := \hat{V}_t^I - \mu_t X - g_t^I \hat{X}_t^U - f_t^I \Theta + \lambda_t \theta_{t-1}^I, \quad P_t^U := \hat{V}_t^U - g_t^U \hat{X}_t^U - f_t^U \Theta. \tag{22}$$

The coefficient μ_t is as defined in equation (11), the coefficient f_t, the weight ω_t, and the price impact of the informed trader λ_t are given as

$$f_t := \left(\frac{1}{\lambda_t + \gamma_t^I} + \frac{N_U}{\gamma_t^U} \right)^{-1}, \quad \omega_t := \frac{f_t}{\lambda_t + \gamma_t^I}, \quad \lambda_t := \frac{\partial P_t}{\partial \theta_t^I} = \frac{\gamma_t^U}{N_U} > 0, \tag{23}$$

where $\gamma_t^i > 0$, $g_t^i < 0$, and $f_t^i > 0$ for $i \in \{I, U\}$ and $t < T$ can be computed recursively using equations (A-132), (A-134), (A-140), and (A-144) in the Appendix. Their values at $t = T$ are $\gamma_T^i = A^i \sigma_{V,T}^i$ and $g_T^i = f_T^i = 0$.

2. The optimal holdings at t are

$$\theta_t^I = \frac{P_t^I - P_t}{\gamma_t^I + \lambda_t}, \quad \theta_t^U = \frac{P_t^U - P_t}{\gamma_t^U}. \tag{24}$$

3. The value functions in equilibrium are

$$\begin{aligned} J_t^I &= -\rho_t^I e^{-A^I \left[W_t^I + \frac{1}{2} (\Phi_t^I)^\top M_t^I \Phi_t^I + (C_t^I)^\top \Phi_t^I \theta_{t-1}^I + \frac{1}{2} m_t^I (\theta_{t-1}^I)^2 \right]}, \\ J_t^{U,j} &= -\rho_t^U e^{-A^U \left[W_t^{U,j} + \frac{1}{2} (\Phi_t^U)^\top M_t^U \Phi_t^U + (C_t^U)^\top \Phi_t^U \theta_{t-1}^I + \frac{1}{2} m_t^U (\theta_{t-1}^I)^2 \right]}, \end{aligned} \tag{25}$$

where the state vectors are defined as $\Phi_t^I := (\hat{V}_t^I \;\; X \;\; \hat{X}_t^U \;\; \Theta)^T$, and $\Phi_t^U := (\hat{V}_t^U \;\; \hat{X}_t^U \;\; \Theta)^T$, the coefficients ρ_t^i, M_t^i, m_t^i, and C_t^i can be computed recursively using equations (A-151)-(A-154) and their values at $t = T$ are as given in equations (A-105) and (A-107) in the Appendix.

Proof. See Appendix A.8. □

Proposition 6 suggests that the equilibrium price at t increases in the reservation value of both informed (P_t^I) and uninformed (P_t^U) traders, and decreases in the total supply of the security. The reservation value represents a critical price threshold: investors will long the risky security if the market price is below this threshold, and short it if the price exceeds it, as detailed in equation (24).

Equation (22) implies that, at period T, the reservation values for informed and uninformed traders are:

$$P_T^I := \hat{V}_T^I - \mu_T X + \lambda_T \theta_{T-1}^I, \quad P_T^U := \hat{V}_T^U. \tag{26}$$

For an informed trader, the reservation value of the asset increases with both the composite signal $\hat{V}_T^I - \mu_T X_T$ and the previous inventory θ_{T-1}^I due to the price impact. An uninformed trader's reservation value increases with his estimate of the asset's liquidation value, $\hat{V}_T^U$. As the final payoff V is realized at period $T+1$, at period T informed traders adjust their positions based on their updated payoff estimates and the realized hedging demand, while uninformed traders make adjustments according to their updated payoff estimates.

For $t < T$, the reservation values of both traders are influenced by the informed trader's composite signal $\hat{V}_t^I - \mu_t X_t$ and the uninformed traders' estimate $\hat{V}_t^U$. This indicates that each trader's reservation value is affected by other traders' estimates due to the presence of both long-term stock positions and short-term speculative positions in their optimal demand. The former relies on the trader's estimate of the asset's final payoff $\hat{V}_t^i$, while the latter is based on the trader's estimate of future trading prices. Since these future prices are influenced by both groups of traders, the reservation price at period t for each trader depends on both $\hat{V}_t^I - \mu_t X_t$ and $\hat{V}_t^U$. By applying the market-clearing condition and the relationship $\hat{V}_t^I - \mu_t X = \hat{V}_t^U - \mu_t \hat{X}_t^U$, we can show that traders' reservation values can be written as in equation (22).

4.2 The Effects on Cost of Capital and Liquidity Dynamics

Proposition 7. *In an imperfectly competitive setting, the precision of future information influences current prices and the quantities of stock holdings, as the coefficients are recursively solved using backward induction.*

Proof. The proof follows directly from Proposition 6. □

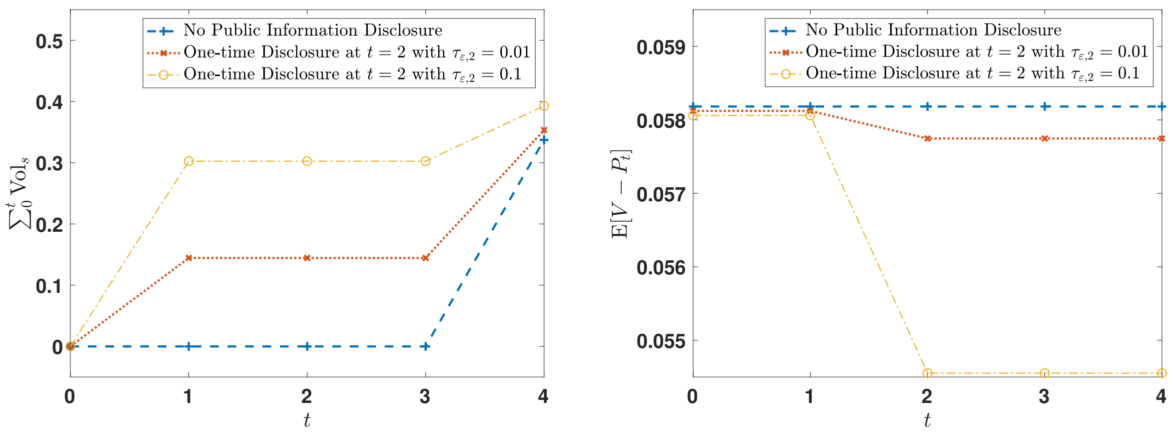
Proposition 7 implies that the equilibrium price P_t and risk premium $\mathrm{E}[V - P_t]$ in an imperfect market are influenced by the precisions of future public information. From Proposition 6, the expected risk premium is:

$$\begin{aligned} \mathrm{E}[V - P_t] &= \left(f_t + \omega_t f_t^I + (1-\omega_t) f_t^U\right)\Theta - \lambda_t \omega_t \mathrm{E}[\theta_{t-1}^I] \\ &= \frac{\left((f_t^I + f_t^U + \gamma_t^I)\lambda_t + f_t^U \gamma_t^I\right)\Theta + N_U \lambda_t^2 \mathrm{E}[\theta_{t-1}^U]}{\gamma_t^I + 2\lambda_t}. \end{aligned} \tag{27}$$

Equation (27) implies that the risk premium is driven by two primary factors. The first is the uncertainty surrounding the fundamental values of the asset, the asymmetry of information, and the uncertainty of liquidity shocks. Specifically, the risk premium demanded by uninformed traders increases when the values of τ_V, τ_η, or τ_X decrease. The second factor, distinguishing an imperfectly competitive market from a competitive one, is the role of price impact costs and traders' inventories, which are affected by the precision and frequency of upcoming public information releases.

As shown in the right panel of Figure 7, in an imperfectly competitive market, the risk premium begins to decline even before any public information is released. The anticipation of forthcoming public disclosures reduces not only the future risk premium but also the current one. This occurs because informed traders, anticipating future disclosures, strategically shift part of their trading—both for private information and hedging purposes—ahead of the announcements to minimize total price impact costs, as explained in more detail below.

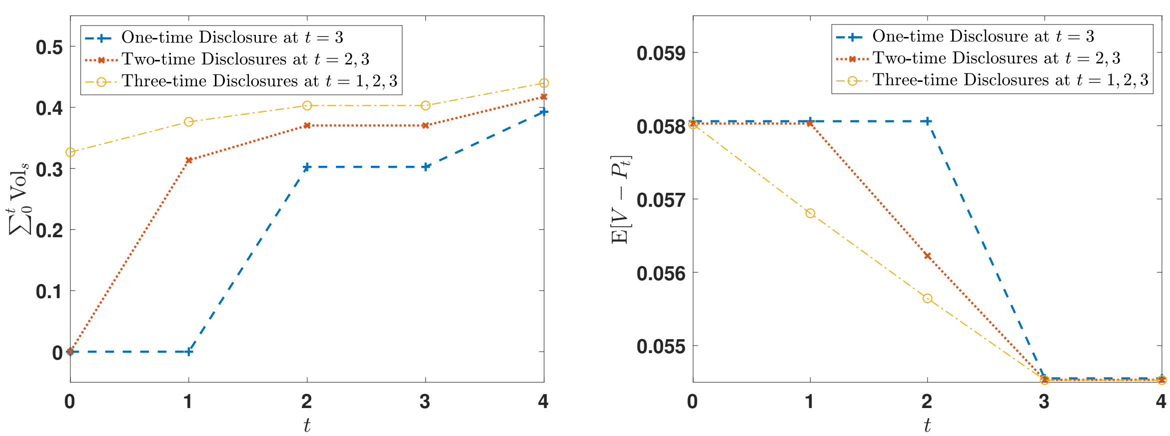
Figure 7 further illustrates that information disclosure leads to an increase in total trading volume (the size effect) and a reduction in the risk premium (the price effect). Since infor-



(a) Cumulative Trading Volume

(b) Risk Premium

Figure 7. Cumulative Trading Volume (left) and Risk Premium (right) with Varying Information Quantity. Parameters: $T = 4$, $\tau_T = 1$, $N_I = 1$, $N_U = 10$, $A^I = A^U = 1$, $\tau_V = 1$, $\tau_\eta = 1$, $\tau_X = 1.1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$.



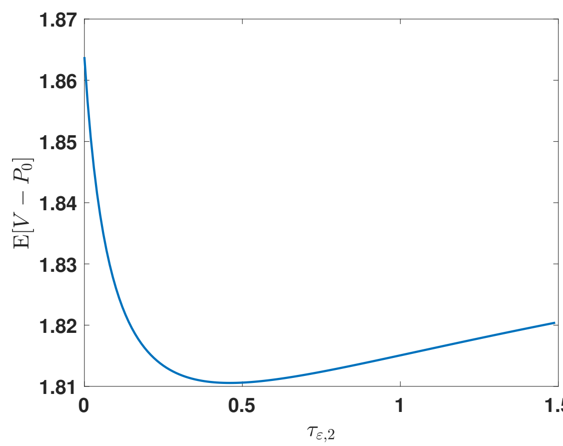
(a) Cumulative Trading Volume

(b) Risk Premium

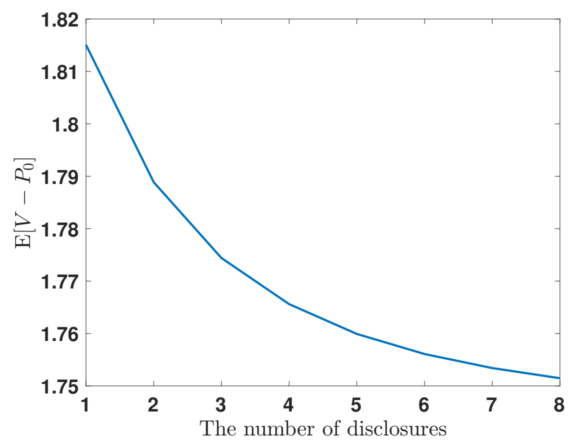
Figure 8. Cumulative Trading Volume (left) and Risk Premium (right) with Varying Information Disclosure Frequency. Parameters: $T = 4$, $\tau_T = 0.1$, $N_I = 1$, $N_U = 10$, $A^I = A^U = 1$, $\tau_V = 1$, $\tau_\eta = 1$, $\tau_X = 1.1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$. Dashed curve represents scenarios with a single disclosure at $t = 3$, with $\tau_{\varepsilon,3} = \tau_T$; dotted curve represents two disclosures at $t = 2, 3$, with $\tau_{\varepsilon,2} = \tau_{\varepsilon,3} = \tau_T/2$; dot-dashed curve represents three information disclosures with $\tau_{\varepsilon,1} = \tau_{\varepsilon,2} = \tau_{\varepsilon,3} = \tau_T/3$.

mation disclosure decreases the level of adverse selection, the size effect generally outweighs the price effect. Thus, information disclosure can still enhance risk-sharing. When trading is predominantly driven by hedging demands, the increase in risk-sharing due to information disclosure is relatively limited because adverse selection is less of an issue in these scenarios. In contrast, when trading is primarily motivated by private information, public information disclosure can significantly mitigate the adverse selection effect and thereby considerably enhance risk-sharing.

Similar to the competitive scenario, Figure 8 demonstrates that dividing the information into multiple pieces, rather than releasing it all at once, leads to an increase in total trading volume. Moreover, when information is released gradually, trades occur at prices that reflect a higher average risk premium, since the risk premium declines more slowly over time. Consequently, the average risk premium associated with gradual information disclosure increases, accompanied by a rise in total trading volume. Hence, a gradual release of public information can still enhance risk-sharing.



(a) Information Quantity

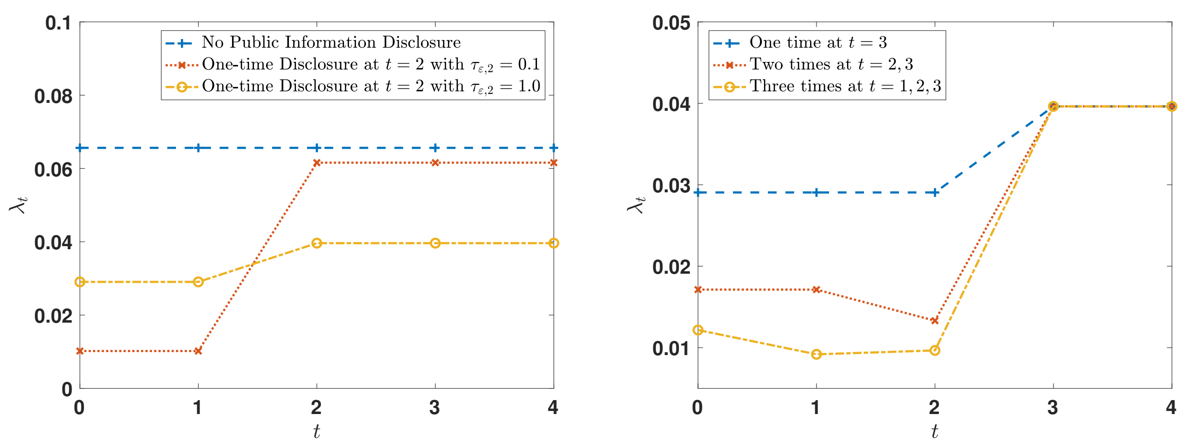


(b) Disclosure Frequency

Figure 9. The Changes of Initial Cost of Capital with Different Information Quantity (left) and Disclosure Frequency (right). Parameters: $T = 10$, $N_I = 1$, $N_U = 5$, $A^I = 1$, $A^U = 2$, $\tau_V = 1$, $\tau_\eta = 1$, $\tau_X = 1$, $\bar{V} = 1$, $\Theta = 10$, and $\theta^I_{-1} = \theta^U_{-1}$. Panel (a): One-time information disclosure at $t = 2$ with different precision. Panel (b): Fixed amount of information $\tau_T = 1$ with different number of disclosures.

The left panel of Figure 9 plots the cost of capital at time 0 against the precision of future information disclosure. Compared to the case with no disclosure (i.e., $\tau_{\varepsilon,2} = 0$), future information disclosure reduces the initial cost of capital. However, this reduction is not monotonic in the precision of future information: the cost of capital initially decreases as precision increases, but then slightly rises. The right panel of Figure 9 shows that the cost of capital at time 0 decreases monotonically with the frequency of information disclosure.

To understand the intuition behind these results, we first explain how public information disclosures affect the dynamics of market liquidity. As illustrated in Figure 10, public disclosures reduce the per-unit price impact, even though they tend to increase total trading volume. Anticipating future public announcements, informed traders strategically shift part of their trading—both for private information and hedging—before the release and trade the rest afterward to minimize overall trading costs. As a result, more frequent information disclosure helps smooth market liquidity over time.



(a) Information Quantity (b) Disclosure Frequency

Figure 10. Price Impact with Varying Information Quantity (left) and Disclosure Frequency (right). Parameters: $T=4$, $N_I=1$, $N_U=10$, $A^I=A^U=1$, $\tau_V=1$, $\tau_\eta=0.1$, $\tau_X=1.1$, $\bar{V}=1$, $\Theta=1$, and $\theta^I_{-1}=\theta^U_{-1}$. Panel (a): One-time information disclosure at $t=2$ with different precision. Panel (b): $\tau_T=1$, dashed curve represents scenarios with a single disclosure at $t=3$, with $\tau_{\varepsilon,3}=\tau_T$; dotted curve represents two disclosures at $t=2,3$, with $\tau_{\varepsilon,2}=\tau_{\varepsilon,3}=\tau_T/2$; dot-dashed curve represents three information disclosures with $\tau_{\varepsilon,1}=\tau_{\varepsilon,2}=\tau_{\varepsilon,3}=\tau_T/3$.

