Liquidity Provision in the Secondary Market for Private Equity Fund Stakes

November 16, 2018

Abstract

We estimate the demand for private equity fund stakes in the secondary market using a broker's proprietary data on bids. We show that the demand response to aggregate liquidity shocks is negatively related to contemporaneous bids, and this relationship is stronger for funds that most likely are put for sale in times of low liquidity. The demand response to aggregate liquidity shocks is unrelated to future NAV-to-NAV returns and to future bidding behaviour. These results suggest a link between the variation in discounts in private equity stakes and the variation in liquidity provision in the secondary market for private equity.

Keywords: Private Equity, Secondary Markets, Liquidity, Liquidity Premium, Net Asset values.
JEL Classification Numbers: G24

1 Introduction

Once marginal due to contractual restrictions on transfers (see Lerner and Schoar (2004)), the secondary market for Private Equity Fund (PEF) stakes took off right after the 2008 financial crisis to an annual turnover of over $30 billion per annum. Behind this growth are the liquidity needs experienced by some investors during the crisis and an increased need of investors to rebalance their private equity exposures more frequently as it constitutes a larger fraction of their total asset holdings. (see Bollen and Sensoy (2016)).

At the same time, substantial capital has been earmarked by various entities to purchase PEF stakes on the secondary market. For instance, secondary funds raised over $160 billion between 2011 and 2016 to purchase such stakes.[1] Despite this influx of capital, Nadauld, Sensoy, Vorkink, and Weisbach (2017) show that, on average, transactions occur at a discount to the Net Asset Value (NAV) of a fund.[2]

A common explanation for the existence of a discount is that of imperfect liquidity provision: when funding liquidity is low and some investors are forced to sell their PEF stakes for cash, buyers who step up to absorb the supply can acquire assets for less than the underlying value of the asset (see Hege and Nuti (2011), Kleymenova, Talmor, and Vasvari (2012) and Nadauld et al. (2017)). In contrast, in a perfectly liquid market, the demand for the asset is perfectly elastic and sellers can place their trades without causing a price drop.

Thus, the first, and main, empirical test for the hypothesis of insufficient liquidity provision is that, for funds experiencing selling pressure, the correlation between the quantity demanded for a fund (number of bids received) and the price bidders are willing to pay is negative, i.e., that the demand for stakes in a given fund is downward sloping. To identify the demand component that responds to selling pressure, i.e., movements along the demand curve, we estimate the demand for each different type of PEF as a function of a set of liquidity variables, PEFs' past performance, and stock market return and volatility. The estimated model allows us to decompose the total demand into the component explained by

[1]Other buyers include funds-of-funds, which raise third party capital to invest in both the primary and secondary market for PEF stakes; and various asset owners (investment banks, hedge funds, endowments, pension funds, sovereign funds), which have their own teams to manage their purchases of PEF stakes.

[2]Nadauld et al. (2017) report average discounts ranging from an average of 46% in 2009 to 7% in 2014.

liquidity variables causing selling pressure, which we denote as *liquidity-driven demand*, and the component due to other trading motives, such as portfolio rebalancing due to aggregate income shocks, or changes in expected returns and risk.[3]

We estimate the demands for different types of PEFs using a unique data set consisting of all the bids submitted to a London-based secondary market intermediary for PEF stakes between September 2009 and December 2016. We form portfolios of PEFs according to their characteristics, and estimate a conditional demand system for each of these PEF types. The demand is jointly determined and modeled as a Poisson distribution with time-varying mean arrival rate of bids. Estimating the demands as a system allows us to control for correlation across fund types with similar characteristics (fund size, age, or location fixed effects) beyond the effect of aggregate liquidity shocks, so as to identify the differential effect that liquidity shocks have on different funds.

To the extent that different PEFs have different exposures to aggregate liquidity shocks, the liquidity provision hypothesis also implies that the strength of the partial correlation between demand and bids should depend on fund characteristics. Accordingly, our second set of empirical tests evaluates the correlation between bid levels and liquidity-driven demand in the cross-section of funds. Consider the age of a fund: as PEF stakes combine an equity claim and a credit line (granted to the fund manager for future investments), investors may be concerned that future capital calls arrives at times of low funding liquidity. This capital call risk decreases as the fund gets older and approaches being fully invested. Hence, we expect the negative correlation between bids and liquidity-driven demand to be present only among young funds.

We also conjecture that PE funds will exhibit different degrees of downward price pressure following liquidity shocks depending on their size. For example, large funds are regarded as less opaque than smaller counterparts and typically sold at a lower discount (see Nadauld et al. (2017)). Therefore, stakes in large funds may be the preferred private equity holdings to sell when the investor is looking to rebalance her portfolio. Such rebalancing needs may follow from a decrease in liquidity in the public equity market that drives equity returns

[3]The identification of the slope of the demand curve for illiquid assets using supply shocks dates from Shleifer (1986). For a recent application to the demand for stocks, see Koijen and Yogo (2017).

down and temporarily overexpose them to private equity. In other words, we would expect to identify a downward sloping demand curve for large funds only because these would be the ones chosen to be offered for sale in equilibrium. Alternatively, it could be that an investor's (over)exposure to PE is driven by her stakes in small funds because these tend to have a narrower investment focus across sectors of the economy. In that case, we could expect a stronger negative correlation for the small funds that picked the over-performing industries. Our empirical strategy aims to determine which fund size is most exposed to liquidity shocks.