From Equation (27), the cost of capital at time 0 is given by:

$$\mathrm{E}[V-P_0]=\frac{\left((f^I_0+f^U_0+\gamma^I_0)\lambda_0+f^U_t\gamma^I_0\right)\Theta+N_U\lambda_0^2\theta^U_{-1}}{\gamma^I_0+2\lambda_0}, \tag{28}$$

where f_0^I, f_0^U, and γ_0^I are endogenous parameters at time 0 defined in Proposition 6 and θ_{-1}^U is the uninformed traders' initial endowment of the risky asset. We show numerically, across a wide range of parameter values, that the cost of capital $\mathrm{E}[V - P_0]$ increases with the price impact cost at time 0 (λ_0).

Thus, the prospect of future public information generally incentivizes informed traders to shift part of their trading ahead of the announcements, thereby reducing the price impact. As a result, the cost of capital prior to the public disclosure decreases.

As future public information becomes more precise, traders are further incentivized to shift more of their trading both for private information and hedging demand to periods before the public disclosure. This shift occurs because greater precision reduces uncertainty and decreases the hedging benefits. As a result, the price impact before the public disclosure begins to rise, as shown in Panel (a) of Figure 10. Therefore, the decline in the initial cost of capital becomes non-monotonic in the precision of future public information.

In contrast, when total precision is held constant, increasing the frequency of future public disclosures reduces the cost of capital monotonically. More frequent disclosure leads informed traders to spread their trading more evenly over time, thereby further reducing the price impact, as shown in Panel (b) of Figure 10. As a result, higher disclosure frequency results in a monotonic decline in the initial cost of capital.

4.3 The Effects on Welfare

This section examines the impact of information disclosure on the welfare of market participants in an imperfectly competitive market.

In a competitive market, trading by any individual investor does not affect market prices. The welfare of both informed and uninformed traders is entirely dependent on the level of risk-sharing achieved. In such a market, both trader types can reach peak welfare concurrently. Conversely, in an imperfectly competitive market, the informed trader imposes price impact while trading. This means that trades by informed traders move prices in a way that is disadvantageous to them but favorable to the uninformed traders who trade on the opposite side. Therefore, the price impact essentially acts as a compensation from the informed trader to the uninformed traders for facilitating trades. The presence of price impact incurs extra costs for the informed trader, reducing his welfare while increasing that of the uninformed traders. Therefore, the effect of information disclosure on traders' welfare differs from that in a competitive market, distinctly affecting informed and uninformed traders.

4.3.1 The Effects of Quantity of Disclosed Information

In this section, we explore the effects of a one-time release of varying amounts of public information. Initially, as more information is disclosed, there's an increase in risk-sharing. However, after reaching a certain threshold, similar to competitive markets, the extent of risk sharing begins to decline.

Similarly, the cost of price impact initially rises with the increase in disclosed information, a consequence of amplified trading needs. As more information is released, the price impact costs begin to decrease due to reduced uncertainty. The peaks of both the scope of risk-sharing and the total price impact costs increase with the precision of the informed trader's private information.

For uninformed traders, who benefit from both risk-sharing and the effects of price impact costs, welfare improves as more public information is disclosed, but only to a certain extent. Beyond this point, their welfare starts to decline, irrespective of the precision levels of private information and liquidity shocks, as depicted in the left panels of Figures 11, 12, and 13. The peak of welfare for uninformed traders is influenced by these precisions; notably, when the informed trader observes more precise private information, the peak of welfare for uninformed traders is reached at a higher level of public information precision.

The impact of information disclosure on the welfare of the informed trader is complex, depending on the interplay between two main factors: the extent of risk-sharing and the level of price impact costs. Public information disclosure leads to increased trading volume, which

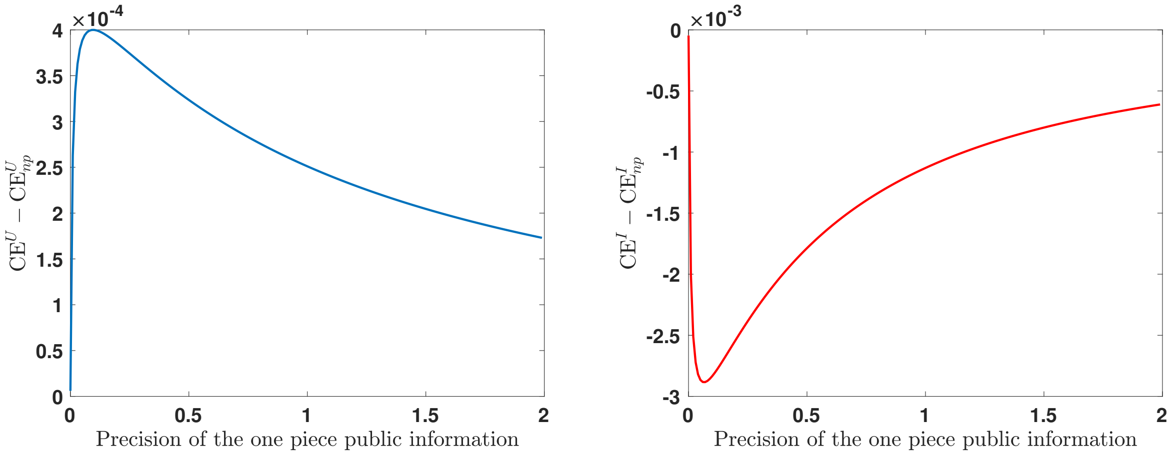


Figure 11. Changes in Certainty Equivalent Welfare for Both Traders (Δ^i_{CE}) Versus the Precision of Public Information. Parameters: $\tau_\eta = 0.1$, $\tau_X = 1.1$, $N_I = 1$, $N_U = 10$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$.

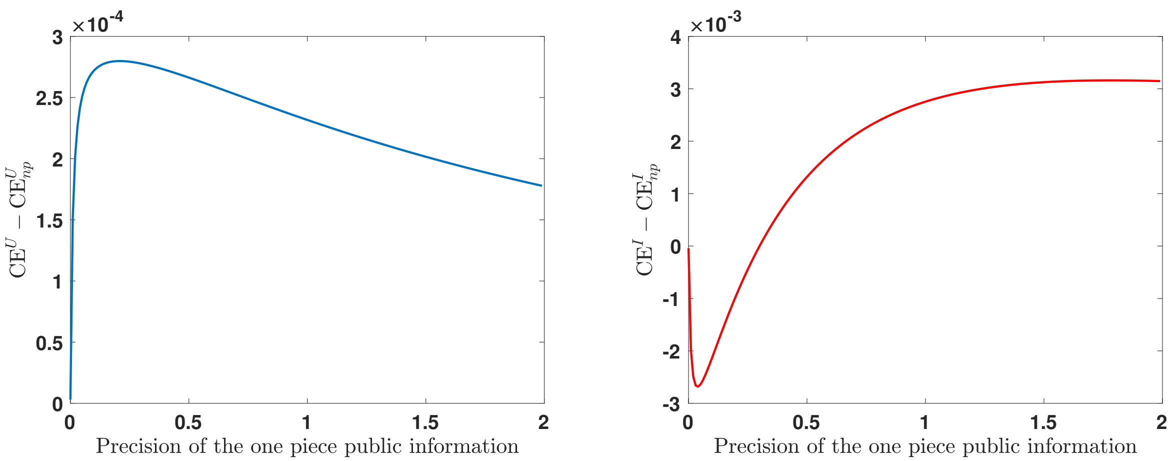


Figure 12. Changes in Certainty Equivalent Welfare for Both Traders (Δ^i_{CE}) Versus the Precision of Public Information. Parameters: $\tau_\eta = 0.5$, $\tau_X = 1.1$, $N_I = 1$, $N_U = 10$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$.

enhances risk-sharing opportunities but also raises total price impact costs. For an informed trader whose trading is primarily driven by hedging needs, the rise in price impact costs outweighs the benefits of increased risk sharing, negatively affecting their welfare. This effect is depicted in the right panel of Figure 11. On the other hand, if the informed trader's trading is mainly based on private information, the expanded scope of risk sharing offsets the increased price impact costs, making information disclosure beneficial to their welfare, as shown in the

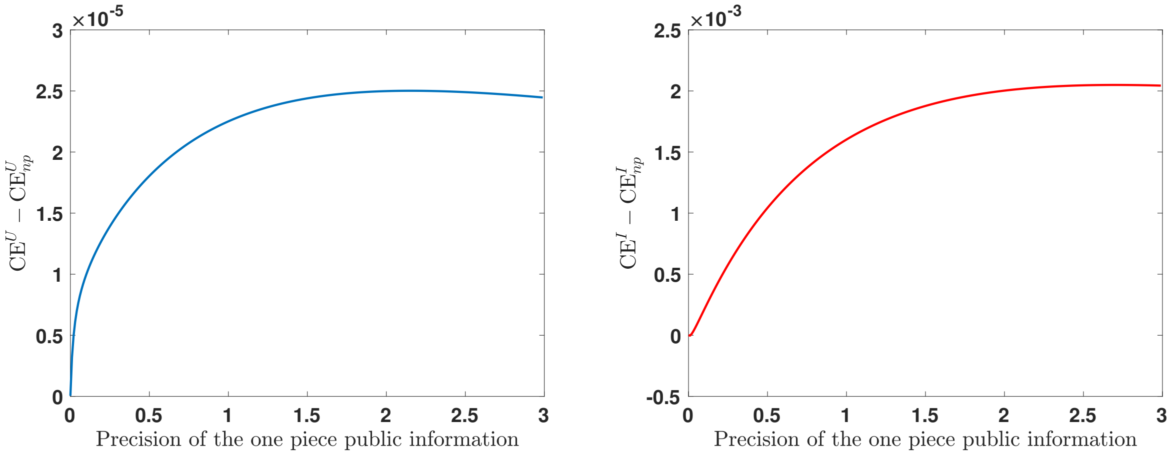


Figure 13. Changes in Certainty Equivalent Welfare for Informed Traders (Δ^i_{CE}) Versus the Precision of Public Information. Parameters: $\tau_\eta = 2$, $\tau_X = 2$, $N_I = 1$, $N_U = 10$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$.

right panel of Figure 13. In situations where hedging needs and trading on private information are more balanced, the overall effect depends on the precision of the public information. High precision in public information tips the balance in favor of expanded risk sharing, whereas lower precision makes the heightened price impact costs more significant, as shown in the right panel of Figure 12.

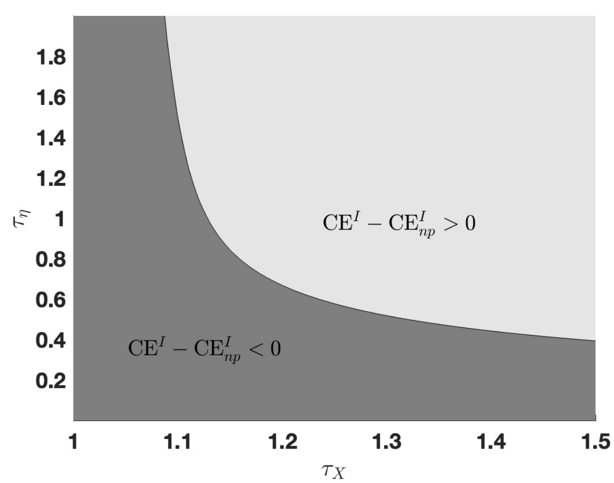


Figure 14. Changes in Certainty Equivalent Welfare for Informed Traders (Δ^I_{CE}) with Varying τ_η and τ_X. Parameters: $\tau_T = 1$, $\tau_T = 1$, $N_I = 1$, $N_U = 5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$.

The impact of information disclosure on the informed trader's welfare is influenced by their trading motives, given the quantity of disclosed information. As illustrated in Figure 14, when τ_X is relatively small, as shown in the dark shaded region of the figure, indicating a substantial liquidity shock, the public information disclosure tends to negatively impact the informed trader. In contrast, when τ_X is large, indicating a less pronounced liquidity, and the informed trader's trading is mainly driven by private information (indicated by a large τ_η), the informed trader generally benefits from the release of public information.

4.3.2 The Effects of Information Disclosure Frequency

We next examine the effects of disclosure frequency on the welfare of both informed and uninformed traders, while keeping the total quantity (τ_T) of disclosed information constant. For uninformed traders, the impact of more frequent disclosures is unequivocally positive. This is because incremental information release broadens the risk-sharing scope and elevates the overall price impact cost of the informed trader. Consequently, the welfare of uninformed traders improves with the increase in the number of disclosures, regardless of the market conditions, as demonstrated in the left panels of Figure 15, Figure 16, and Figure 17. This implies that a gradual information disclosure always benefits uninformed market participants.

The impact of information disclosure frequency on the welfare of informed trader is more complicated and largely influenced by his trading motivation. If the informed trader's trading is primarily driven by hedging demands, his welfare tends to decrease with more frequent disclosures, as illustrated in the right panel of Figure 15. In such cases, the increase in overall price impact costs due to gradual disclosure outweighs the benefits of enhanced risk sharing. Conversely, when the informed trader's trading is primarily driven by his private information, his welfare improves with an increase in disclosure frequency, as depicted in the right panel of Figure 17. In this case, the benefits from enhanced risk sharing due to gradual disclosure outweigh the increased price impact costs. For the informed trader with more balanced trading from hedging and private information, there is an initial increase in welfare with increasing disclo-

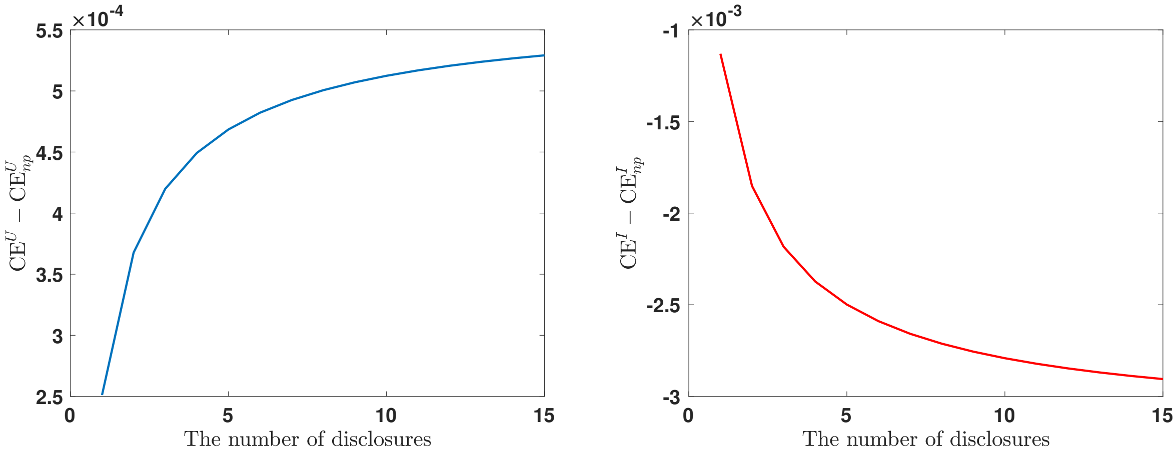


Figure 15. Changes in Certainty Equivalent Welfare for Informed Traders (Δ^i_{CE}) Versus Varying Information Disclosure Frequency. Parameters: $\tau_\eta = 0.1$, $\tau_X = 1.1$, $\tau_T = 1$, $N_I = 1$, $N_U = 10$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$.

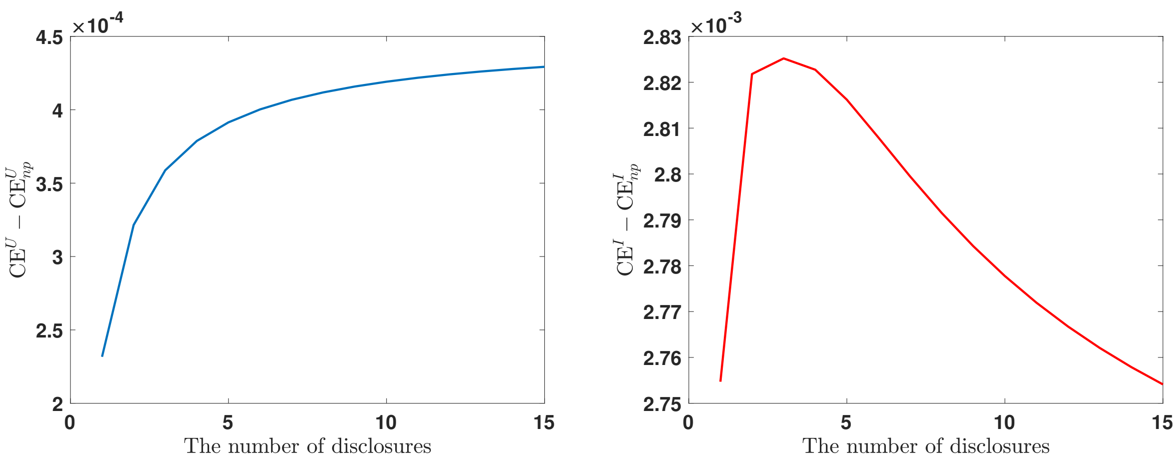


Figure 16. Changes in Certainty Equivalent Welfare for Informed Traders (Δ^i_{CE}) Versus Varying Information Disclosure Frequency. Parameters: $\tau_\eta = 0.5$, $\tau_X = 1.1$, $\tau_T = 1$, $N_I = 1$, $N_U = 10$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$.

sure frequency, followed by a decrease. This indicates the presence of an optimal frequency of disclosure for such traders, where the expanded risk sharing scope and heightened price impact costs balance each other out, as demonstrated in the right panel of Figure 16.

In summary, unlike in a competitive market, the effects of disclosure frequency on the welfare of informed and uninformed traders are different in an imperfectly competitive market. Uninformed traders consistently benefit from more frequent information releases, due to broader

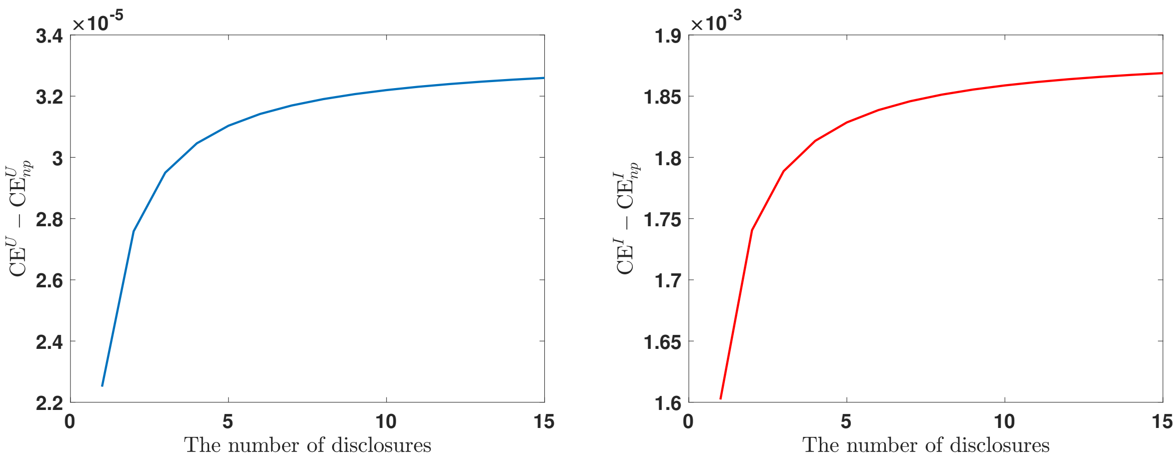


Figure 17. Changes in Certainty Equivalent Welfare for Informed Traders (Δ^i_{CE}) Versus Varying Information Disclosure Frequency. Parameters: $\tau_\eta = 2$, $\tau_X = 2$, $\tau_T = 1$, $N_I = 1$, $N_U = 10$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\bar{V} = 1$, $\Theta = 1$, and $\theta^I_{-1} = \theta^U_{-1}$.

risk-sharing and higher price impact costs imposed on informed traders. However, the impact on informed traders varies: those driven by hedging demands see a decline in welfare due to increased price impact costs, whereas traders driven by private information experience an increase in welfare as the benefits of extended risk sharing outweigh the costs. For traders with balanced dual motives, there is an initial welfare improvement, followed by a decline, indicating an optimal level of disclosure frequency.

4.4 Model Implications

Our model illuminates the impact of institutional investor ownership on a firm's approach to information disclosure. It reveals that firms with highly concentrated institutional ownership—characterized by a few investors holding a large share of the equity—might favor less frequent disclosure. This tendency likely stems from the significant voting power and monopolistic influence these investors possess, which allows them to substantially influence the firm's disclosure strategies.

In contrast, firms with a more competitive and diverse institutional investor base are likely to benefit from a gradual approach to information disclosure. These firms should aim to release information progressively as it becomes available, rather than infrequently.

Therefore, our study highlights the critical need to consider the composition of traders, their trading motivations, and the level of market competitiveness when analyzing the effects of a firm's disclosure practices. Such an understanding is crucial for regulators tasked with fostering risk-sharing among traders and enhancing the overall welfare of market participants.

5 Conclusion

This paper studies the effects of information disclosure frequency on cost of capital and the welfare of market participants across various market settings. It distinguishes between competitive and non-competitive markets, exploring the impact of gradual information release on market dynamics and participants' welfare. Our paper highlights that the implications of information disclosure frequency critically depend on the composition of investors, traders' trading incentives, and the degree of market competitiveness.

In competitive market environments, future public disclosure does not affect the initial cost of capital. Both informed and uninformed traders benefit from gradual information disclosure, as it mitigates uncertainty over time and enhances risk-sharing. This suggests that, in competitive markets, firms should adopt a gradual information release strategy.

In contrast, in non-competitive markets, where informed traders generate price impact through their trading, increasing disclosure frequency reduces the initial cost of capital, as informed traders spread their trading over time. While more frequent disclosure continues to benefit uninformed traders, it may negatively impact informed traders, who trade based on both private information and liquidity shocks, as higher disclosure frequency increases the cumulative costs of hedging. Consequently, our paper suggests that in scenarios involving institutional investors with monopolistic influence, firms may disclose information less frequently.

References

Armstrong, C. S., K. Balakrishnan, and D. Cohen. 2012. Corporate governance and the information environment: Evidence from state antitakeover laws. *Journal of Accounting and Economics* 53:185–204.

Baik, B., J.-M. Kim, K. Kim, and S. Patro. 2020. Hedge fund ownership and voluntary disclosure. *Review of Quantitative Finance and Accounting* 54:877–910.

Bird, A., and S. A. Karolyi. 2016. Do institutional investors demand public disclosure? *The Review of Financial Studies* 29:3245–3277.

Boone, A. L., and J. T. White. 2015. The effect of institutional ownership on firm transparency and information production. *Journal of Financial Economics* 117:508–533.

Brennan, M., and H. Cao. 1996. Information, trade, and derivative securities. *The Review of Financial Studies* 9:163–208.

Bushee, B. J., and C. F. Noe. 2000. Corporate disclosure practices, institutional investors, and stock return volatility. *Journal of accounting research* pp. 171–202.

Chapman, K. L., N. Reiter, H. D. White, and C. D. Williams. 2019. Information overload and disclosure smoothing. *Review of Accounting Studies* 24:1486–1522.

Chen, J., and M. J. Jung. 2016. Activist hedge funds and firm disclosure. *Review of Financial Economics* 29:52–63.

Dugast, J., and T. Foucault. 2018. Data abundance and asset price informativeness. *Journal of Financial economics* 130:367–391.

Ge, X., P. Bilinski, and A. Kraft. 2021. Institutional blockholders and voluntary disclosure. *European Accounting Review* 30:1013–1042.

Gigler, F., and T. Hemmer. 1998. On the frequency, quality, and informational role of mandatory financial reports. *Journal of Accounting Research* 36:117–147.

Goldstein, I., Y. Li, and L. Yang. 2014. Speculation and hedging in segmented markets. *Review of Financial Studies* 27:881–922.

Goldstein, I., and L. Yang. 2019. Good disclosure, bad disclosure. *Journal of Financial Economics* 131:118–138.

Higgins, K. 2014. Disclosure effectiveness: Remarks before the American bar association business law section spring meeting. *Security and Exchange Commission* .

Hirshleifer, J. 1978. The private and social value of information and the reward to inventive activity. In *Uncertainty in economics*, pp. 541–556. Elsevier.

Jiang, X., X. Wei, L. Yang, and X. Zhang. 2023. The effect of frequent disclosure on price informativeness and cost of capital in a Keynesian beauty contest setting. *Working Paper* .

Jo, H., and Y. Kim. 2007. Disclosure frequency and earnings management. *Journal of Financial Economics* 84:561–590.

Liptser, R. S., A. N. Shiriaev, and A. N. Shiryaev. 2001. *Statistics of random processes II: II. Applications*, vol. 2. Springer Science & Business Media.

McMullin, J. L., B. P. Miller, and B. J. Twedt. 2019. Increased mandated disclosure frequency and price formation: Evidence from the 8-K expansion regulation. *Review of Accounting Studies* 24:1–33.

Rostek, M., and M. Weretka. 2015. Dynamic thin markets. *The Review of Financial Studies* 28:2946–2992.

Vayanos, D., and J. Wang. 2012. Liquidity and asset returns under asymmetric information and imperfect competition. *Review of Financial Studies* 25:1339–1365.

White, M. 2013. The Importance of independence. Securities and Exchange Commission.

A Appendix

A.1 Proof of Lemma 1

To derive the filtering equations, we use the results in the following lemma, the proof of which can be found in Liptser, Shiriaev, and Shiryaev (2001).

Lemma 2. *Let*

$$x_t = A_t x_{t-1} + B_t \varepsilon_{x,t}, \quad y_t = H_t x_t + \varepsilon_{y,t}, \quad t = 1, 2, \ldots$$

x_t is the n-vector of state variables at t, y_t is the m-vector of observations at t. A_t, B_t and H_t are , respectively, $(n \times n)$, $(n \times k)$, $(m \times n)$ constant matrices. $\{\varepsilon_{x,t}, t = 1, 2, \ldots\}$ and $\{\varepsilon_{y,t}, t = 1, 2, \ldots\}$ are respectively a k-vector and an m-vector white Gaussian sequence. $\varepsilon_{x,t} \sim \mathcal{N}(0, Q_t)$, $\varepsilon_{y,t} \sim \mathcal{N}(0, R_t)$, and $x_0 \sim \mathcal{N}(\bar{x}_0, \Sigma_{x,0})$. x_0, $\{\varepsilon_{x,t}\}$ and $\{\varepsilon_{y,t}\}$ are independent. Let

$$\hat{x}_t = \hat{x}_{t|t} = \mathrm{E}[x_t | y_\tau : 1 \le \tau \le t], \quad O_t = O_{t,t} = \mathrm{E}[(x_t - \hat{x}_t)(x_t - \hat{x}_t)^\top | y_\tau : 1 \le \tau \le t].$$

Then,

$$\begin{aligned} \hat{x}_t &= A_t \hat{x}_{t-1} + K_t (y_t - H_t A_t \hat{x}_{t-1}), \quad O_t = (I_n - K_t H_t)(A_t O_{t-1} A_t^\top + B_t Q_t B_t^\top), \\ K_t &= (A_t O_{t-1} A_t^\top + B_t Q_t B_t^\top) H_t^\top [H_t (A_t O_{t-1} A_t^\top + B_t Q_t B_t^\top) H_t^\top + R_t]^{-1}. \end{aligned}$$

where I_n is the $(n \times n)$ identity matrix.

We can now solve for the informed filters $\hat{V}_t^I$ by applying this lemma. Make the following substitution: $x_t = x_0 = V$, $\varepsilon_{x,t} = 0$, $y_t = v_t$, $\varepsilon_{y,t} = \varepsilon_t$, so the constant matrices are

$$A_t = 1, \quad H_t = 1, \quad Q_t = 0, \quad R_t = \tau_{\varepsilon,t}^{-1}. \tag{A-1}$$

By definition $\hat{x}_t = \hat{V}_t^I$, $O_t = o_{V,t}^I$, so by the above lemma, we have $o_{V,t}^I = (1 - K_t^I) o_{V,t-1}^I$ and

$$\frac{1}{o_{V,t}^I} = \frac{1}{o_{V,t-1}^I} + \tau_{\varepsilon,t}, \qquad K_t^I = \frac{o_{V,t-1}^I}{o_{V,t-1}^I + \tau_{\varepsilon,t}^{-1}} = o_{V,t}^I \tau_{\varepsilon,t}, \tag{A-2}$$

i.e., K_t^I can be expressed in terms of $o_{V,t-1}^I$ and $\{o_{V,t}^I\}$ can be expressed recursively with the initial value $o_{V,0}^I = \tau_V^{-1}$. Now we can express informed investors' expectation as follows:

$$\hat{V}_t^I = \hat{V}_{t-1}^I + K_t^I e_t^I, \qquad e_t^I = s_t - \hat{V}_{t-1}^I \sim \mathcal{N}(0, \Sigma_t^I), \text{ for } t = 1, \ldots, T, \tag{A-3}$$

where $\Sigma_t^I = o_{V,t-1}^I + \tau_{\varepsilon,t}^{-1}$ and $e_0^I = v - \bar{V}$.

We can now solve for the uninformed filters $\hat{V}_t^U$ and $\hat{X}_t^U$ by applying this lemma. Make the following substitution:

$$x_t = x_0 = \begin{pmatrix} V \\ X \end{pmatrix}, \quad \varepsilon_{x,t} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}; \quad y_t = s_t, \quad \varepsilon_{y,t} = \varepsilon_t. \tag{A-4}$$

The constant matrices are

$$A_t = I_2 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad H_t = \begin{pmatrix} 1 & -h_t \end{pmatrix}, \quad Q_t = 0, \quad R_t = \tau_{\varepsilon,t}^{-1}. \tag{A-5}$$

Since $s_t = v_t - h_t X = V - h_t X + \varepsilon_t$, by definition

$$\hat{x}_t = \begin{pmatrix} \hat{V}_t^U \\ \hat{X}_t^U \end{pmatrix}, \quad O_t = \begin{pmatrix} o_{V,t}^U & o_{VX,t}^U \\ o_{VX,t}^U & o_{X,t}^U \end{pmatrix},$$

where $o_{VX,t}^U = \mathrm{Cov}_t^U(V, X)$. Therefore, by the above lemma,

$$K_t^U = \frac{1}{\Sigma_t^U} O_{t-1} \begin{pmatrix} 1 \\ -h_t \end{pmatrix} = \frac{1}{\Sigma_t^U} \begin{pmatrix} o_{V,t-1}^U - h_t o_{VX,t-1}^U \\ o_{VX,t-1}^U - h_t o_{X,t-1}^U \end{pmatrix}, \tag{A-6}$$

$$O_t = O_{t-1} - \frac{1}{\Sigma_t^U} O_{t-1} \begin{pmatrix} 1 & -h_t \\ -h_t & h_t^2 \end{pmatrix} O_{t-1}, \tag{A-7}$$

where Σ_t^U is a scalar determined by O_{t-1} and h_t

$$\Sigma_t^U = H_t O_{t-1} H_t^\top + R_t = o^U_{V,t-1} - 2h_t o^U_{VX,t-1} + h_t^2 o^U_{X,t-1} + \tau_{\varepsilon,t}^{-1} \equiv \mathrm{Var}^U_{t-1}(s_t). \tag{A-8}$$

Thus, the elements of O_t can be determined by the elements of O_{t-1}:

$$\begin{aligned} o^U_{V,t} &= o^U_{V,t-1} - \frac{1}{\Sigma_t^U}(o^U_{V,t-1} - h_t o^U_{VX,t-1})^2, \qquad o^U_{X,t} = o^U_{X,t-1} - \frac{1}{\Sigma_t^U}(o^U_{VX,t-1} - h_t o^U_{X,t-1})^2, \\ o^U_{VX,t} &= o^U_{VX,t-1} - \frac{1}{\Sigma_t^U}(o^U_{V,t-1} - h_t o^U_{VX,t-1})(o^U_{VX,t-1} - h_t o^U_{X,t-1}). \end{aligned}$$

Reversely, O_{t-1} can be determined by O_t as

$$o^U_{V,t-1} = \frac{[o^U_{V,t} o^U_{X,t} - (o^U_{VX,t})^2] h_t^2 - o^U_{V,t}\tau_{\varepsilon,t}^{-1}}{o^U_{V,t} - 2o^U_{VX,t} h_t + o^U_{X,t} h_t^2 - \tau_{\varepsilon,t}^{-1}}, \tag{A-9}$$

$$o^U_{X,t-1} = \frac{o^U_{V,t} o^U_{X,t} - (o^U_{VX,t})^2 - o^U_{X,t}\tau_{\varepsilon,t}^{-1}}{o^U_{V,t} - 2o^U_{VX,t} h_t + o^U_{X,t} h_t^2 - \tau_{\varepsilon,t}^{-1}}, \tag{A-10}$$

$$o^U_{VX,t-1} = \frac{[o^U_{V,t} o^U_{X,t} - (o^U_{VX,t})^2] h_t - o^U_{VX,t}\tau_{\varepsilon,t}^{-1}}{o^U_{V,t} - 2o^U_{VX,t} h_t + o^U_{X,t} h_t^2 - \tau_{\varepsilon,t}^{-1}}. \tag{A-11}$$

Thus, if we take a guess of O^U_{T-1}, then all the $\{O^U_t\}$ for $t < T-1$ can be computed recursively, and $\{O^U_{T-1}\}$ can be pinned down recursively with initial value $O_0 = \begin{pmatrix} \tau_V^{-1} & 0 \\ 0 & \tau_X^{-1} \end{pmatrix}$.

Moreover,

$$\Sigma_t^U \equiv \mathrm{Var}^U_{t-1}(s_t) = \frac{\tau_{\varepsilon,t}^{-2}}{\tau_{\varepsilon,t}^{-1} - (o^U_{V,t} - 2o^U_{VX,t} h_t + o^U_{X,t} h_t^2)}, \tag{A-12}$$

which can be proved positive.

Similarly, $K_t^U \equiv \begin{pmatrix} K^U_{V,t} \\ K^U_{X,t} \end{pmatrix}$ can be expressed by O_t^U:

$$K^U_{V,t} = \frac{o^U_{V,t} - h_t o^U_{VX,t}}{\tau_{\varepsilon,t}^{-1}}, \quad K^U_{X,t} = \frac{o^U_{VX,t} - h_t o^U_{X,t}}{\tau_{\varepsilon,t}^{-1}}, \quad K_t^I = o^I_{V,t}\tau_{\varepsilon,t}. \tag{A-13}$$

Thus, $\{O_t^U\}$ and $\{K_t^U\}$ for $t < T-1$ can be determined by the initial guess of O^U_{T-1} and $\{h_t\}$

(where $h_0 = h$ and $h_t = 0$ for $t = 1, ..., T$), and so does

$$\begin{pmatrix} \hat{V}_t^U \\ \hat{X}_t^U \end{pmatrix} = \begin{pmatrix} \hat{V}_{t-1}^U \\ \hat{X}_{t-1}^U \end{pmatrix} + \begin{pmatrix} K_{V,t}^U \\ K_{X,t}^U \end{pmatrix} [s_t - (\hat{V}_{t-1}^U - h_t \hat{X}_{t-1}^U)]. \tag{A-14}$$

Let $e_t^U = s_t - (\hat{V}_{t-1}^U - h_t \hat{X}_{t-1}^U)$. So $e_t^U \sim \mathcal{N}(0, \Sigma_t^U)$, where $\Sigma_t^U = o_{V,t-1}^U + \tau_{\varepsilon,t}^{-1} + h_t^2 \tau_X^{-1}$.