Another plausible explanation for discounts is that they are a compensation for asymmetric information: sellers are likely to have better information about the value of the asset than potential buyers because they monitor the stakes and receive extensive information about fund value from the fund's managers. As in Yuan (2005), potential buyers will absorb any supply only at low enough prices to accommodate the possibility that the seller knows when the stake is overpriced.[4] Moreover, periods of low funding liquidity may be associated with greater uncertainty and greater asymmetric information and thus an increase in demand that is contemporaneous with lower bid values could be the result of asymmetric information.

To ensure that any negative correlation between bid and liquidity-driven demand is not driven by asymmetric information, we propose an additional empirical test following Llorente, Michaely, Saar, and Wang (2002). We reason that if selling is due to an adverse liquidity shock, then the future PEF's cash flows should not be affected. As a result, bids should eventually revert to their pre-shock values. If, on the other hand, selling is due to information, then bids adjust downwards gradually as they incorporate this private information. Moreover, given that reported NAVs cannot incorporate discounts to liquidity considerations under U.S. and European fair value rules (SFAS 157 and IFRS 13, respectively), the liquidity provision hypothesis, therefore, implies that current liquidity-driven demand flows have no predictability over the fund's NAV-to-NAV returns. But the asymmetric information hypothesis predicts NAVs will eventually reach a lower value as information is revealed.

[4]An asymmetric information effect may also occur if *bidders*, not investors, receive private information about the fund's value. Following Hasbrouck (1991), any private information known to some bidders, and, therefore, unobservable to the econometrician, is captured in the residual of the demand model. The liquidity-driven demand component is a projection of demand on publicly observable factors and does not contain, by construction, any information known only to the bidder.

Therefore, high levels of our measure of liquidity-driven demand would predict *lower future* NAV-to-NAV returns.

Our dataset consists of 4,365 bids on 497 LBO funds by 144 bidders. Our data provider is an important sell-side agent for investors wishing to exit via the secondary market. Its bid book is comprehensive and representative of the global market: the average bid in our sample each year matches very closely the average annual transaction price reported by Greenhill-Cogent. Most of the bids are made at a discount to NAV, and the discounts vary over time as well as cross-sectionally.[5]

To give a flavor of the estimated demand model, we find that demands for different types of funds and by different types of bidders respond differently to aggregate liquidity shocks. For example, the overall number of bids decreases in months where the yield curve steepens, yet this total effect comprises a decrease in bidding for old and middle-aged funds but a small increase for young funds. Similarly, an increase in the Fontaine-Garcia (Fontaine and Garcia (2012)) index of bond market illiquidity, which captures a tightening of current aggregate funding liquidity, is associated with an overall decrease in demand, but an increase in demand for young funds. We observe similarly heterogeneous patterns across fund sizes, where demand flows towards the largest funds in response to liquidity shocks. While preliminary, this evidence is consistent with liquidity provision in that demand responds positively to liquidity shocks mostly for funds that should be vulnerable to selling pressure.

Equipped with this estimated demand model, we project the demands for different fund types onto aggregate liquidity proxies to construct the estimated liquidity-driven demand flows. As predicted by the liquidity provision hypothesis, we find: (i) a significant negative correlation between bid levels and the liquidity-driven demand; (ii) the negative correlation is strongest for the youngest and the largest funds; (iii) the negative correlation does not appear to represent private information: past demand shocks are not correlated with current bids and liquidity-driven demand flows do not predict future NAV-to-NAV fund returns.

[5] Our descriptive statistics closely match those by Nadauld et al. (2017) even though that study uses transaction prices from a US-based financial intermediary. This similarity is not surprising given that i) bidders submit the same bid to several intermediaries to maximize the probability of matching a selling interest; and ii) submitting bids is costly both in due diligence and in terms of reputation. Hence, bids are informative of the bidder's true intended price.

To quantify the effects described above, we provide estimates of the rent expected for liquidity provision by fund type. We also identify which type of investors, if any, act systematically as liquidity providers. To measure the discount required for providing liquidity, we use our estimate of the sensitivity of bid levels to the liquidity-driven demand. Indeed, the slope of the asset's demand curve is akin to the illiquidity measure in Kyle (1985) ('Kyle's lambda'). We find, for example, that a one standard deviation increase in the liquidity-driven demand for young funds is associated with an average drop in bid values by up to 2.5 percentage points. This associated drop in bids can reach up to 3.6% in times when aggregate liquidity is lowest. For the very large funds the same estimate is 2.3 percentage points. While these differences are important, we note that these are measures of the impact on bids and not necessarily on *prices*. Since the seller can choose to hold and find higher bidders, we interpret them as an upper bound to this component of expected liquidity discounts.

To identify the liquidity providers, we estimate the liquidity-driven demand by investor type: secondary funds, funds-of-funds, and other asset owners (pension funds, endowments, banks). Secondary funds are responsible for three quarters of the bids in our sample and their demand respond the strongest to most macroeconomic factors. Yet we find that asset owners, not secondary funds, are those that systematically increase their bidding in times of low aggregate liquidity and whose bids are negatively correlated with liquidity-driven demand. Asset owners submit bids that are on average one percentage point lower when their liquidity-driven demand increases by one sample standard deviation. One possible explanation is that asset owners are not constrained to invest exclusively in the secondary market. These investors, therefore, have the flexibility to bid in this market when it is most lucrative. Other types of investors have a more restricted mandate in that they basically have a pre-committed amount to deploy over an investment period. A complementary explanation is that secondary funds and funds-of-funds lose their ability to provide liquidity when aggregate liquidity is low, becoming liquidity demanders themselves like the slow moving merger arbitrageurs in Mitchell, Pedersen, and Pulvino (2007).

Our study is one of three empirical studies of the secondary market for PEF stakes. Like us, Kleymenova et al. (2012) utilize bid-level data. They explore the determinants of bids in auctions of PEF stakes, finding that bids are lower if fewer bidders participate in an auction. The liquidity proxies they use, e.g., number of bids, are fund specific and may confound the liquidity motive with other motives for trading. In contrast, in our work, we decompose the number of bids that a fund receives into what is caused by liquidity shocks and what is driven by other trading motives. Our analysis also *excludes* the financial crisis, which is needed in order to characterize discounts in normal market conditions. Like us they find that young funds (i.e., funds with less undrawn capital) seem to be the most illiquid ones and that non-traditional investors, as opposed to dedicated secondary funds, increase market liquidity. Nadauld et al. (2017) use transaction price data and find that buyers of PEF stakes earn a higher return than sellers, a difference they attribute to compensation for liquidity provision. Our analysis, complements theirs by identifying the liquidity provision channel using demand data, and by studying how shocks to liquidity shift demand for PEF stakes. We use our study of the demand for PEF stakes to study the cross-sectional response of funds to liquidity shocks. In the cross section of funds, liquidity shocks hit young funds the most. The response of investors to the shocks varies, with only asset owners actively demanding these young funds.

In Lerner and Schoar (2004), private equity investments are illiquid by design so as to screen investors with deep pockets, who are therefore likely to fund randomly timed capital calls or future financing rounds. The costs of exiting private equity investments via the secondary market are modeled and calibrated by Bollen and Sensoy (2016). Our empirical specification of the determinants of aggregate liquidity shocks follows from the motives for selling in their model. We contribute to this literature by showing that shocks to public equities, corporate bonds, and treasuries markets spillover to the private equity secondary market via the liquidity provision channel, quantifying their price impact, and identifying the funds most affected by this risk.

Recent theories of over-the-counter markets describe how certain investors endogenously take up intermediation roles in the absence of market makers. In Lagos and Rocheteau (2009), Hugonnier, Lester, and Weill (2014), and Chang and Zhang (2015), liquidity suppliers are not the agents with the highest current valuation. Our evidence suggests that the liquidity providers are not the investors specialized in the secondary market acquisitions but other asset owners (e.g., investment banks, pension funds, endowments, sovereign wealth funds), i.e., those not limited to invest in private equity and possibly with a lower valuation.[6]

2 Liquidity Shocks in Private Equity

Bollen and Sensoy (2016) explicitly model the cost from LPs potentially having to sell at a discount in the secondary market. In their model, investors (called limited partners or LPs for short) demand immediacy to sell after receiving a liquidity shock, idiosyncratic or aggregate. These exogenous shocks capture, in a reduced form, three different motives for liquidity selling. First, the costs of funding uncalled capital commitments, whose timing is random, may increase unexpectedly. Second, the investor may suddenly have an increased demand for cash. Third, unexpected poor performance in public equity markets or, conversely, overperformance in private equity investments, may cause investors' portfolios to be overweighted in private equity with respect to their institutional targets.

Below we discuss theories that will guide our specification of liquidity shocks via the three selling channels above. We add to the work of Bollen and Sensoy (2016) by hypothesizing which characteristics of private equity funds make them more sensitive to these types of shocks and, therefore, more likely to be put for sale. We discuss potential drivers of the demand operating via different channels in Section 4, when we present the empirical specification of our demand model.

2.1 The Capital Call Risk Motive

Brunnermeier and Pedersen (2009) show that investors that are financially constrained prefer to sell, or at least avoid owning, capital-intensive assets to reduce the likelihood of being constrained in the future. In this case, assets can be sold below their fundamental value. Private equity investments are capital intensive because they represent a capital commitment where the LPs cede control over the timing of investments to the GP and the capital calls

[6] Our work also adds to the literature on risk and return of illiquid or thinly traded assets; see, for example Acharya and Pedersen (2005), Albuquerque and Schroth (2015), Ang, Papanikolaou, and Westerfield (2014), Bongaerts, De Jong, and Driessen (2011), Brunnermeier and Pedersen (2009), Duffie, Gârleanu, and Pedersen (2005), Franzoni, Nowak, and Phalippou (2012), Gârleanu (2009), Jurek and Stafford (2015), Longstaff (2009), Sagi (2017), and, for a more private equity focused modelling of illiquidity, Sorensen, Wang, and Yang (2014).

are stochastic. Moreover, investors face a risk of *clustering of capital calls* if several of the invested funds call their capital at the same time. For example, capital calls cluster when GPs use them to pass-through the cash needs of financially constrained or distressed portfolio companies in recessions (Hege and Nuti (2011)).