For $t = T, T-1, ..., 1$, the recursive expression of state variables is

$$H_t^I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & K_{X,t}^U \mu_{t-1} & 1 - K_{X,t}^U \mu_{t-1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad F_t^I = \begin{pmatrix} K_{V,t}^I \\ 0 \\ K_{X,t}^U \\ 0 \end{pmatrix}, \quad F_t^U = \begin{pmatrix} K_{V,t}^U \\ K_{X,t}^U \\ 0 \end{pmatrix}. \tag{A-15}$$

A.2 Proof of Proposition 1

To derive the optimal holdings and price, we use the following lemma.

Lemma 3. *Let u be an $n \times 1$ normal vector with mean $\bar{u}$ and covariance matrix Σ, A a scalar, B an $n \times 1$ vector, C an $n \times n$ symmetric matrix, I the $n \times n$ identity matrix, and $|M|$ the determinant of a matrix M. Then,*

$$\begin{aligned} \mathrm{E}_u \exp\{-\rho[A + B^\top u + \frac{1}{2} u^\top C u]\} = \frac{1}{\sqrt{|I + \rho C \Sigma|}} \exp\big\{ &- \rho[A + B^\top \bar{u} + \frac{1}{2} \bar{u}^\top C \bar{u} \\ &- \frac{1}{2} \rho (B + C\bar{u})^\top (\Sigma^{-1} + \rho C)^{-1} (B + C\bar{u})]\big\}. \end{aligned}$$

We solve the model backward. The final wealth of informed trader i is

$$W_{T+1}^{I,i} = W_T^{I,i} + \theta_T^{I,i}(V - P_T) + (V - \bar{V})X, \tag{A-16}$$

The expected utility of informed traders at $t = T$ is

$$\mathrm{E}_T^I[-e^{-A^I W_{T+1}^I}] = -e^{-A^I\left[W_T^{I,i} + \theta_T^{I,i}(\hat{V}_T^I - P_T) + (\hat{V}_T^I - \bar{V})X - \frac{1}{2} A^I o_{V,T}^I (\theta_T^{I,i} + X)^2\right]}. \tag{A-17}$$

Taking the first order derivative with respect to $\theta_T^{I,i}$ yields

$$\theta_T^{I,i} + X = \frac{\hat{V}_T^I - P_T}{A^I o_{V,T}^I}, \tag{A-18}$$

and the second order condition, $A^I o_{V,T}^I > 0$, is satisfied automatically.

Similarly, the final wealth of uninformed trader j is

$$W_{T+1}^{U,j} = W_T^{U,j} + \theta_T^{U,j}(V - P_T). \tag{A-19}$$

Thus, the expected utility of uninformed traders at $t = T$ is

$$\mathrm{E}_T^U[-e^{-A^U W_{T+1}^{U,j}}] = -e^{-A^U\left[W_T^{U,j} + \theta_T^{U,j}(\hat{V}_T^U - P_T) - \frac{1}{2}A^U o_{V,T}^U (\theta_T^{U,j})^2\right]}. \tag{A-20}$$

The first order condition with respect to $\theta_T^{U,j}$ yields

$$\theta_T^{U,j} = \frac{\hat{V}_T^U - P_T}{A^U o_{V,T}^U}, \tag{A-21}$$

and the second order condition, $A^U o_{V,T}^U > 0$, is satisfied automatically. By market clearing condition, $\sum_{i=1}^{N_I} \theta_T^{I,i} + \sum_{j=1}^{N_U} \theta_T^{U,j} = \Theta$, the equilibrium price can be written as

$$P_T = \omega_T \left(\hat{V}_T^I - \mu_T X\right) + (1 - \omega_T)\hat{V}_T^U - \omega_T \mu_T \frac{\Theta}{N_I}, \tag{A-22}$$

where

$$\mu_T = A^I o_{V,T}^I, \quad \omega_T = \frac{N_I/(A^I o_{V,T}^I)}{N_I/(A^I o_{V,T}^I) + N_U/(A^U o_{V,T}^U)}. \tag{A-23}$$

Since $\hat{V}_T^I - \mu_T X = \hat{V}_T^U - \mu_T \hat{X}_T^U$, the price at T can be written as

$$P_T = \hat{V}_T^U - \omega_T \mu_T (\hat{X}_T^U + \frac{\Theta}{N_I}) = \hat{V}_T^I - \mu_T X + (1 - \omega_T)\mu_T \hat{X}_T^U - \omega_T \mu_T \frac{\Theta}{N_I}, \tag{A-24}$$

and

$$\theta_T^{I,i} = \frac{\Theta}{N_I} - \frac{f_T}{N_I}(\hat{X}_T^U + \frac{\Theta}{N_I}), \quad \theta_T^{U,j} = \frac{f_T}{N_U}(\hat{X}_T^U + \frac{\Theta}{N_I}), \quad \text{where} \quad f_T = N_I(1 - \omega_T). \tag{A-25}$$

Substituting the equilibrium price into the expected utility functions, we have informed traders' value function at $t = T$ as a function of state variable Φ_T

$$
\begin{aligned}
J_T^{I,i} &= -e^{-A^I\left[W_T^{I,i}+\hat{V}_T^I X+\frac{1}{2}\mu_T(\theta_T^{I,i})^2-\bar{V}X-\frac{1}{2}\mu_T X^2\right]} \\
&= -\rho_T^I e^{-A^I\left[W_T^{I,i}+\hat{V}_T^I X+\frac{1}{2}m_T^I\phi_T^2+\frac{1}{2}\mu_T(1-\omega_T)(\hat{X}_T^U)^2\right]},
\end{aligned} \tag{A-26}
$$

where

$$
m_T^I = -\mu_T\omega_T(1-\omega_T) = -\mu_T\omega_T\frac{f_T}{N_I}, \quad \rho_T^I = e^{A^I\left[\bar{V}X+\frac{1}{2}\mu_T X^2-\frac{1}{2}\mu_T\omega_T(\frac{\Theta}{N_I})^2\right]}. \tag{A-27}
$$

Similarly, uninformed traders' value function at $t = T$ can be written as:

$$
J_T^{U,j} = -e^{-A^U\left[W_T^{U,j}+\frac{1}{2}\gamma_T^U(\theta_T^{U,j})^2\right]} = -\rho_T^U e^{-A^U\left[W_T^{U,j}+\frac{1}{2}m_T^U\phi_T^2\right]}, \tag{A-28}
$$

where

$$
m_T^U = \frac{N_I}{N_U}\mu_T\omega_T(1-\omega_T) = \mu_T\omega_T\frac{f_T}{N_U}, \quad \rho_T^U = 1. \tag{A-29}
$$

Note that $N_I m_T^I + N_U m_T^U = 0$.

We next conjecture that traders' value function at t are given as (A-26) and (A-28) where the subscript T is replaced by t. We then solve the problem backward from t to $t-1$ and verify our conjecture. For uninformed traders,

$$
\hat{V}_t^U = \hat{V}_{t-1}^U + K_{V,t}^U e_t^U, \quad \phi_t = \phi_{t-1} + K_{X,t}^U e_t^U, \quad P_t - P_{t-1} = a_t\phi_{t-1} + b_t e_t^U, \tag{A-30}
$$

where

$$
a_t = \omega_{t-1}\mu_{t-1} - \omega_t\mu_t, \quad b_t = K_{V,t}^U - \omega_t\mu_t K_{X,t}^U, \tag{A-31}
$$

and

$$
J_t^{U,j} = -\rho_t^U e^{-A^U\left[W_{t-1}^{U,j}+\theta_{t-1}^{U,j}(P_t-P_{t-1})+\frac{1}{2}m_t^U\phi_t^2\right]}, \tag{A-32}
$$

the expected value function can be rewritten as

$$
\mathrm{E}_{t-1}[J_t^{U,j}] = -\rho_t^U\sqrt{\Xi_t^U/\Sigma_t^U}\, e^{-A^U\left[W_{t-1}^{U,j}+a_t\phi_{t-1}\theta_{t-1}^{U,j}+\frac{1}{2}m_t^U\phi_{t-1}^2-\frac{1}{2}A^U\Xi_t^U\left(m_t^U K_{X,t}^U\phi_{t-1}+b_t\theta_{t-1}^{U,j}\right)^2\right]}, \tag{A-33}
$$

where

$$\Sigma_t^U = o_{V,t-1}^U + \tau_{\varepsilon,t}^{-1}, \quad \Xi_t^U = \left(\left(\Sigma_t^U \right)^{-1} + A^U m_t^U \left(K_{X,t}^U \right)^2 \right)^{-1}. \tag{A-34}$$

The first order condition with respect to $\theta_{t-1}^{U,j}$ gives

$$\theta_{t-1}^{U,j} = \frac{f_{t-1}}{N_U} \phi_{t-1}, \quad \text{where} \quad f_{t-1} = N_U \left(\frac{a_t}{A^U \Xi_t^U b_t^2} - \frac{m_t^U K_{X,t}^U}{b_t} \right). \tag{A-35}$$

The value function at time $t-1$ can be written as

$$J_{t-1}^{U,j} = -\rho_{t-1}^U e^{-A^U \left[W_{t-1}^{U,j} + \frac{1}{2} m_{t-1}^U \phi_{t-1}^2 \right]}, \tag{A-36}$$

where

$$\rho_{t-1}^U = \rho_t^U \sqrt{\Xi_t^U / \Sigma_t^U}, \quad m_{t-1}^U = m_t^U \left(1 - \frac{a_t K_{X,t}^U}{b_t} \right) + \frac{f_{t-1}}{N_U} a_t. \tag{A-37}$$

The second order condition requires that $A^U \Xi_t^U b_t > 0$.

For informed traders, since $\hat{V}_t^I - \mu_t X = \hat{V}_t^U - \mu_t \hat{X}_t^U$, and

$$\hat{V}^I = \hat{V}_{t-1}^I + K_t^I e_t^I, \quad \phi_t = \phi_{t-1} + K_{X,t}^U \mu_{t-1} (X - \hat{X}_{t-1}^U) + K_{X,t}^U e_t^I, \tag{A-38}$$

we have,

$$P_t - P_{t-1} = a_t \phi_{t-1} + b_t \mu_{t-1} (X - \hat{X}_{t-1}^U) + b_t e_t^I. \tag{A-39}$$

Since the value function of informed trader at t can be written as

$$J_t^{I,i} = -\rho_t^I e^{-A^I \left[W_{t-1}^{I,i} + \theta_{t-1}^{I,i} (P_t - P_{t-1}) + \hat{V}_t^I X + \frac{1}{2} m_t^I \phi_t^2 + \frac{1}{2} \mu_t (1-\omega_t) (\hat{X}_t^U)^2 \right]}, \tag{A-40}$$

the expected value function can be rewritten as

$$\begin{aligned} \mathrm{E}_{t-1}^I [J_t^{I,i}] = -\rho_t^I \sqrt{\Xi_t^I / \Sigma_t^I} & e^{-A^I \left[W_{t-1}^{I,i} + \hat{V}_{t-1}^I X + \theta_{t-1}^{I,i} \left(a_t \phi_{t-1} + b_t \mu_{t-1} (X - \hat{X}_{t-1}^U) \right) \right]} \\ & \times e^{-A^I \left[\frac{1}{2} m_t^I \left(\phi_{t-1} + K_{X,t}^U \mu_{t-1} (X - \hat{X}_{t-1}^U) \right)^2 + \frac{1}{2} \mu_t (1-\omega_t) \left(\hat{X}_{t-1}^U + K_{X,t}^U \mu_{t-1} (X - \hat{X}_{t-1}^U) \right)^2 \right]} \\ & \times e^{-A^I \left[-\frac{1}{2} A^I \Xi_t^I \left[\theta_{t-1}^{I,i} b_t + K_t^I X + m_t^I K_{X,t}^U \left(\phi_{t-1} + K_{X,t}^U \mu_{t-1} (X - \hat{X}_{t-1}^U) \right) + \mu_t (1-\omega_t) \left(\hat{X}_{t-1}^U + K_{X,t}^U \mu_{t-1} (X - \hat{X}_{t-1}^U) \right) \right]^2 \right]}, \end{aligned}$$

where

$$\Sigma_t^I = o_{V,t-1}^I + \tau_{\varepsilon,t}^{-1}, \quad \Xi_t^I = \left(\left(\Sigma_t^I\right)^{-1} + A^I \left(K_{X,t}^U\right)^2 \left(m_t^I + \mu_t(1-\omega_t)\right) \right)^{-1}. \tag{A-41}$$

The first order condition with respect to $\theta_{t-1}^{I,i}$ yields

$$\theta_{t-1}^{I,i} = \left(\frac{a_t}{A^I \Xi_t^I b_t^2} - \frac{m_t^I K_{X,t}^U}{b_t} \right) \phi_{t-1} - \hat{X}_{t-1}^U = \frac{\Theta}{N_I} - \frac{f_{t-1}}{N_I} \phi_{t-1}, \tag{A-42}$$

which is due to the fact that

$$\mu_{t-1} = \frac{A^I \Xi_t^I K_t^I}{1 - A^I \Xi^I (K_{X,t}^U)^2 (m_t^I + \mu_t(1-\omega_t))} = A^I \Sigma_t^I K_t^I = A^I o_{V,t-1}^I, \tag{A-43}$$

and

$$K_t^I = K_{V,t}^U - \mu_t K_{X,t}^U, \quad K_t^I \mu_{t-1} = \mu_{t-1} - \mu_t. \tag{A-44}$$

The second order condition requires that $A^I \Xi_t^I b_t > 0$. In equilibrium, the market clearing condition gives

$$1 - \frac{f_{t-1}}{N_I} = 1 - \frac{N_U}{N_I} \left(\frac{a_t}{A^U \Xi_t^U b_t^2} - \frac{m_t^U K_{X,t}^U}{b_t} \right) = \frac{a_t}{A^I \Xi_t^I b_t^2} - \frac{m_t^I K_{X,t}^U}{b_t}, \tag{A-45}$$

which determines a_t, so ω_{t-1} can be determined.

Substituting the the equilibrium holdings, the informed traders' value function at time $t-1$ and the parameter m_{t-1}^I can be expressed as

$$J_{t-1}^{I,i} = -\rho_{t-1}^I e^{-A^I \left[W_{t-1}^{I,i} + \hat{V}_{t-1}^I X + \frac{1}{2} m_{t-1}^I \phi_{t-1}^2 + \frac{1}{2} \mu_{t-1}(1-\omega_{t-1})(\hat{X}_{t-1}^U)^2 \right]}, \tag{A-46}$$

where

$$\begin{aligned} \rho_{t-1}^I &= \rho_t^I \sqrt{\Xi_t^I / \Sigma_t^I} e^{A^I \left[\frac{1}{2}(\mu_{t-1} - \mu_t) X^2 - \frac{1}{2}(\mu_{t-1}\omega_{t-1} - \mu_t \omega_t) \left(\frac{\Theta}{N_I} \right)^2 \right]}, \\ m_{t-1}^I &= m_t^I \left(1 - \frac{a_t K_{X,t}^U}{b_t} \right) - \frac{f_{t-1}}{N_I} a_t. \end{aligned} \tag{A-47}$$

We have

$$N_I m_{t-1}^I + N_U m_{t-1}^U = \left(N_I m_t^I + N_U m_t^U\right) \left(1 - \frac{a_t K_{X,t}^U}{b_t} \right) = 0, \tag{A-48}$$

since $N_I m_T^I + N_U m_T^U = 0$. By using the fact that $N_I m_t^I + N_U m_t^U = 0$, we have

$$a_t = \frac{N_I b_t^2}{\frac{N_I}{A^I \Xi_t^I} + \frac{N_U}{A^U \Xi_t^U}} = \frac{N_I \left(\omega_t K_t^I + (1-\omega_t) K_{V,t}^U\right)^2}{\frac{N_I}{A^I \Sigma_t^I} + \frac{N_U}{A^U \Sigma_t^U} + \frac{N_I(1-\omega_t)}{\mu_t}(K_{V,t}^U - K_t^I)^2}. \tag{A-49}$$

Substituting the expression of ω_T, we show recursively

$$\omega_t = \frac{\frac{N_I}{A^I o_{V,t}^I}}{\frac{N_I}{A^I o_{V,t}^I} + \frac{N_U}{A^U o_{V,t}^U}}. \tag{A-50}$$

Substituting ω_t into the recursive expressions of m_t^I and m_t^U yields

$$m_t^I = -\mu_t \omega_t (1-\omega_t), \quad m_t^U = \frac{N_I}{N_U} \mu_t \omega_t (1-\omega_t). \tag{A-51}$$

Plugging m_t back to the formula of f_t, then we have $f_t = N_I(1-\omega_t)$.