The capital call risk motive predicts that a tightening of current or future funding liquidity, which increases the expected costs of funding future capital calls, may trigger liquidity sales by LPs in order to ease the pressure from capital requirements. And LPs are likely to sell precisely the assets with the largest amount of uncalled capital, i.e., young funds.

2.2 The Preference for Cash Motive

A preference for cash may arise when cash constrained investors cannot borrow externally to pursue better investment opportunities. Albuquerque and Schroth (2015) proxy such states of liquidity with the combination of high funding costs and high availability of investment opportunities. The preference for cash motive predicts that in case of a liquidity shock during such periods, LPs will choose to sell their more liquid PE funds, as they satisfy the need for immediacy of cash at the smallest discount. Nadauld et al. (2017) find that the probability that a fund gets traded in the secondary market, and the price paid for it, are increasing in fund size, implying that cash pressured LPs may attempt to sell the larger funds in their portfolio. They also find that middle-aged funds are more likely to be sold and experience lower discounts.

2.3 The Portfolio Rebalancing Motive

Institutional investors set targets ex ante for the allocation of private equity in their portfolio. Their portfolios may suddenly become overweighted following periods where the relative performance of private (public) equity has been unusually high (low). As PE funds last for a long time, sales through the secondary market may be required to decrease the allocation back to its target. The portfolio rebalancing motive predicts that negative shocks to stock market excess returns trigger sales of PEFs. To reduce their overall exposure to private equity at the least cost, LPs may attempt to sell first their stakes in the larger funds.

The exposure of particular funds to sudden large rebalancing needs may also depend on the degree of diversification in their investments. Large funds tend to invest across industries and are therefore more exposed to aggregate economy-wide and stock market fluctuations. Smaller funds typically focus on single industies and, as a result, display higher volatility and cause LPs to be overinvested in top performing industries. Therefore, a cross-sectional prediction of the portfolio rebalancing motive is that larger funds are more exposed to aggregate economy-wide fluctuations whereas smaller funds may see more selling pressure when industry performance displays greater dispersion.[7]

2.4 Summary

The three motives for selling private equity in the secondary market and causing a price impact can be proxied by indices of aggregate funding liquidity, relative performance between the public and private equity classes, the dispersion in returns across industries and the state of investment opportunities combined with funding liquidity in the economy. Young funds are exposed to such shocks via the capital call risk, whereas large and middle-aged funds may be sold when cash is scarce and to rebalance a portfolio suddenly overinvested in PE after a stock market downturn. Small funds may be vulnerable to industry performance heterogeneity also via the portfolio rebalancing motive.

We investigate these hypotheses via the specification of the demand model in Section 4. We discuss, and control for, determinants of demand other than liquidity.

3 Data

3.1 Institutional setup

Investors in a private equity fund (simply referred to as *fund*) are called Limited Partners (LPs). They can use an Over The Counter (OTC) secondary market to transfer their limited

[7] We note that rebalancing motive for trading refers to large portfolio adjustments. As such, it is different from marginal adjustments to update the optimal allocations within portoflio targets. Marginal rebalancing, which would respond to smaller changes in expected returns and variances of individual funds, may well be a driver of demand and our empirical specification will control for such fund-specific measures of expected returns and risk.

partnership stake (fund stake) to other investors. The fund manager – called General Partner (GP) – needs to approve the transfer. The buying LP pays the selling LP an amount expressed as a fraction of the fund's latest Net Asset Value (NAV). As NAVs are reported at a quarterly frequency, any cash flow occurring from the date of the latest reported NAV to the transaction date are taken into account when determining the actual cash transfer between the two parties: the purchase price is reduced by the net cash flows (capital distributions minus capital investments) that occurred in that lapse of time.

NAVs are an estimate made by the GP of the fair value of all ongoing investments in a fund. According to the related fair value accounting rules (SFAS 157 for the U.S. and IFRS 13 for Europe), a NAV is an "estimate of the price at which an orderly transaction to sell these assets would take place between market participants under current market conditions." Implicit in the assumption of an orderly transaction is that the GP should not factor in the NAV the illiquidity of the fund's underlying assets. Crain and Law (2017) show that since the implementation of these rules (years 2006-2007), NAVs have been, on average, close to the subsequent aggregate fund net cash flows. We shall use this technical aspect of fair value accounting rules in our identification strategy below.

At fund inception – the year of which this occurs is referred to as vintage year – LPs commit to provide a certain amount of capital to the fund over a fixed period of time (referred to as investment period).[8] The amount, number, and timing of *capital calls* are left to the discretion of the GP and are unknown at inception. GPs call capital as they need it, up to the total amount committed at inception. LPs buying a PEF stake inherit any remaining capital commitments. Sellers on the secondary market, therefore, sell a combination of a stake in on-going investments and unfunded commitments.

Although the first organization specializing in buying fund stakes in the secondary market was created in 1984, transaction volumes did not increase markedly until 2007. One reason behind the birth of this market has been the realization that LPs need to rebalance their private equity portfolio as it became a large fraction of their overall portfolio. At the same

[8] The aggregate commitments to a fund across all of its LPs is referred to as fund size. The difference between fund size and the sum of all capital calls to a given date is referred to as unfunded commitments, or dry powder. Dry powder is maximal at the time of fund inception and minimal past the investment period.

time, GPs appeared to no longer view secondary market trades as bad signals about their funds. As a result, numerous private equity funds-of-funds and other asset owners (investment banks, hedge funds, endowments, pension funds, sovereign funds) started to actively trade in this market, alongside specialized secondary funds.[9] Greenhill Cogent estimates that yearly transaction volumes doubled in 2007 and, except for 2009, then increased smoothly from $18 billion in 2007 to $37 billion in 2016.

3.2 Dataset

This secondary market is intermediated by specialized organizations. One such financial intermediary, based in London, and operating on that market since September 2009 (its other operations are older), gave us access to its entire database. Most of the requests it receives are from LPs offering to buy fund stakes at an indicative price (the bid), i.e. the demand side of the market. When bids are quoted as a range (e.g. 80%-85%), we use the midpoint.

Importantly, this financial intermediary is viewed as a market leader for *individual* fund stake transactions (in contrast to transactions on *portfolios* of funds.) This is an attractive feature because it is problematic to infer the valuation of individual fund stakes from the pricing of a portfolio of fund stakes. To illustrate, a report by Cogent (July 2016) states that "supply/demand mismatch for newer funds has begun to incentivize opportunistic sellers to include some recent vintage funds in sale portfolios in order to achieve their pricing objectives on portfolios that also include older and less desirable funds." As a result, some funds may appear to be more demanded than they actually are and some funds may receive a higher price than they would have had if sold on their own.

For each bid received, we observe the name of the fund, but not that of the bidders (only their type is given to us).[10] We then use Preqin datasets to construct the characteristics of individual funds.[11] We observe that 75% of bids are for buyout funds, and that 94%

[9] Paris-based Ardian raised the largest secondary fund to date with $10.8 billion of committed capital in 2016. These secondary funds (and all funds-of-funds) are also structured as limited partnerships.

[10] The funds most represented in our dataset are well-known funds. Those with the highest numbers of bids are: Apax Europe VII, Bain Capital IX, Blackstone V, and Thomas H Lee VI. They received more than 30 bids each.

[11] We match the funds in our dataset to two databases provided by Preqin. The first database contains fund

of the bids are for funds focusing on Western Europe (including Scandinavia and UK) and North America. In order to work with a homogeneous sample, we include in our sample only buyout funds focusing on these geographies.[12]

We collect all the bids received up until December 2016, and use them to measure the demand for each type of fund in the secondary market. Demand, not executed trades, is necessary for our tests.[13] Specifically, demand is measured by the total number of bids received in a given month for a given type of fund. Figure 1 plots the demand for all the funds in our sample between September 2009 and December 2016. In total, there are 4,365 bids. We do not observe a time trend in demand, but note some marked cycles and high volatility.

Bidders are classified into one of three types. Secondary funds (SF) are the most common type of bidders in our dataset. Funds-of-funds (FoFs) engage in both primary and secondary transactions; they are the second most common type in our dataset. We pool together the ten other types under the label asset owners (AOs). AOs includes banks, pension funds, endowments, sovereign wealth funds, and family offices. The countries with most bidders are the U.K. (35%), the U.S. (23%), Switzerland (13%), France (12%), Germany (4%), Norway (4%), Spain (3%), Netherlands (2%), and Canada (1%).

Insert Figure 1 Here

We expect the bids received by our financial intermediary to be the best estimate of the market value of a given fund stake at that given point in time. Although we can never rule out strategic behavior, it is important to note that submitting bids is costly both in terms of due diligence and in terms of reputation. A bidder attempting to manipulate the perception of demand, submitting unrealistic bids, or reneging on a submitted bid, would be quickly

characteristics such as size, vintage year, fund type (e.g., buyout, venture, infrastructure), and geographic focus. The second Preqin database contains data on fund cash flows and NAVs. These data allow us to calculate future fund performance. Note that we compute a fund's performance in the currency the fund is raised in.

[12] Venture capital funds and emerging market funds in our sample have much larger discounts on average.

[13] Demand data are also more informative about the variation in bidders valuation over time than actual transaction prices. The latter omit the lowest bids when patient sellers can afford to wait for a price recovery. Demand data allows LPs or the econometrician to predict the future state of demand, which is useful to pin down the timing of the sale, as a function of forecastable state variables.

spotted and excluded from this market (financial intermediaries would ignore future demand or supply emanating from that organization).

In addition, empirically, we observe a number of completed transactions and do not find any significant deviation between initial bids and transaction prices. Moreover we compare the time-series of average bids to the time-series of average transaction prices as reported by the US market leader on the secondary market (Greenhill Cogent). The correlation between the two time-series is 0.94, with the two averages being almost the same; see Figure 2 for a plot.[14] The average bid in both samples is low in the early part of the period and displays a slight upward trend at the end of the sample. Similarly, we compare the time-series of yearly averages in our sample to the transaction prices in Nadauld et al. (2017). The correlation is 0.99, and the averages are the same.