A.3 Expected Trading Volume in the Competitive Market

The trading quantity is normally distributed, specifically,

$$N_U(\theta_t^U - \theta_{t-1}^U) = (f_t - f_{t-1})\left(\hat{X}_{t-1}^U + \frac{\Theta}{N_I}\right) + f_t K_{X,t}^U e_t^U, \tag{A-52}$$

with mean

$$\mu_{vol} = \mathrm{E}\left[N_U(\theta_t^U - \theta_{t-1}^U)\right] = (f_t - f_{t-1})\frac{\Theta}{N_I}. \tag{A-53}$$

The variance can be computed by the law of total variance,

$$\begin{aligned}
\mathrm{Var}\left(N_U(\theta_t^U - \theta_{t-1}^U)\right) &= \mathrm{E}\left[\mathrm{Var}_{t-1}^U\left((f_t - f_{t-1})\hat{X}_{t-1}^U + f_t K_{X,t}^U e_t^U\right)\right] + \mathrm{Var}\left(\mathrm{E}_{t-1}^U\left[(f_t - f_{t-1})\hat{X}_{t-1}^U + f_t K_{X,t}^U e_t^U\right]\right) \\
&= \left(f_t K_{X,t}^U\right)^2 \Sigma_t^U + (f_t - f_{t-1})^2 \mathrm{Var}(\hat{X}_{t-1}^U),
\end{aligned}$$

where

$$\mathrm{Var}(\hat{X}_{t-1}^U) = \sum_{s=1}^{t-1}\left(K_{X,s}^U\right)^2 \Sigma_s^U + \mathrm{Var}(\hat{X}_0^U),$$

can also be computed by the law of total variance. Thus, the variance of the trading quantity is

$$\sigma_{vol}^2 = \mathrm{Var}\left(N_U(\theta_t^U - \theta_{t-1}^U)\right) = \left(f_t K_{X,t}^U\right)^2 \Sigma_t^U + (f_t - f_{t-1})^2 \left[\sum_{s=1}^{t-1} \left(K_{X,s}^U\right)^2 \Sigma_s^U + \mathrm{Var}(\hat{X}_0^U)\right].$$

Therefore,

$$\mathrm{Vol}_t = \sqrt{\frac{2}{\pi}}\sigma_{vol} e^{-\frac{\mu_{vol}^2}{2\sigma_{vol}^2}} + \mu_{vol}\,\mathrm{erf}\left(\frac{\mu_{vol}}{\sqrt{2}\sigma_{vol}}\right), \tag{A-54}$$

where erf$(\cdot)$ is the error function.

A.4 Welfare with No Public Information in a Competitive Market

In this section, we compute the welfare with no public information in the competitive market for informed and uninformed traders,

$$\begin{aligned}
\mathscr{W}_{np}^I &= \mathrm{E}\left[-e^{-A^I\left[\theta_{-1}^I P_0 + X(\hat{V}_0^I - \bar{V}) + \frac{1}{2}m_0^I\phi_0^2 + \frac{1}{2}\mu_0\left((1-\omega_0)(\hat{X}_0^U)^2 - X^2\right) + \frac{1}{2}\mu_0\omega_0(\frac{\Theta}{N_I})^2\right]}\right], \\
\mathscr{W}_{np}^U &= \mathrm{E}\left[-e^{-A^U[\theta_{-1}^U P_0 + \frac{1}{2}m_0^U\phi_0^2]}\right].
\end{aligned} \tag{A-55}$$

Recall $\mathscr{F}_0^I = \{v, X\}$ and $\mathscr{F}_0^U = \{s_0\}$, and $v = V + \eta$, $s_0 = V + \eta - hX$, $h = \frac{A^I}{\tau_\eta}$, and $\mathrm{E}[V] = \bar{V}$, and $\mathrm{E}[X] = 0$. We can rewrite the state variables at time $t = 0$ as

$$\hat{V}_0^I = \frac{\tau_\eta}{\tau_V + \tau_\eta}(V + \eta) + \frac{\tau_V}{\tau_V + \tau_\eta}\bar{V}, \quad \hat{V}_0^U = \frac{\tau_0}{\tau_V + \tau_0}(V + \eta - hX) + \frac{\tau_V}{\tau_V + \tau_0}\bar{V}, \tag{A-56}$$

$$\hat{X}_0^U = \frac{\tau_s}{\tau_X + \tau_s}X - \frac{\tau_s/h}{\tau_X + \tau_s}(V + \eta) + \frac{\tau_s/h}{\tau_X + \tau_s}\bar{V}, \tag{A-57}$$

where

$$\tau_0 = \frac{1}{\tau_\eta^{-1} + h^2\tau_X^{-1}}, \quad \tau_s = \frac{h^2}{\tau_\eta^{-1} + \tau_V^{-1}}. \tag{A-58}$$

Define

$$u := \begin{pmatrix} V \\ X \\ \eta \end{pmatrix} \sim \mathcal{N}(\bar{u}, \Sigma), \quad \text{where} \quad \bar{u} := \begin{pmatrix} \bar{V} \\ 0 \\ 0 \end{pmatrix}, \quad \Sigma := \begin{pmatrix} \tau_V^{-1} & 0 & 0 \\ 0 & \tau_X^{-1} & 0 \\ 0 & 0 & \tau_\eta^{-1} \end{pmatrix}. \tag{A-59}$$

Let

$$\phi_0^I := \begin{pmatrix} \hat{V}_0^I \\ X \\ \hat{X}_0^U \end{pmatrix} = F_0^I u + \ell_I, \quad \phi_0^U := \begin{pmatrix} \hat{V}_0^U \\ \hat{X}_0^U \end{pmatrix} = F_0^U u + \ell_U, \tag{A-60}$$

where

$$\ell_I := \bar{V} \begin{pmatrix} \frac{\tau_V}{\tau_V+\tau_\eta} \\ 0 \\ \frac{\tau_s/h}{\tau_X+\tau_s} \end{pmatrix}, \quad \ell_U := \bar{V} \begin{pmatrix} \frac{\tau_V}{\tau_V+\tau_0} \\ \frac{\tau_s/h}{\tau_X+\tau_s} \end{pmatrix}, \tag{A-61}$$

and

$$F_0^I := \begin{pmatrix} \frac{\tau_\eta}{\tau_V+\tau_\eta} & 0 & \frac{\tau_\eta}{\tau_V+\tau_\eta} \\ 0 & 1 & 0 \\ -\frac{\tau_s/h}{\tau_X+\tau_s} & \frac{\tau_s}{\tau_X+\tau_s} & -\frac{\tau_s/h}{\tau_X+\tau_s} \end{pmatrix}, \quad F_0^U := \begin{pmatrix} \frac{\tau_0}{\tau_V+\tau_0} & -\frac{\tau_0 h}{\tau_V+\tau_0} & \frac{\tau_0}{\tau_V+\tau_0} \\ -\frac{\tau_s/h}{\tau_X+\tau_s} & \frac{\tau_s}{\tau_X+\tau_s} & -\frac{\tau_s/h}{\tau_X+\tau_s} \end{pmatrix}. \tag{A-62}$$

Thus,

$$\begin{aligned} \mathscr{W}_{np}^I =& \mathrm{E}\left[-e^{-A^I\left[a_I+b_I^\top\phi_0^I+\frac{1}{2}(\phi_0^I)^\top c_I\phi_0^I\right]}\right] = \mathrm{E}\left[-e^{-A^I\left[a_I+b_I^\top\ell_I+\frac{1}{2}\ell_I^\top c_I\ell_I+\left[b_I^\top F_0^I+\ell_I^\top c_I F_0^I\right]u+\frac{1}{2}u^\top(F_0^I)^\top c_I F_0^I u\right]}\right] \\ =& -\sqrt{\frac{|\Xi_I|}{|\Sigma|}}e^{-A^I\left[a_I+b_I^\top\ell_I+\frac{1}{2}\ell_I^\top c_I\ell+\left[b_I^\top F_0^I+\ell_I^\top c_I F_0^I\right]\bar{u}+\frac{1}{2}\bar{u}^\top(F_0^I)^\top c_I F_0^I\bar{u}\right]} \\ &\times e^{-A^I\left[-\frac{1}{2}A^I\left[b_I+c_I^\top\ell_I+c_I F_0^I\bar{u}\right]^\top F_0^I\Xi_I(F_0^I)^\top\left[b_I+c_I^\top\ell_I+c_I F_0^I\bar{u}\right]\right]}, \end{aligned} \tag{A-63}$$

where

$$\Xi_I = \left(\Sigma^{-1} + A^I(F_0^I)^\top c_I F_0^I\right)^{-1}, \quad a_I = \frac{\Theta^2}{2N_I^2}(m_0^I + \mu_0\omega_0) - \frac{\Theta\mu_0\omega_0}{N_I}\theta_{-1}^I, \tag{A-64}$$

$$b_I = \begin{pmatrix} \theta_{-1}^I \\ -\mu_0\theta_{-1}^I - \bar{V} \\ \frac{\Theta}{N_I}m_0^I + \mu_0(1-\omega_0)\theta_{-1}^I \end{pmatrix}, \quad c_I = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -\mu_0 & 0 \\ 0 & 0 & m_0^I + \mu_0(1-\omega_0) \end{pmatrix}. \tag{A-65}$$

Similarly, the uninformed trader's welfare without public information is

$$\begin{aligned}\mathscr{W}_{np}^U &= \mathrm{E}\left[-e^{-A^U\left[a_U + b_U^\top \phi_0^U + \frac{1}{2}(\phi_0^U)^\top c_U \phi_0^U\right]}\right] \\ &= -\sqrt{\frac{|\Xi_U|}{|\Sigma|}} e^{-A^U\left[a_U + b_U^\top \ell_U + \frac{1}{2}\ell_U^\top c_U \ell_U + [b_U^\top F_0^U + \ell_U^\top c_U F_0^U]\bar{u} + \frac{1}{2}\bar{u}^\top (F_0^I)^\top c_I F_0^I \bar{u}\right]} \\ &\quad \times e^{-A^U\left[-\frac{1}{2}A^U[b_U + c_U^\top \ell_U + c_U F_0^U \bar{u}]^\top F_0^U \Xi_U (F_0^U)^\top [b_U + c_U^\top \ell_U + c_U F_0^U \bar{u}]\right]},\end{aligned} \tag{A-66}$$

where

$$\Xi_U = \left(\Sigma^{-1} + A^U (F_0^U)^\top c_U F_0^U\right)^{-1}, \quad a_U = \frac{m_0^U}{2}\left(\frac{\Theta}{N_I}\right)^2 - \frac{\Theta \mu_0 \omega_0}{N_I}\theta_{-1}^I, \tag{A-67}$$

$$b_U = \begin{pmatrix} \theta_{-1}^U \\ m_0^U \frac{\Theta}{N_I} - \omega_0 \mu_0 \theta_{-1}^U \end{pmatrix}, \quad c_U = \begin{pmatrix} 0 & 0 \\ 0 & m_0^U \end{pmatrix}. \tag{A-68}$$

We denote the informed trader's no-trade welfare as

$$\mathscr{W}_{nt}^I = \mathrm{E}\left[-e^{-A^I[\theta_{-1}^I V + X(V - \bar{V})]}\right] = -\sqrt{\frac{\tau_V \tau_X}{\tau_V \tau_X - (A^I)^2}} e^{-A^I\left[\bar{V}\theta_{-1}^I - \frac{A^I}{2}\frac{\tau_X(\theta_{-1}^I)^2}{\tau_V\tau_X - (A^I)^2}\right]}, \tag{A-69}$$

where θ_{-1}^I is the informed trader's original asset holding before trading. We denote the uninformed trader's no-trade welfare as

$$\mathscr{W}_{nt}^U = \mathrm{E}\left[-e^{-A^U[\theta_{-1}^U V]}\right] = -e^{-A^U\left[\bar{V}\theta_{-1}^U - \frac{A^U}{2}\frac{(\theta_{-1}^U)^2}{\tau_V}\right]}, \tag{A-70}$$

where θ_{-1}^U is the informed trader's original asset holding before trading.

A.5 Proof of Proposition 3

If there is only one piece of public information, then, regardless of the timing of its release,

$$\rho^U = \left(1 + \frac{\Gamma}{\tau_V + \tau_0}\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I} + \frac{N_U}{A^U}} \frac{\tau_\eta - \tau_0}{\Gamma + \tau_V + \bar{\tau}}\right)^2\right)^{-1/2}, \quad \rho^I = \left(1 + \frac{\Gamma}{\tau_V + \tau_\eta}\left(\frac{\frac{N_U}{A^U}}{\frac{N_I}{A^I} + \frac{N_U}{A^U}} \frac{\tau_\eta - \tau_0}{\Gamma + \tau_V + \bar{\tau}}\right)^2\right)^{-1/2}. \tag{A-71}$$

Note that the welfare is a concave function of Γ. Thus, there exists a global maximizer of Γ, which can be obtained using the first order condition,

$$\Gamma = \tau_V + \bar{\tau}, \quad \text{where} \quad \bar{\tau} = \frac{\frac{N_I}{A^I}\tau_\eta + \frac{N_U}{A^U}\tau_0}{\frac{N_I}{A^I} + \frac{N_U}{A^U}}. \tag{A-72}$$

A.6 Proof of Proposition 4

Given that the timing of information release has no impact, our comparison is focused on the welfare outcomes of releasing all information at once versus splitting it into two parts and releasing them sequentially. The distinction arises solely from ρ^I and ρ^U, which are functions of Π^I and Π^U, respectively,

$$\rho^I = \left(\Pi^I\right)^{-\frac{1}{2}}, \quad \rho^U = \left(\Pi^U\right)^{-\frac{1}{2}}, \quad \text{where} \tag{A-73}$$

$$\Pi^I = \prod_{t=1}^{T}\left[1 + A^I \Sigma_t^I \mu_t (1-\omega_t)^2 (K_{X,t}^U)^2\right], \quad \Pi^U = \prod_{t=1}^{T}\left[1 + \frac{N_I}{N_U} A^U \Sigma_t^U \mu_t \omega_t (1-\omega_t)(K_{X,t}^U)^2\right].$$

Thus, the impact of public information on welfare arises solely from Π^I and Π^U,

$$\Pi^I = \prod_{t=1}^{T}\left[1 + \frac{\tau_{\varepsilon,t}}{\tau_V + \tau_\eta + \tau_t - \tau_{\varepsilon,t}}\left(\frac{\frac{N_U}{A^U}}{\frac{N_I}{A^I} + \frac{N_U}{A^U}}\frac{\tau_\eta - \tau_0}{\tau_V + \tau_t + \bar{\tau}}\right)^2\right], \tag{A-74}$$

$$\Pi^U = \prod_{t=1}^{T}\left[1 + \frac{\tau_{\varepsilon,t}}{\tau_V + \tau_0 + \tau_t - \tau_{\varepsilon,t}}\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I} + \frac{N_U}{A^U}}\frac{\tau_\eta - \tau_0}{\tau_V + \tau_t + \bar{\tau}}\right)^2\right]. \tag{A-75}$$

We first prove the case for uninformed traders. A higher Π^U leads to a higher welfare of uninformed traders. If there is only one piece of public information, from Appendix A.5, we have

$$\Pi^U_{one-piece} = 1 + c\frac{\tau_T}{\tau_V + \tau_0}, \quad \text{where} \quad c = \left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I} + \frac{N_U}{A^U}}\frac{\tau_\eta - \tau_0}{\tau_T + \tau_V + \bar{\tau}}\right)^2. \tag{A-76}$$

If we split the information with total precision τ_T into two pieces of information with precisions $\tau_{\varepsilon,1}$ and $\tau_{\varepsilon,2}$, $\tau_{\varepsilon,1} + \tau_{\varepsilon,2} = \tau_T$, then

$$\Pi^U_{two-piece} = \left[1 + c\frac{\tau_T - \tau_{\varepsilon,1}}{\tau_V + \tau_0 + \tau_{\varepsilon,1}}\right]\left[1 + c\frac{\tau_{\varepsilon,1}}{\tau_V + \tau_0}\left(\frac{\tau_V + \bar{\tau} + \tau_T}{\tau_V + \bar{\tau} + \tau_{\varepsilon,1}}\right)^2\right]. \tag{A-77}$$

Define

$$\Delta^U = \Pi^U_{two-piece} - \Pi^U_{one-piece} = \frac{c(a+\tau_T)}{a}\frac{\tau_{\varepsilon,1}(\tau_T - \tau_{\varepsilon,1})}{(b+\tau_{\varepsilon,1})^2}, \tag{A-78}$$

where

$$a = \tau_V + \tau_0, \quad b = \tau_V + \bar{\tau}. \tag{A-79}$$

Therefore, for $0 < \tau_{\varepsilon,1} < \tau_T$, we have $\Delta^U > 0$, i.e.,

$$\Pi^U_{two-piece} > \Pi^U_{one-piece}. \tag{A-80}$$

Therefore, releasing information all at once yields lower welfare for uninformed traders compared to splitting the information into two parts and releasing it sequentially. Now the questions is how to split the information benefits uninformed traders the most. The first order condition for the optimal $\tau^U_{\varepsilon,1}$ gives

$$\tau^U_{\varepsilon,1} = \tau_T \frac{b}{\tau_T + 2b} = \tau_T \frac{\tau_V + \bar{\tau}}{\tau_T + 2(\tau_V + \bar{\tau})} < \frac{\tau_T}{2}. \tag{A-81}$$

We now prove the case for informed traders. Similarly, we have

$$\Pi^I_{one-piece} = 1 + d\frac{\tau_T}{\tau_V + \tau_\eta}, \quad \text{where} \quad d = \left(\frac{\frac{N_U}{A^U}}{\frac{N_I}{A^I} + \frac{N_U}{A^U}} \frac{\tau_\eta - \tau_0}{\tau_T + \tau_V + \bar{\tau}} \right)^2. \tag{A-82}$$

If we split the information with total precision τ_T into two pieces of information with precisions $\tau_{\varepsilon,1}$ and $\tau_{\varepsilon,2}$, $\tau_{\varepsilon,1} + \tau_{\varepsilon,2} = \tau_T$, then

$$\Pi^I_{two-piece} = \left[1 + d\frac{\tau_T - \tau_{\varepsilon,1}}{\tau_V + \tau_\eta + \tau_{\varepsilon,1}}\right]\left[1 + d\frac{\tau_{\varepsilon,1}}{\tau_V + \tau_\eta}\left(\frac{\tau_V + \bar{\tau} + \tau_T}{\tau_V + \bar{\tau} + \tau_{\varepsilon,1}}\right)^2\right]. \tag{A-83}$$

Define

$$\Delta^I = \Pi^I_{two-piece} - \Pi^I_{one-piece} = \frac{d(\alpha+\tau_T)}{\alpha}\frac{\tau_{\varepsilon,1}(\tau_T - \tau_{\varepsilon,1})}{(b+\tau_{\varepsilon,1})^2}, \tag{A-84}$$

where

$$\alpha = \tau_V + \tau_\eta, \quad b = \tau_V + \bar{\tau}. \tag{A-85}$$

Therefore, for $0 < \tau_{\varepsilon,1} < \tau_T$, we have $\Delta^I > 0$, i.e.,

$$\Pi^I_{two-piece} > \Pi^I_{one-piece}. \tag{A-86}$$

Therefore, releasing information all at once yields lower welfare for informed traders compared to splitting the information into two parts and releasing it sequentially. Now the questions is how to split the information benefits informed traders the most. The first order condition for the optimal $\tau^I_{\varepsilon,1}$ gives

$$\tau^I_{\varepsilon,1} = \tau^U_{\varepsilon,1} = \frac{b\tau_T}{\tau_T + 2b} = \frac{\tau_T}{2 + \frac{\tau_T}{\tau_V + \bar{\tau}}} < \frac{\tau_T}{2}. \tag{A-87}$$

Thus, the optimal release scheme for both informed and uninformed traders should involve disclosing an increasing amount of information to the market over time. For a larger ratio $\frac{\tau_T}{\tau_V+\bar{\tau}}$, the portion that is disclosed at the beginning should be smaller.