Most importantly, our identification strategy relies on a good estimate of the demand for stakes by liquidity providers. As a result, the unit of analysis is naturally the number of bids received by our financial intermediary rather than the price at which transactions have closed, or the number of transactions closed. It is also worth noting that once a potential buyer has identified the funds it wishes to bid for (and the offered price), it is costless to submit that demand to all financial intermediaries. This may explain why we observe such a high correlation between statistics from US-based intermediaries and those from our London-based intermediary.

Insert Figure 2 Here

3.3 Descriptive statistics

Table 1 summarizes the bids in our data set. All variables are defined in Appendix A. The average (median) bid is 88.3% (90.0%) of the latest reported NAV (henceforth referred to as NAV). We observe a surprisingly wide variation in the bids. About one-in-five bids is made at or above NAV, and the same proportion is made below 75% of NAV (see Figure 3 for a

[14] Greenhill Cogent, in its (*Secondary Market Trends & Outlook*, reports semi-annual statistics from 2010 to 2013 and annual statistics for 2009, 2014, 2015, 2016. The figure averages our bids at the same frequency for ease of comparison.

histogram).[15]

Insert Figure 3 Here

Table 1 also shows that there are 144 unique bidders, evenly spread out between our three bidder types. However, SFs place more bids (61 bids per bidder), and therefore account for the majority of the bids in our sample (about 75%). This is consistent with SF being specialized and solely acting on the secondary market. We also note that FoFs place the lowest average bids (82.5%, median = 85%). AOs place the highest bids (93.9%, median = 97.5%) and on average target funds younger than those targeted by FoFs or SFs, i.e., 5.9 vs. 6.5 or 6.8 years old, respectively.

Insert Table 1 Here

Data on the fund's cash payouts and, therefore, their actual performance, are available only for a sub-set of our data. When we use these characteristics the sample is reduced to 3,093 bids. However, a comparison of the summary statistics between Panels A and B of Table 1 does not reveal any systematic differences in fund age or size between the full sample and that sub-sample.

3.4 Fund classification

Some funds in our data concentrate a majority of bids whereas others of very similar size and age receive very few or no bids for given spells of time. By aggregating the number of bids per month over all funds of similar characteristics we guarantee that the demand for each group (i.e., type) of funds has a Poisson distribution. If done correctly, this grouping of funds into types not only has the advantage of a Poisson distribution's tractability in the estimation of a large demand system but also of including all funds that are homogeneous from the perspective of a bidder providing liquidity to LPs of *any* fund that is equally affected by aggregate liquidity shocks.

[15] We winsorize the bid distribution at the 1st and 99th percentiles, which correspond to bids of 30% and 120% of NAV, respectively. These bids seem extreme and may reflect an unusual situation for a given fund. As the bids are all correctly entered by the financial intermediary, we do not drop them out of our sample, but winsorize them to limit their influence in our results.

We use a statistical approach to determine the classification procedure. We estimate a piece-wise Logit model for the likelihood of a fund with a certain set of characteristics to receive at least one bid in a given quarter. This analysis reveals that i) three characteristics stand out: fund age, fund size and region of investment focus; and that ii) the effects of fund size and fund age are non-linear. The classification procedure is detailed in Appendix B.

We identify four breaking points in fund size and three breaking points in fund age. Funds are then assigned to one of four size categories (bottom tercile [Small], inter-tercile [Mid-size], 66th to 90th percentile [Large], larger than 90th percentile [Very large]), of three age categories (under four years old [Young], between four and seven years old [Mid-life], and over eight years old [Old]), and of two regions of investment focus (Europe, US). Panel A of Table 2 summarizes the total number of bids per month for each of these 24 (= 4 × 3 × 2) fund groups.

Insert Table 2 Here

The number of bids per month is on average higher and more volatile for the Large and Very Large funds. The average number of bids per month per fund type does not vary significantly across age categories, although the demand for Young funds is the most volatile. In consistency with the Poisson assumption, which we test formally below, the distribution of the number of bids per month is right skewed for each size or age category.

4 An empirical model of bid arrivals

4.1 Demand model

Let $X_{i,t}$ be the total number of bids for funds of type $i = 1, ..., 24$ during month $t = 1, ..., 88$. We assume that $X_{i,t}$ has a Poisson distribution in which the average number of bids per month, $\lambda_{i,t}$, is conditional on a set of time-varying observable variables, $\mathbf{Z}_t$, that are common to all the funds, e.g., measures of the state of aggregate liquidity, and a vector of time-varying type-specific control variables, $\mathbf{W}_{i,t}$, e.g., the average past performance of funds of type i. The Poisson assumption is natural given that the number of bids can only be positive and that the distribution of bids per month is positively skewed for each fund type. Moreover, even if the Poisson assumption about the arrival of bids was more natural for an individual fund than for a group of funds, the sum of individual arrivals, i.e., the total bids for all funds in a group, will also have a Poisson distribution if the arrivals of bids across funds within the group were independent conditional on the mean arrival rate.