For disclosure with multiple period T, we conjecture that the optimal information disclosure precision is

$$\tau_{t-1} = \frac{t-1}{t + \frac{\tau_t}{b}}\tau_t = \frac{t-1}{\frac{1}{b} + \frac{t}{\tau_t}}, \quad \text{for} \quad t = 2, 3, \ldots T. \tag{A-88}$$

As a result, τ_t is a generalized continued fraction,

$$\tau_t = \frac{t}{\frac{1}{b} + \frac{t+1}{\tau_{t+1}}}, \tag{A-89}$$

and plugging this into the equation of τ_{t-1} gives

$$\tau_{t-1} = \frac{t-1}{\frac{2}{b} + \frac{t+1}{\tau_{t+1}}}. \tag{A-90}$$

By keeping doing this, we obtain

$$\tau_{t-1} = \frac{t-1}{\frac{T+1-t}{b} + \frac{T}{\tau_T}} = \tau_T \frac{t-1}{T + (T+1-t)\frac{\tau_T}{b}}. \tag{A-91}$$

Similarly,

$$\tau_t = \frac{t}{\frac{T-t}{b} + \frac{T}{\tau_T}} = \tau_T \frac{t}{T + (T-t)\frac{\tau_T}{b}}. \tag{A-92}$$

Note that

$$\tau_t = \sum_{s=1}^{t} \tau_{\varepsilon,s}, \tag{A-93}$$

and τ_T is the total information disclosed to the market. Therefore,

$$\tau_{\varepsilon,t} = \tau_t - \tau_{t-1} = \tau_T \frac{T\left(1+\frac{\tau_T}{b}\right)}{\left[T + (T-t)\frac{\tau_T}{b}\right]\left[T + (T+1-t)\frac{\tau_T}{b}\right]}. \tag{A-94}$$

Obviously,

$$\tau_{\varepsilon,t} < \tau_{\varepsilon,t+1}. \tag{A-95}$$

It is also easy to verify that

$$\tau_{\varepsilon,t} - \tau_{\varepsilon,t-1} < \tau_{\varepsilon,t+1} - \tau_{\varepsilon,t}. \tag{A-96}$$

Hence, the optimal $\tau_{\varepsilon,t}$ is an increasing convex function of time.

A.7 Proof of Proposition 5

Since the welfare depends on the public information only through Π^U and a higher Π^U leads to a higher welfare, we only need to study how T affects Π^U, which can be expressed as

$$\begin{aligned}\Pi^U &= \prod_{t=1}^{T}\left[1 + \frac{\tau_{\varepsilon,t}}{\tau_V+\tau_0+\tau_t-\tau_{\varepsilon,t}}\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\tau_V+\tau_t+\bar{\tau}}\right)^2\right] = \prod_{t=1}^{T}\left[1 + \frac{\frac{\tau_T}{T}}{\tau_V+\tau_0+\frac{t-1}{T}\tau_T}\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\tau_V+\bar{\tau}+\frac{t}{T}\tau_T}\right)^2\right] \\ &= e^{\sum_{t=1}^{T}\ln\left[1+\frac{\frac{\tau_T}{T}}{\tau_V+\tau_0+\frac{t-1}{T}\tau_T}\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\tau_V+\bar{\tau}+\frac{t}{T}\tau_T}\right)^2\right]}.\end{aligned}$$

By Taylor expansion,

$$\ln\left[1 + \frac{\frac{\tau_T}{T}}{\tau_V+\tau_0+\frac{t-1}{T}\tau_T}\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\tau_V+\bar{\tau}+\frac{t}{T}\tau_T}\right)^2\right] = \sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}\left(\frac{\frac{\tau_T}{T}}{\tau_V+\tau_0+\frac{t-1}{T}\tau_T}\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\tau_V+\bar{\tau}+\frac{t}{T}\tau_T}\right)^2\right)^{n+1},$$

and since

$$\lim_{T\to\infty}\sum_{t=1}^{T}\frac{\frac{\tau_T}{T}}{\tau_V+\tau_0+\frac{t-1}{T}\tau_T}\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\tau_V+\bar{\tau}+\frac{t}{T}\tau_T}\right)^2=\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\bar{\tau}-\tau_0}\right)^2\left[\frac{\tau_T(\tau_0-\bar{\tau})}{(\tau_V+\bar{\tau})(\tau_T+\tau_V+\bar{\tau})}+\ln\frac{(\tau_V+\bar{\tau})(\tau_T+\tau_V+\tau_0)}{(\tau_V+\tau_0)(\tau_T+\tau_V+\bar{\tau})}\right],$$

$$\lim_{T\to\infty}\sum_{t=1}^{T}\left(\frac{\frac{\tau_T}{T}}{\tau_V+\tau_0+\frac{t-1}{T}\tau_T}\left(\frac{\frac{N_I}{A^I}}{\frac{N_I}{A^I}+\frac{N_U}{A^U}}\frac{\tau_\eta-\tau_0}{\tau_V+\bar{\tau}+\frac{t}{T}\tau_T}\right)^2\right)^{n+1}=0,\quad n\geq 1,$$

we can obtain the upper bound of ρ^U. Similarly, we also obtain the upper bound of ρ^I.

A.8 Proof of Proposition 6

We solve the model backward. The optimal holdings for the informed trader and uninformed traders at the last period T are as follows:

$$\theta_T^I=\frac{\hat{V}_T^I-P_T-A^I o_{V,T}^I X+\lambda_T\theta_{T-1}^I}{A^I o_{V,T}^I+\lambda_T},\quad \theta_T^{U,j}=\frac{\hat{V}_T^U-P_T}{A^U o_{V,T}^U}. \tag{A-97}$$

The market clearing condition yields,

$$\theta_T^I+\sum_{j=1}^{N_U}\frac{\hat{V}_T^U-P_T}{A^U o_{V,T}^U}=\Theta, \tag{A-98}$$

from (A-98), expressing P_T as a function of θ_T^I yields,

$$\lambda_T:=\frac{\partial P_T}{\partial\theta_T^I}=\frac{\gamma_T^U}{N_U}, \tag{A-99}$$

where $\gamma_T^I:=A^I o_{V,T}^I$ and $\gamma_T^U:=A^U o_{V,T}^U$. Therefore, the equilibrium price at time T is

$$\begin{aligned}P_T&=\omega_T(\hat{V}_T^I-\mu_T X)+(1-\omega_T)\hat{V}_T^U+\lambda_T\omega_T\theta_{T-1}^I-f_T\Theta\\&=\hat{V}_T^I-\mu_T X+(1-\omega_T)\mu_T\hat{X}_T^U+\lambda_T\omega_T\theta_{T-1}^I-f_T\Theta\\&=\hat{V}_T^U-\omega_T\mu_T\hat{X}_T^U+\lambda_T\omega_T\theta_{T-1}^I-f_T\Theta,\end{aligned} \tag{A-100}$$

where coefficients μ_T, f_T, and ω_T are given as

$$\mu_T:=A^I o_{V,T}^I,\quad f_T:=\left(\frac{1}{\gamma_T^I+\lambda_T}+\frac{N_U}{\gamma_T^U}\right)^{-1},\quad \omega_T:=\frac{1}{\gamma_T^I+\lambda_T}f_T. \tag{A-101}$$

Hence, the optimal holding of the informed trader is

$$\theta_T^I = -\frac{\mu_T(1-\omega_T)}{\gamma_T^I+\lambda_T}\hat{X}_T^U + \omega_T\Theta + \frac{\lambda_T(1-\omega_T)}{\gamma_T^I+\lambda_T}\theta_{T-1}^I, \tag{A-102}$$

and optimal holding of an uninformed trader is

$$\theta_T^{U,j} = \frac{\mu_T\omega_T}{\gamma_T^U}\hat{X}_T^U + \frac{1-\omega_T}{N_U}\Theta - \frac{\lambda_T\omega_T}{\gamma_T^U}\theta_{T-1}^I. \tag{A-103}$$

It is easy to verify that $\theta_T^I + \sum_{j=1}^{N_U}\theta_T^{U,j} = \Theta$.

Therefore, the informed trader's value function at $t=T$ is

$$J_T^I = \mathrm{E}_T^I[-e^{-A^I W_{T+1}^I}] = -\rho_T^I e^{-A^I\left[W_T^I + \frac{1}{2}(\Phi_T^I)^\top M_T^I \Phi_T^I + (C_T^I)^\top \Phi_T^I \theta_{T-1}^I + \frac{1}{2}m_T^I(\theta_{T-1}^I)^2\right]}, \tag{A-104}$$

where

$$\rho_T^I = e^{A^I\bar{V}X}, \quad m_T^I = \lambda_T\omega_T, \quad M_T^I = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & -\gamma_T^I & 0 & 0 \\ 0 & 0 & \frac{\mu_T^2}{2\lambda_T+\gamma_T^I} & -\mu_T\omega_T \\ 0 & 0 & -\mu_T\omega_T & \frac{\lambda_T^2}{2\lambda_T+\gamma_T^I} \end{pmatrix}, \quad C_T^I = \begin{pmatrix} 1 \\ -\mu_T \\ \mu_T(1-\omega_T) \\ -\lambda_T(1-\omega_T) \end{pmatrix}. \tag{A-105}$$

Similarly, uninformed traders' value functions at $t=T$ are

$$J_T^{U,j} = \mathrm{E}_T^U[-e^{-A^U W_{T+1}^{U,j}}] = -\rho_T^U e^{-A^U\left[W_T^{U,j} + \frac{1}{2}(\Phi_T^U)^\top M_T^U \Phi_T^U + (C_T^U)^\top \Phi_T^U \theta_{T-1}^I + \frac{1}{2}m_T^U(\theta_{T-1}^I)^2\right]}, \tag{A-106}$$

where

$$\rho_T^U = 1, \quad m_T^U = \frac{\lambda_T^2\omega_T^2}{\gamma_T^U}, \quad M_T^U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (\mu_T\omega_T)^2/\gamma_T^U & \mu_T\omega_T f_T/\gamma_T^U \\ 0 & \mu_T\omega_T f_T/\gamma_T^U & f_T^2/\gamma_T^U \end{pmatrix}, \quad C_T^U = \begin{pmatrix} 0 \\ -\frac{\lambda_T\mu_T\omega_T^2}{\gamma_T^U} \\ -\frac{\lambda_T\omega_T f_T}{\gamma_T^U} \end{pmatrix}. \tag{A-107}$$

The optimal holdings can be rewritten as

$$\theta_T^I = \frac{P_T^I - P_T}{\gamma_T^I + \lambda_T}, \quad \theta_T^{U,j} = \frac{P_T^U - P_T}{\gamma_T^U}, \tag{A-108}$$

where

$$P_T^I = \hat{V}_T^I - \mu_T X - g_T^I \hat{X}_T^U - f_T^I \Theta + \lambda_T \theta_{T-1}^I, \qquad P_T^U = \hat{V}_T^U - g_T^U \hat{X}_T^U - f_T^U \Theta, \tag{A-109}$$

and

$$g_T^I = g_T^U = 0, \quad f_T^I = f_T^U = 0. \tag{A-110}$$

Thus, the price can be rewritten as

$$\begin{aligned} P_T &= \omega_T P_T^I + (1-\omega_T) P_T^U - f_T \Theta = \left(H_{P,T}^I\right)^\top \Phi_T^I + \omega_T \lambda_T \theta_{T-1}^I \\ &= \left(H_{P,T}^U\right)^\top \Phi_T^U + \omega_T \lambda_T \theta_{T-1}^I, \end{aligned} \tag{A-111}$$

where

$$H_{P,T}^I = \begin{pmatrix} 1 \\ -\mu_T \\ (1-\omega_T)\mu_T \\ -f_T \end{pmatrix}, \qquad H_{P,T}^U = \begin{pmatrix} 1 \\ -\omega_T \mu_T \\ -f_T \end{pmatrix}. \tag{A-112}$$

We next conjecture that at any time t, traders' value function, stock holdings, and prices can be expressed similarly to those at $t = T$, with the subscript T replaced by t. We then solve the problem backward from t to $t-1$ and verify our conjecture.

At time t, the optimal holdings, prices, and value functions can be expressed in terms of informed and uninformed traders' state vectors:

$$\Phi_t^I = \begin{pmatrix} \hat{V}_t^I \\ X \\ \hat{X}_t^U \\ \Theta \end{pmatrix}, \quad \Phi_t^U = \begin{pmatrix} \hat{V}_t^U \\ \hat{X}_t^U \\ \Theta \end{pmatrix}. \tag{A-113}$$

The state vectors can be expressed recursively,

$$\Phi_t^I = H_t^I \Phi_{t-1}^I + F_t^I e_t^I, \quad \Phi_t^U = \Phi_{t-1}^U + F_t^U e_t^U, \tag{A-114}$$

where

$$H_t^I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & K_{X,t}^U \mu_{t-1} & 1 - K_{X,t}^U \mu_{t-1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad F_t^I = \begin{pmatrix} K_{V,t}^I \\ 0 \\ K_{X,t}^U \\ 0 \end{pmatrix}, \quad F_t^U = \begin{pmatrix} K_{V,t}^U \\ K_{X,t}^U \\ 0 \end{pmatrix}, \tag{A-115}$$

and

$$e_t^I = s_t - \hat{V}_{t-1}^I \sim \mathcal{N}(0, \sigma_t^I), \quad e_t^U = s_t - (\hat{V}_{t-1}^U - h_t \hat{X}_{t-1}^U) \sim \mathcal{N}(0, \sigma_t^U), \tag{A-116}$$

where $\sigma_t^I = o_{V,t-1}^I + \tau_{\varepsilon,t}^{-1}$, $\sigma_t^U = o_{V,t-1}^U + \tau_{\varepsilon,t}^{-1} + h_t^2 \tau_X^{-1}$, $h_0 = h$ and $h_t = 0$ for $t = 1, ..., T$.