The conditional density of X_{it} is therefore given by

$$\begin{aligned} f(x_{it}|t, \mathbf{Z}_t, \mathbf{W}_{it}, \mathbf{1}_i) &\equiv \Pr(X_{it} = x_{it}|\mathbf{Z}_t, \mathbf{W}_{it}, \mathbf{1}_i) \\ &= \frac{\lambda_{it}^{x_{it}} \exp(-\lambda_{it})}{x_{it}!}, \end{aligned}$$

where $\mathbf{1}_i$ is a vector of nine binary variables for the characteristics of each fund type i, e.g., Small (yes/no), or Old (yes/no), or US (yes/no), etc. The bid arrival rates are written parametrically as

$$\lambda_{it} = \exp(\tau t + \mathbf{Z}_t' \boldsymbol{\beta}_i^Z + \mathbf{W}_{it}' \boldsymbol{\beta}_i^W + \sum_{c=1}^{9} 1\{c = i\}\gamma_c) \ \forall i, t, \tag{1}$$

The exponential function guarantees that the number of arrivals is positive. Note that $\boldsymbol{\beta}_i^Z$ is type-specific, allowing for demand for different fund types to respond differently to macroeconomic shocks. The parameter τ is a common time trend and the parameters $\boldsymbol{\gamma}$ are fixed effects.

Assuming that bids across fund types are independently distributed conditional on $\mathbf{Z}_t$ and $\mathbf{W}_{it}$, then the log-likelihood function for the number of bids $x_{i,t}$ for each fund type in each month is as follows:

$$\ln L(\boldsymbol{\beta}^Z, \boldsymbol{\beta}^W, \tau, \boldsymbol{\gamma}|\{x_{i,t}\}_{i,t}, \{\mathbf{Z}_t\}_t, \{\mathbf{W}_{i,t}\}_{i,t}) = \sum_{t=1}^{88} \left(\sum_{i=1}^{24} x_{i,t} \ln \lambda_{i,t} - \sum_{i=1}^{24} \lambda_{i,t} - \sum_{i=1}^{24} \ln x_{it}! \right).$$

We estimate $\boldsymbol{\beta}^Z, \boldsymbol{\beta}^W, \tau$, and $\boldsymbol{\gamma}$ by maximizing this expression. Note that even if x_{it} is drawn independently from $x_{i't}$ for any $i \neq i'$ conditional on $\mathbf{Z}_t, \mathbf{W}_{it}$, and $\mathbf{1}_i$, the two demands x_{it} and $x_{i't}$ are unconditionally correlated. For this reason, the demands are jointly estimated as a system using the full panel rather than as independent time-series.

The assumption that the Poisson arrival rate of bids, λ, is an exponential function of its determinants $\mathbf{Z}_t$ and parameters $\boldsymbol{\beta}$ (equation 1), guarantees that the maximum likelihood estimator of $\boldsymbol{\beta}$ is unique.[16] The intuition for the estimated sign of each coefficient in $\boldsymbol{\beta}^Z$ is similar to that of OLS and discussed in more detail in Appendix C. Essentially, the maximum likelihood estimator infers positive (negative) loadings for each variable in $\mathbf{Z}_t$ whenever this variable is positively (negatively) associated to the demand for the given fund type, conditional on the demands for other funds of similar size, age, or location.

4.2 Specification

This sub-section describes the set of explanatory variables used to model the average bid arrival rate for each fund type. The economy-wide variables used as explanatory variables ($\mathbf{Z}_t$) are described in Table 2 - Panel B.[17] All the variables are defined in Appendix A.

4.2.1 Aggregate liquidity conditions and capital call risk

We include in $\mathbf{Z}_t$ the monthly changes in the logarithm of the Federal Reserve System's balance sheet ($\Delta \ln$(FED's Total Assets)) as changes to the borrowing cost.[18] To proxy for time-varying aggregate borrowing constraints, we include the change in the logarithm of the outstanding volume of commercial paper ($\Delta \ln$(Commercial paper)). Following Fontaine and Garcia (2012), we also include the monthly changes in the bond premium attributed to funding liquidity risk (Δ(Fontaine-Garcia)).

The specification includes the monthly changes in the slope of the yield curve (term spread), measured as the difference between the 10-year and 3-month yields on US Treasury bills (Δ(Yield curve slope)), to capture changing expectations of future borrowing costs. Higher future interest rates may force limited partners to sell in anticipation of increasingly

[16]It is straightforward to show that the system of first-order conditions to maximize the likelihood function is monotonically decreasing on each $\beta_{i,k}$.

[17] Statistics on the portfolio specific variables – which all relate to the fund portfolio relative past performance – are not tabulated.

[18]We choose this variable over the Federal Funds overnight rate because it varies more over time and probably describes the liquidity state better than the interest rate at its nominal lower bound. Moreover, the FED's quantitative easing policy targeted assets of maturities closer to PE's than to financial instruments associated to the Fed Funds rate.

expensive funding of future capital calls.

Temporary bond market illiquidity may also entice limited partners to sell PEF stakes by expecting a strong downward price impact from selling bonds when cash is needed or by suddenly being underweighted in bonds relative to private equity when bond prices are low. Therefore, we complement the Δ(Fontaine-Garcia) index, which incorporates bond market illiquidity shocks, with the monthly differences in the spread between BAA- and AAA-rated corporate bond yields (ΔCorporate spread). As Driessen (2005) shows, the corporate bond spread not only contains default risk information, but is also a priced macroeconomic liquidity risk factor.