The asset price can be expressed in terms of state vectors as

$$P_t = \omega_t P_t^I + (1 - \omega_t) P_t^U - f_t \Theta = \left(H_{P,t}^I\right)^\top \Phi_t^I + \omega_t \lambda_t \theta_{t-1}^I = \left(H_{P,t}^U\right)^\top \Phi_t^U + \omega_t \lambda_t \theta_{t-1}^I. \tag{A-117}$$

The optimal holdings are

$$\theta_t^I = \frac{P_t^I - P_t}{\gamma_t^I + \lambda_t}, \qquad \theta_t^{U,j} = \frac{P_t^U - P_t}{\gamma_t^U}, \tag{A-118}$$

where

$$P_t^I = \hat{V}_t^I - \mu_t X - g_t^I \hat{X}_t^U - f_t^I \Theta + \lambda_t \theta_{t-1}^I, \qquad P_t^U = \hat{V}_t^U - g_t^U \hat{X}_t^U - f_t^U \Theta, \tag{A-119}$$

$$f_t = \left(\frac{1}{\lambda_t + \gamma_t^I} + \frac{N_U}{\gamma_t^U}\right)^{-1}, \quad \omega_t = \frac{f_t}{\lambda_t + \gamma_t^I}, \tag{A-120}$$

and

$$H_{P,t}^I = \begin{pmatrix} 1 \\ -\mu_t \\ (1 - \omega_t)\mu_t - \left(\omega_t g_t^I + (1 - \omega_t) g_t^U\right) \\ -\left(f_t + \omega_t f_t^I + (1 - \omega_t) f_t^U\right) \end{pmatrix}, \quad H_{P,t}^U = \begin{pmatrix} 1 \\ -\omega_t \mu_t - \left(\omega_t g_t^I + (1 - \omega_t) g_t^U\right) \\ -\left(f_t + \omega_t f_t^I + (1 - \omega_t) f_t^U\right) \end{pmatrix}. \tag{A-121}$$

Hence, the optimal holdings can be expressed in terms of state vectors

$$\theta_t^I = (H_{\theta,t}^I)^\top \Phi_t^I + \frac{(1-\omega_t)\lambda_t}{\lambda_t + \gamma_t^I}\theta_{t-1}^I = (H_{\Theta,t})^\top \Phi_t^U + \omega_t \theta_{t-1}^I, \tag{A-122}$$

$$\theta_t^{U,j} = (H_{\theta,t}^U)^\top \Phi_t^U - \frac{\omega_t \lambda_t}{\gamma_t^U}\theta_{t-1}^I = (H_{\theta,t}^U)^\top \Phi_t^U - \frac{\omega_t}{N_U}\theta_{t-1}^I, \tag{A-123}$$

noting that

$$\lambda_t = \frac{\gamma_t^U}{N_U}, \quad \frac{(1-\omega_t)\lambda_t}{\lambda_t + \gamma_t^I} = \omega_t, \quad f_t = \frac{\lambda_t(\lambda_t + \gamma_t^I)}{2\lambda_t + \gamma_t^I}, \tag{A-124}$$

$$\omega_t = \frac{f_t}{\lambda_t + \gamma_t^I} = \frac{\lambda_t}{2\lambda_t + \gamma_t^I}, \quad 1 - \omega_t = \frac{f_t}{\lambda_t} = \frac{\lambda_t + \gamma_t^I}{2\lambda_t + \gamma_t^I}, \tag{A-125}$$

and

$$H_{\theta,t}^I = \begin{pmatrix} 0 \\ 0 \\ -\frac{1-\omega_t}{\lambda_t+\gamma_t^I}\left(\mu_t + g_t^I - g_t^U\right) \\ -\frac{1-\omega_t}{\lambda_t+\gamma_t^I}\left(f_t^I - f_t^U\right) + \frac{f_t}{\lambda_t+\gamma_t^I} \end{pmatrix}, \quad H_{\theta,t}^U = \begin{pmatrix} 0 \\ \frac{\omega_t}{\gamma_t^U}\left(\mu_t + g_t^I - g_t^U\right) \\ \frac{\omega_t}{\gamma_t^U}\left(f_t^I - f_t^U\right) + \frac{f_t}{\gamma_t^U} \end{pmatrix}, \quad H_{\Theta,t} = \begin{pmatrix} 0 \\ -\frac{\omega_t}{\lambda_t}\left(\mu_t + g_t^I - g_t^U\right) \\ \omega_t - \frac{\omega_t}{\lambda_t}\left(f_t^I - f_t^U\right) \end{pmatrix}.$$

The value functions at t can be written as

$$\begin{aligned} J_t^I &= -\rho_t^I e^{-A^I\left[W_t^I + \frac{1}{2}(\Phi_t^I)^\top M_t^I \Phi_t^I + (C_t^I)^\top \Phi_t^I \theta_{t-1}^I + \frac{1}{2} m_t^I (\theta_{t-1}^I)^2\right]}, \\ J_t^{U,j} &= -\rho_t^U e^{-A^U\left[W_t^{U,j} + \frac{1}{2}(\Phi_t^U)^\top M_t^U \Phi_t^U + (C_t^U)^\top \Phi_t^U \theta_{t-1}^I + \frac{1}{2} m_t^U (\theta_{t-1}^I)^2\right]}, \end{aligned} \tag{A-126}$$

and all the coefficients can be computed recursively. We next solve the uninformed traders' problem at $t-1$. Rewrite uninformed traders' value function at t as

$$J_t^{U,j} = -\rho_t^U e^{-\lambda^U\left[W_{t-1}^{U,j} - \theta_{t-1}^{U,j} P_{t-1} + \theta_{t-1}^{U,j} P_t + \frac{1}{2}(\Phi_t^U)^\top M_t^U \Phi_t^U + (C_t^U)^\top \Phi_t^U \theta_{t-1}^I + \frac{1}{2} m_t^U (\theta_{t-1}^I)^2\right]}. \tag{A-127}$$

Thus, we can rewrite the value function as

$$\begin{aligned} J_t^{U,j} =& -\rho_t^U e^{-A^U\left[W_{t-1}^{U,j} - \theta_{t-1}^{U,j} P_{t-1} + \theta_{t-1}^{U,j}\left((H_{P,t}^U)^\top \Phi_{t-1}^U + \omega_t \lambda_t \theta_{t-1}^I\right) + (C_t^U)^\top \Phi_{t-1}^U \theta_{t-1}^I + \frac{1}{2}(\Phi_{t-1}^U)^\top M_t^U \Phi_{t-1}^U + \frac{1}{2} m_t^U (\theta_{t-1}^I)^2\right]} \\ & \times e^{-A^U\left[e_t^U\left((H_{P,t}^U)^\top F_t^U \theta_{t-1}^{U,j} + (C_t^U)^\top F_t^U \theta_{t-1}^I + (F_t^U)^\top M_t^U \Phi_{t-1}^U\right) + \frac{1}{2}(F_t^U)^\top M_t^U F_t^U (e_t^U)^2\right]}. \end{aligned} \tag{A-128}$$

Hence, the expectation at $t-1$ is

$$\begin{aligned}\mathrm{E}_{t-1}^{U}[J_t^{U,j}] = &-\rho_t^U e^{-A^U\left[W_{t-1}^{U,j}-\theta_{t-1}^{U,j}P_{t-1}+\theta_{t-1}^{U,j}\left((H_{P,t}^U)^\top\Phi_{t-1}^U+\omega_t\lambda_t\theta_{t-1}^I\right)+(C_t^U)^\top\Phi_{t-1}^U\theta_{t-1}^I+\frac{1}{2}(\Phi_{t-1}^U)^\top M_t^U\Phi_{t-1}^U+\frac{1}{2}m_t^U(\theta_{t-1}^I)^2\right]} \\ &\times\sqrt{\frac{\Xi_t^U}{\Sigma_t^U}}e^{-A^U\left[-\frac{1}{2}A^U\Xi_t^U\left((H_{P,t}^U)^\top F_t^U\theta_{t-1}^{U,j}+(C_t^U)^\top F_t^U\theta_{t-1}^I+(F_t^U)^\top M_t^U\Phi_{t-1}^U\right)^2\right]},\end{aligned} \tag{A-129}$$

where $\Xi_t^U = \left[(\Sigma_t^U)^{-1}+A^U(F_t^U)^\top M_t^U F_t^U\right]^{-1}$. Taking the first order condition with respect to $\theta_{t-1}^{U,j}$ yields

$$-P_{t-1}+(H_{P,t}^U)^\top\Phi_{t-1}^U+\omega_t\lambda_t\theta_{t-1}^I = A^U\Xi_t^U(H_{P,t}^U)^\top F_t^U(F_t^U)^\top\left[H_{P,t}^U\theta_{t-1}^{U,j}+C_t^U\theta_{t-1}^I+M_t^U\Phi_{t-1}^U\right], \tag{A-130}$$

and the second order condition requires $A^U\Xi_t^U\left[(H_{P,t}^U)^\top F_t^U\right]^2>0$, which is satisfied automatically. By the symmetry and market clearing condition, we have

$$\theta_{t-1}^I = \Theta - N_U\theta_{t-1}^{U,j}, \quad \theta_{t-1}^{U,j} = \frac{P_{t-1}^U - P_{t-1}}{\gamma_{t-1}^U}, \tag{A-131}$$

where

$$\begin{aligned}\gamma_{t-1}^U &= \omega_t\gamma_t^U + A^U\Xi_t^U(H_{P,t}^U)^\top F_t^U(F_t^U)^\top\left(H_{P,t}^U - N_U C_t^U\right), \\ P_{t-1}^U &= \left[(H_{P,t}^U)^\top - A^U\Xi_t^U(H_{P,t}^U)^\top F_t^U(F_t^U)^\top M_t^U\right]\Phi_{t-1}^U \\ &\quad + \left[\omega_t\lambda_t - A^U\Xi_t^U(H_{P,t}^U)^\top F_t^U(F_t^U)^\top C_t^U\right]\Theta.\end{aligned} \tag{A-132}$$

Comparing with

$$P_{t-1}^U = \hat{V}_{t-1}^U - g_{t-1}^U\hat{X}_{t-1}^U - f_{t-1}^U\Theta, \tag{A-133}$$

gives

$$\begin{aligned}g_{t-1}^U &= -\left[(H_{P,t}^U)^\top - A^U\Xi_t^U(H_{P,t}^U)^\top F_t^U(F_t^U)^\top M_t^U\right]_{1,2}, \\ f_{t-1}^U &= -\left[(H_{P,t}^U)^\top - A^U\Xi_t^U(H_{P,t}^U)^\top F_t^U(F_t^U)^\top M_t^U\right]_{1,3} - \left[\omega_t\lambda_t - A^U\Xi_t^U(H_{P,t}^U)^\top F_t^U(F_t^U)^\top C_t^U\right].\end{aligned} \tag{A-134}$$

It can be verified that

$$1 = \left[(H_{P,t}^U)^\top - A^U\Xi_t^U(H_{P,t}^U)^\top F_t^U(F_t^U)^\top M_t^U\right]_{1,1}. \tag{A-135}$$

We next solve the informed trader's problem. Rewrite the informed trader's value function at t as

$$J_t^I = -\rho_t^I e^{-A^I\left[W_{t-1}^I-(\theta_{t-1}^I-\theta_{t-2}^I)P_{t-1}+\frac{1}{2}(\Phi_t^I)^\top M_t^I\Phi_t^I+(C_t^I)^\top\Phi_t^I\theta_{t-1}^I+\frac{1}{2}m_t^I(\theta_{t-1}^I)^2\right]}. \tag{A-136}$$

Thus, the value function can be rewritten as

$$\begin{aligned} J_t^I =& -\rho_t^I e^{-A^I\left[W_{t-1}^I-(\theta_{t-1}^I-\theta_{t-2}^I)P_{t-1}+(C_t^I)^\top H_t^I\Phi_{t-1}^I\theta_{t-1}^I+\frac{1}{2}m_t^I(\theta_{t-1}^I)^2+\frac{1}{2}(\Phi_{t-1}^I)^\top(H_t^I)^\top M_t^I H_t^I\Phi_{t-1}^I\right]} \\ &\times e^{-A^I[e_t^I((C_t^I)^\top F_t^I\theta_{t-1}^I+(F_t^I)^\top M_t^I H_t^I\Phi_{t-1}^I)+\frac{1}{2}(F_t^I)^\top M_t^I F_t^I(e_t^I)^2]}. \end{aligned} \tag{A-137}$$

Hence, the expectation at $t-1$ is

$$\begin{aligned} \mathrm{E}_{t-1}^I[J_t^I] =& -\rho_t^I e^{-A^I\left[W_{t-1}^I-(\theta_{t-1}^I-\theta_{t-2}^I)P_{t-1}+(C_t^I)^\top H_t^I\Phi_{t-1}^I\theta_{t-1}^I+\frac{1}{2}m_t^I(\theta_{t-1}^I)^2+\frac{1}{2}(\Phi_{t-1}^I)^\top(H_t^I)^\top M_t^I H_t^I\Phi_{t-1}^I\right]} \\ &\times\sqrt{\frac{\Xi_t^I}{\Sigma_t^I}}e^{-A^I\left[-\frac{1}{2}A^I\Xi_t^I((C_t^I)^\top F_t^I\theta_{t-1}^I+(F_t^I)^\top M_t^I H_t^I\Phi_{t-1}^I)^2\right]}, \end{aligned} \tag{A-138}$$

where $\Xi_t^I = \left[(\Sigma_t^I)^{-1}+A^I(F_t^I)^\top M_t^I F_t^I\right]^{-1}$. Taking the first order condition with respect to θ_{t-1}^I yields

$$\theta_{t-1}^I = \frac{P_{t-1}^I - P_{t-1}}{\lambda_{t-1}+\gamma_{t-1}}, \tag{A-139}$$

where

$$\lambda_{t-1} = \frac{\partial P_{t-1}}{\partial\theta_{t-1}^I} = \frac{\gamma_{t-1}^U}{N_U}, \quad \gamma_{t-1}^I = A^I\Xi^I\left((C_t^I)^\top F_t^I\right)^2 - m_t^I, \tag{A-140}$$

and the second order condition requires that

$$\lambda_{t-1}+\gamma_{t-1}^I > 0. \tag{A-141}$$

Comparing

$$P_{t-1}^I = \left[(C_t^I)^\top H_t^I - A^I\Xi_t^I(C_t^I)^\top F_t^I(F_t^I)^\top M_t^I H_t^I\right]\Phi_{t-1}^I + \lambda_{t-1}\theta_{t-2}^I, \tag{A-142}$$

with

$$P_{t-1}^I = \hat{V}_{t-1}^I - \mu_{t-1}X - g_{t-1}^I\hat{X}_{t-1}^U - f_{t-1}^I\Theta + \lambda_{t-1}\theta_{t-2}^I, \tag{A-143}$$

we obtain

$$
\begin{aligned}
g^I_{t-1} &= -\left[(C^I_t)^\top H^I_t - A^I \Xi^I_t (C^I_t)^\top F^I_t (F^I_t)^\top M^I_t H^I_t\right]_{1,3}, \\
f^I_{t-1} &= -\left[(C^I_t)^\top H^I_t - A^I \Xi^I_t (C^I_t)^\top F^I_t (F^I_t)^\top M^I_t H^I_t\right]_{1,4}.
\end{aligned} \tag{A-144}
$$

It can be verified that

$$
\begin{aligned}
1 &= \left[(C^I_t)^\top H^I_t - A^I \Xi^I_t (C^I_t)^\top F^I_t (F^I_t)^\top M^I_t H^I_t\right]_{1,1}, \\
\mu_{t-1} &= -\left[(C^I_t)^\top H^I_t - A^I \Xi^I_t (C^I_t)^\top F^I_t (F^I_t)^\top M^I_t H^I_t\right]_{1,2}.
\end{aligned} \tag{A-145}
$$

Thus, the asset price at $t-1$ has the similar form as t.

$$
\begin{aligned}
P_{t-1} &= \omega_t P^I_{t-1} + (1-\omega_{t-1}) P^U_{t-1} - f_{t-1}\Theta \\
&= \left(H^I_{P,t-1}\right)^\top \Phi^I_{t-1} + \omega_{t-1}\lambda_{t-1}\theta^I_{t-2} \\
&= \left(H^U_{P,t-1}\right)^\top \Phi^U_{t-1} + \omega_{t-1}\lambda_{t-1}\theta^I_{t-2}.
\end{aligned} \tag{A-146}
$$

Hence, the optimal holdings can be expressed in terms of state vectors

$$
\theta^I_{t-1} = (H^I_{\theta,t-1})^\top \Phi^I_{t-1} + \omega_{t-1}\theta^I_{t-2} = (H_{\Theta,t-1})^\top \Phi^U_{t-1} + \omega_{t-1}\theta^I_{t-2}, \tag{A-147}
$$

$$
\theta^{U,j}_{t-1} = (H^U_{\theta,t-1})^\top \Phi^U_{t-1} - \frac{\omega_{t-1}}{N_U}\theta^I_{t-2}. \tag{A-148}
$$

Lastly, we need to verify the value functions at time $t-1$,

$$
J^I_{t-1} = -\rho^I_{t-1} e^{-A^I\left[W^I_{t-1} + \frac{1}{2}(\Phi^I_{t-1})^\top M^I_{t-1}\Phi^I_{t-1} + (C^I_{t-1})^\top \Phi^I_{t-1}\theta^I_{t-2} + \frac{1}{2} m^I_{t-1}(\theta^I_{t-2})^2\right]}, \tag{A-149}
$$

$$
J^{U,j}_{t-1} = -\rho^U_{t-1} e^{-A^U\left[W^{U,j}_{t-1} + \frac{1}{2}(\Phi^U_{t-1})^\top M^U_{t-1}\Phi^U_{t-1} + (C^U_{t-1})^\top \Phi^U_{t-1}\theta^I_{t-2} + \frac{1}{2} m^U_{t-1}(\theta^I_{t-2})^2\right]}, \tag{A-150}
$$

where

$$
\rho^I_{t-1} = \rho^I_t \sqrt{\Xi^I_t / \Sigma^I_t}, \quad \rho^U_{t-1} = \rho^U_t \sqrt{\Xi^U_t / \Sigma^U_t}, \tag{A-151}
$$

$$
\begin{aligned}
M^I_{t-1} &= (H^I_t)^\top M^I_t H^I_t + m^I_t H^I_{\theta,t-1}(H^I_{\theta,t-1})^\top + H^I_{\theta,t-1}(C^I_t)^\top H^I_t \\
&\quad + (H^I_t)^\top C^I_t (H^I_{\theta,t-1})^\top - H^I_{\theta,t-1}(H^I_{P,t-1})^\top - H^I_{P,t-1}(H^I_{\theta,t-1})^\top \\
&\quad - A^I \Xi^I_t \left[(C^I_t)^\top F^I_t (H^I_{\theta,t-1})^\top + (F^I_t)^\top M^I_t H^I_t\right]^\top \left[(C^I_t)^\top F^I_t (H^I_{\theta,t-1})^\top + (F^I_t)^\top M^I_t H^I_t\right], \\
M^U_{t-1} &= M^U_t + m^U_t H_{\Theta,t-1}(H_{\Theta,t-1})^\top + H_{\Theta,t-1}(C^U_t)^\top + C^U_t (H_{\Theta,t-1})^\top \\
&\quad + H^U_{\theta,t-1}\left[H^U_{P,t} - H^U_{P,t-1} + \omega_t \lambda_t H_{\Theta,t-1}\right]^\top + \left[H^U_{P,t} - H^U_{P,t-1} + \omega_t \lambda_t H_{\Theta,t-1}\right](H^U_{\theta,t-1})^\top \\
&\quad - A^U \Xi^U_t \left[(H^U_{P,t})^\top F^U_t (H^U_{\theta,t-1})^\top + (C^U_t)^\top F^U_t (H_{\Theta,t-1})^\top + (F^U_t)^\top M^U_t\right]^\top \\
&\quad \times \left[(H^U_{P,t})^\top F^U_t (H^U_{\theta,t-1})^\top + (C^U_t)^\top F^U_t (H_{\Theta,t-1})^\top + (F^U_t)^\top M^U_t\right],
\end{aligned}
\tag{A-152}
$$

$$
\begin{aligned}
m^I_{t-1} &= \omega^2_{t-1} m^I_t + 2(1-\omega_{t-1})\omega_{t-1}\lambda_{t-1} - A^I \Xi^I_t \omega^2_{t-1}(C^I_t)^\top F^I_t (F^I_t)^\top C^I_t, \\
m^U_{t-1} &= \omega^2_{t-1} m^U_t + \frac{2\omega^2_{t-1}}{N_U}(\lambda_{t-1} - \omega_t \lambda_t) - A^U \Xi^U_t \omega^2_{t-1}\left[\left(C^U_t - \frac{1}{N_U} H^U_{P,t}\right)^\top F^U_t\right]^2,
\end{aligned}
\tag{A-153}
$$

$$
\begin{aligned}
(C^I_{t-1})^\top &= \omega_{t-1}(C^I_t)^\top H^I_t + \omega_{t-1}(m^I_t - \lambda_{t-1})(H^I_{\theta,t-1})^\top + (1-\omega_{t-1})(H^I_{P,t-1})^\top \\
&\qquad - A^I \Xi^I_t \omega_{t-1}(C^I_t)^\top F^I_t \left[(C^I_t)^\top F^I_t (H^I_{\theta,t-1})^\top + (F^I_t)^\top M^I_t H^I_t\right], \\
(C^U_{t-1})^\top &= \omega_{t-1}(C^U_{t-1})^\top + m^U_t \omega_{t-1}(H_{\Theta,t-1})^\top - \frac{\omega_{t-1}}{N_U}\left[H^U_{P,t} - H^U_{P,t-1} + \omega_t \lambda_t H_{\Theta,t-1}\right]^\top \\
&\qquad + \omega_{t-1}(\omega_t \lambda_t - \lambda_{t-1})(H^U_{\theta,t-1})^\top - A^U \Xi^U_t \omega_{t-1}\left(C^U_t - \frac{1}{N_U} H^U_{P,t}\right)^\top F^U_t \\
&\qquad \times \left[(H^U_{P,t})^\top F^U_t (H^U_{\theta,t-1})^\top + (C^U_t)^\top F^U_t (H_{\Theta,t-1})^\top + (F^U_t)^\top M^U_t\right].
\end{aligned}
\tag{A-154}
$$

A.9 Expected Trading Volume in the Imperfectly Competitive Market

The mean and variance of the informed trader's trading are

$$
\begin{aligned}
\mathrm{E}[\theta^I_t - \theta^I_{t-1}] &= \left[\left(\omega_t - \frac{\omega_t}{\lambda_t}(f^I_t - f^U_t)\right) - (1-\omega_t)\sum_{s=0}^{t-1}\left(\prod_{i=s+1}^{t-1}\omega_i\right)\left(\omega_s - \frac{\omega_s}{\lambda_s}(f^I_s - f^U_s)\right)\right]\Theta \\
&\qquad - (1-\omega_t)\left(\prod_{i=0}^{t-1}\omega_i\right)\theta^I_{-1} := \mu_{vol},
\end{aligned}
\tag{A-155}
$$

and

$$\begin{aligned}
\mathrm{Var}(\theta_t^I - \theta_{t-1}^I) &= \mathrm{Var}\left(\frac{\omega_t}{\lambda_t}\left(\mu_t + g_t^I - g_t^U\right)\hat{X}_t^U - (1-\omega_t)\sum_{s=0}^{t-1}\left(\prod_{i=s+1}^{t-1}\omega_i\right)\frac{\omega_s}{\lambda_s}\left(\mu_s + g_s^I - g_s^U\right)\hat{X}_s^U\right) \\
&= \left(\frac{\omega_t}{\lambda_t}\left(\mu_t + g_t^I - g_t^U\right) - (1-\omega_t)\sum_{s=0}^{t-1}\left(\prod_{i=s+1}^{t-1}\omega_i\right)\frac{\omega_s}{\lambda_s}\left(\mu_s + g_s^I - g_s^U\right)\right)^2 \mathrm{Var}(\hat{X}_0^U) \\
&+ \sum_{n=1}^{t-1}\left(\frac{\omega_t}{\lambda_t}\left(\mu_t + g_t^I - g_t^U\right) - (1-\omega_t)\sum_{s=n}^{t-1}\left(\prod_{i=s+1}^{t-1}\omega_i\right)\frac{\omega_s}{\lambda_s}\left(\mu_s + g_s^I - g_s^U\right)\right)^2 \left(K_{X,n}^U\right)^2 \Sigma_n^U \\
&+ \left(\frac{\omega_t}{\lambda_t}\left(\mu_t + g_t^I - g_t^U\right)\right)^2 \left(K_{X,t}^U\right)^2 \Sigma_t^U := \sigma_{vol}^2.
\end{aligned} \tag{A-156}$$

Therefore,

$$\mathrm{Vol}_t = \sqrt{\frac{2}{\pi}}\sigma_{vol} e^{-\frac{\mu_{vol}^2}{2\sigma_{vol}^2}} + \mu_{vol}\mathrm{erf}\left(\frac{\mu_{vol}}{\sqrt{2}\sigma_{vol}}\right), \tag{A-157}$$

where erf(·) is the error function. If there is no public information, then the risk premium at time zero is

$$\mathrm{E}[V - P_{np}] = f_{np}\Theta - \lambda_{np}\omega_{np}\theta_{-1}^I, \tag{A-158}$$

and the expected trading volume at time zero is

$$\mathrm{E}[|\,\theta_{np}^I - \theta_{-1}^I\,|] = \sqrt{\frac{2}{\pi}}\sigma_{np} e^{-\frac{\mu_{np}^2}{2\sigma_{np}^2}} + \mu_{np}\mathrm{erf}\left(\frac{\mu_{np}}{\sqrt{2}\sigma_{np}}\right), \tag{A-159}$$

where

$$\begin{aligned}
f_{np} &= \left(\frac{1}{A^I o_{V,0}^I + A^U o_{V,0}^U / N_U} + \frac{1}{A^U o_{V,0}^U / N_U}\right)^{-1}, \\
\omega_{np} &= \frac{1}{A^I o_{V,0}^I + A^U o_{V,0}^U / N_U} f_{np}, \quad \lambda_{np} = A^U o_{V,0}^U / N_U, \\
\mu_{np} &= \omega_{np}\Theta - (1-\omega_{np})\theta_{-1}^I, \quad \sigma_{np}^2 = \left(A^I o_{V,0}^I \frac{\omega_{np}}{\lambda_{np}}\right)^2 \mathrm{Var}(\hat{X}_0^U).
\end{aligned} \tag{A-160}$$

A.10 Market Participants' Welfare in the Imperfectly Competitive Market

Before trading, the informed trader has an initial holding denoted by θ_{-1}^{I}, thus the initial holding of uninformed traders can be denoted by

$$\theta_{-1}^{U,j} = \frac{\Theta - \theta_{-1}^{I}}{N_U}. \tag{A-161}$$

Assume before $t = 0$, traders have no incentive to trade and thus

$$\theta_{-1}^{I} = \theta_{-1}^{U,j} = \frac{\Theta}{N_U + 1}. \tag{A-162}$$

Also note that $\mathrm{E}[V] = \bar{V}$ and $\mathrm{E}[X] = 0$. We can rewrite the state vector at time $t = 0$ as

$$\begin{pmatrix} \hat{V}_0^I \\ X \\ \hat{X}_0^U \end{pmatrix} = F_0^I u + \ell_I, \quad \begin{pmatrix} \hat{V}_0^U \\ \hat{X}_0^U \end{pmatrix} = F_0^U u + \ell_U, \tag{A-163}$$

where

$$\ell_I = \bar{V} \begin{pmatrix} \frac{\tau_V}{\tau_V + \tau_\eta} \\ 0 \\ \frac{\tau_s/h}{\tau_X + \tau_s} \end{pmatrix}, \quad \ell_U = \bar{V} \begin{pmatrix} \frac{\tau_V}{\tau_V + \tau_0} \\ \frac{\tau_s/h}{\tau_X + \tau_s} \end{pmatrix}, \tag{A-164}$$

and

$$F_0^I = \begin{pmatrix} \frac{\tau_\eta}{\tau_V + \tau_\eta} & 0 & \frac{\tau_\eta}{\tau_V + \tau_\eta} \\ 0 & 1 & 0 \\ -\frac{\tau_s/h}{\tau_X + \tau_s} & \frac{\tau_s}{\tau_X + \tau_s} & -\frac{\tau_s/h}{\tau_X + \tau_s} \end{pmatrix}, \quad F_0^U = \begin{pmatrix} \frac{\tau_0}{\tau_V + \tau_0} & -\frac{\tau_0 h}{\tau_V + \tau_0} & \frac{\tau_0}{\tau_V + \tau_0} \\ -\frac{\tau_s/h}{\tau_X + \tau_s} & \frac{\tau_s}{\tau_X + \tau_s} & -\frac{\tau_s/h}{\tau_X + \tau_s} \end{pmatrix}. \tag{A-165}$$

We also rewrite the matrices of M_0^I and M_0^U as

$$M_0^I = \begin{pmatrix} a_{3\times 3}^I & b_{3\times 1}^I \\ (b^I)^\top & c^I \end{pmatrix}, \quad M_0^U = \begin{pmatrix} a_{2\times 2}^U & b_{2\times 1}^U \\ (b^U)^\top & c^U \end{pmatrix}, \tag{A-166}$$

and rewrite the vectors of C_0^I, C_0^U, $H_{P,0}^U$, and define vector ℓ as

$$C_0^I = \begin{pmatrix} \alpha_{3\times 1}^I \\ \beta^I \end{pmatrix}, \quad C_0^U = \begin{pmatrix} \alpha_{2\times 1}^U \\ \beta^U \end{pmatrix}, \quad H_{P,0}^U = \begin{pmatrix} d_{2\times 1}^U \\ k^U \end{pmatrix}, \quad \ell = \begin{pmatrix} 0 \\ \bar{V} \\ 0 \end{pmatrix}. \tag{A-167}$$

Now we substitute $\rho_T^I = e^{A^I \bar{V} X}$ back to the exponential, and let $\hat{\rho}_0^I = \rho_0^I / \rho_T^I$, which is a constant. Therefore, the value functions at $t = 0$ are

$$\begin{aligned} J_0^I &= -\hat{\rho}_0^I e^{-A^I\left[\frac{1}{2}(\Phi_0^I)^\top M_0^I \Phi_0^I + (C_0^I)^\top \Phi_0^I \theta_{-1}^I + \frac{1}{2} m_0^I (\theta_{-1}^I)^2 - \bar{V} X\right]}, \\ &= -\hat{\rho}_0^I e^{-A^I[\frac{1}{2} c^I \Theta^2 + \frac{1}{2} m_0^I (\theta_{-1}^I)^2 + \beta^I \Theta \theta_{-1}^I + (\alpha^I)^\top \ell_I \theta_{-1}^I + (b^I)^\top \ell_I \Theta + \frac{1}{2} \ell_I^\top a^I \ell_I]} \\ &\quad \times e^{-A^I[[(\alpha^I)^\top F_0^I \theta_{-1}^I + (b^I)^\top F_0^I \Theta + \ell_I^\top a^I F_0^I - \ell^\top] u + \frac{1}{2} u^\top (F_0^I)^\top a^I F_0^I u]}, \end{aligned} \tag{A-168}$$

$$\begin{aligned} J_0^U &= -\rho_0^U e^{-A^U\left[\frac{1}{2}(\Phi_0^U)^\top M_0^U \Phi_0^U + \left(H_{P,0}^U + C_0^U\right)^\top \Phi_0^U \theta_{-1}^I + \left(\frac{1}{2} m_0^U + \lambda_0 \omega_0\right)(\theta_{-1}^I)^2\right]} \\ &= -\rho_0^U e^{-A^U[\frac{1}{2} c^U \Theta^2 + (\frac{1}{2} m_0^U + \lambda_0 \omega_0)(\theta_{-1}^I)^2 + (\beta^U + k^U)\Theta \theta_{-1}^I + (\alpha^U + d^U)^\top \ell_U \theta_{-1}^I + (b^U)^\top \ell_U \Theta + \frac{1}{2} \ell_U^\top a^U \ell_U]} \\ &\quad \times e^{-A^U[[(\alpha^U + d^U)^\top F_0^U \theta_{-1}^I + (b^U)^\top F_0^U \Theta + \ell_U^\top a^U F_0^U] u + \frac{1}{2} u^\top (F_0^U)^\top a^U F_0^U u]}. \end{aligned} \tag{A-169}$$

In the competitive case, the coefficients in the quadratic term m_0 does not depend on the precisions of future public information $\{\tau_{\varepsilon,t}\}$, hence the welfare in the competitive case can be expressed as a product of two parts, with only one part depends on $\{\tau_{\varepsilon,t}\}$. While in the imperfectly competitive case, the coefficient matrices M_0, C_0, $H_{P,0}^U$, and m_0 all depend on $\{\tau_{\varepsilon,t}\}$. Thus, the part depending $\{\tau_{\varepsilon,t}\}$ cannot be separated from the welfare.

Hence, the welfare of informed trader is

$$\begin{aligned} \mathscr{W}^I =& \mathrm{E}[J_0^I] \\ =& -\sqrt{\frac{|\Xi_0^I|}{|\Sigma|}} \hat{\rho}_0^I e^{-A^I[\frac{1}{2} c^I \Theta^2 + \frac{1}{2} m_0^I (\theta_{-1}^I)^2 + \beta^I \Theta \theta_{-1}^I + (\alpha^I)^\top \ell_I \theta_{-1}^I + (b^I)^\top \ell_I \Theta + \frac{1}{2} \ell_I^\top a^I \ell_I]} \\ & \times e^{-A^I[[(\alpha^I)^\top F_0^I \theta_{-1}^I + (b^I)^\top F_0^I \Theta + \ell_I^\top a^I F_0^I - \ell^\top]\bar{u} + \frac{1}{2}(\bar{u})^\top (F_0^I)^\top a^I F_0^I \bar{u}]} \\ & \times e^{-A^I\left[-\frac{1}{2} A^I \left((F_0^I)^\top \left(\alpha^I \theta_{-1}^I + b^I \Theta + (a^I)^\top \ell_I + a^I F_0^I \bar{u}\right) - \ell\right)^\top \Xi_0^I \left((F_0^I)^\top \left(\alpha^I \theta_{-1}^I + b^I \Theta + (a^I)^\top \ell_I + a^I F_0^I \bar{u}\right) - \ell\right)\right]}, \end{aligned}$$

where

$$\Xi_0^I = \left[\Sigma^{-1} + A^I (F_0^I)^\top a^I F_0^I\right]^{-1}. \tag{A-170}$$

Similarly, the welfare of uninformed traders is

$$\begin{aligned}
\mathscr{W}^U =& \mathrm{E}[J_0^U] \\
=& -\sqrt{\frac{|\Xi_0^U|}{|\Sigma|}} \rho_0^U e^{-A^U\left[\frac{1}{2}c^U\Theta^2 + \left(\frac{1}{2}m_0^U + \lambda_0\omega_0\right)(\theta_{-1}^I)^2 + (\beta^U + k^U)\Theta\theta_{-1}^I + (\alpha^U + d^U)^\top \ell_U \theta_{-1}^I + (b^U)^\top \ell_U \Theta + \frac{1}{2}\ell_U^\top a^U \ell_U\right]} \\
& \times e^{-A^U\left[\left[(\alpha^U + d^U)^\top F_0^U \theta_{-1}^I + (b^U)^\top F_0^U \Theta + \ell_U^\top a^U F_0^U\right]\bar{u} + \frac{1}{2}(\bar{u})^\top (F_0^U)^\top a^U F_0^U \bar{u}\right]} \\
& \times e^{-A^U\left[-\frac{1}{2}A^U\left((\alpha^U + d^U)\theta_{-1}^I + b^U\Theta + (a^U)^\top \ell_U + a^U F_0^U \bar{u}\right)^\top F_0^U \Xi_0^U (F_0^U)^\top \left((\alpha^U + d^U)\theta_{-1}^I + b^U\Theta + (a^U)^\top \ell_U + a^U F_0^U \bar{u}\right)\right]},
\end{aligned}$$

where

$$\Xi_0^U = \left[\Sigma^{-1} + A^U (F_0^U)^\top a^U F_0^U\right]^{-1}. \tag{A-171}$$

Let $T = 0$ and shut down the public information, so that the welfare $\mathscr{W}_{np}^i$ without public information can be obtained,

$$\mathscr{W}_{np}^I = \mathscr{W}^I(T = 0), \quad \mathscr{W}_{np}^U = \mathscr{W}^U(T = 0). \tag{A-172}$$

The no-trade welfare for informed and uninformed traders is the same as that in the competitive market,

$$\mathscr{W}_{nt}^I = -\sqrt{\frac{\tau_V\tau_X}{\tau_V\tau_X - (A^I)^2}} e^{-A^I\left[\bar{V}\theta_{-1}^I - \frac{A^I}{2}\frac{\tau_X(\theta_{-1}^I)^2}{\tau_V\tau_X - (A^I)^2}\right]}, \quad \mathscr{W}_{nt}^U = -e^{-A^U\left[\bar{V}\theta_{-1}^U - \frac{A^U}{2}\frac{(\theta_{-1}^U)^2}{\tau_V}\right]}. \tag{A-173}$$