4.2.2 Aggregate investment opportunities and the preference for cash

To measure the time variation in the preference for cash, we follow Albuquerque and Schroth (2015) and specify two variables in $\mathbf{Z}_t$ that capture states of the economy when investment opportunities are high but cash is scarce. We include the product of the contemporaneous real OECD GDP growth rate and a binary variable that equals one in months where the Federal Reserve System's balance sheet is contracting.[19] We expect more selling pressure due to a sudden preference for cash when the economy is booming and the FED is tightening the money supply.

To control for increased preferences for cash coming from investment opportunities in the stock market, we also include the product of a FED contraction monthly indicator and the contemporaneous monthly return of the value-weighted S&P 500 index ($R_{\text{S\&P500}}$).

4.2.3 Portfolio rebalancing motives

Positive shocks to the overall returns of private equity may cause investors to be overweighted in this asset class, forcing them to sell. Therefore, we also include, as a proxy for the changing portfolio rebalancing motives, the changes in the logarithm of Private Equity's investment multiple ($\Delta \ln$(Private Equity's TVPI)).

[19] The OECD GDP figures are reported at a quarterly frequency. We impute the same growth rate to each month belonging to the same quarter.

Exceptional performance by some industries in the economy would impact more the values of funds heavily invested in those industries. Since small private equity funds tend to focus on particular sectors of the economy, a higher volatility of past returns across industries skews the performance of funds focused on the winning overperforming industries, making investors in these funds over-weighted in private equity and, therefore, pressured to sell. We include the cross-sectional standard deviation of the Fama and French 49 industry portfolio returns (Cross-Industry Volatility) as a proxy for the rebalancing motive channel of the liquidity-driven demand.

4.2.4 Other controls

We control for the OECD GDP growth rate and the monthly value-weighted S&P 500 index return to account for income effects shifting the private equity demand curve. Similarly, we allow for the PE demand to be affected by current perceptions of future uncertainty, which, as in standard in the literature, can be proxied by the CBOE's implied volatility index, VIX.[20]

The vector $\mathbf{W}_{i,t}$ includes fund type-month specific determinants of demand. We expect demand for PEF stakes to contain also a component related to the the need to marginally adjust optimal allocations, as a function of the past performance of the fund itself. To control for a fund's cross-sectional relative performance, we include the average performance of all funds of the same type in excess of the performance by all other funds over the last six months. To control for its relative recent past performance, we include a binary variable indicating whether the average fund type performance over the last months is above the third quartile over the last 3 years.

4.3 Demand model: results

4.3.1 Goodness-of-fit

Panel A of Table 3 shows that our Poisson model, together with the retained set of explanatory variables, fit well the observed demand for most fund types. The average correlation

[20] Note that the VIX index could also affect demand via its effect on current pricing dislocations (see Pasquariello (2014)).

between the actual and predicted demand is 0.53 (across the 24 fund types). The average and median p-values of the Wald statistic – null hypothesis of which is that all parameters of the demand for a given fund type are zero – are both below 0.01 (across the 24 fund types).

Insert Table 3 Here

The average p-value of the binomial deviance statistic, under the null joint hypotheses that all the observed and predicted demand are equal and that demand has a Poisson distribution conditional on the explanatory variables, is 0.2.[21] On average, we cannot reject that the model is correctly specified and that the Poisson assumption is met.

In addition to these standard statistics, we compare the correlation structure of the observed demand with that of the predicted demand. We thereby assess the extent to which our model can explain the correlation between the demand of, say, US large young funds and that of US small old funds. Figure 4 plots each pair of predicted versus actual correlations. There are $\frac{24\times(24-1)}{2}$ pairwise correlations, and thus as many pairs plotted. We observe that the data points cluster around the 45-degree line, which indicates that the correlation structure between observed demand time-series is well-captured by the set of explanatory variables together with the Poisson modeling structure we have used. We note that the maximum likelihood estimator does not target to match these correlations directly, yet the fitted model predicts them very well.

Insert Figure 4 Here

4.3.2 Demand responses by fund type

For each fund type, we estimate the loading of demand, $\boldsymbol{\beta}_i^Z$, on each liquidity state variable in $\mathbf{Z}_t$. Panel B of Table 3 summarizes the demand response across all fund types to each liquidity state variable implied by the estimated 24 loadings, i.e., the change in demand given a one-sample standard deviation of each variable about its mean. The estimated

[21] The binomial deviance statistic for the Poisson responses follows a χ^2 distribution and is given by

$$D_i = 2\sum_t \{x_{i,t} \ln \frac{x_{i,t}}{\hat{\lambda}_{i,t}} - (x_{i,t} - \hat{\lambda}_{i,t})\}.$$

loadings and implied demand responses to any state variable are statistically significant for all 24 demands, showing that each specified state variable plays a role in explaining demand fluctuations.

The results in Panel B show that the demand response to shocks to some liquidity variables is very heterogeneous across fund types. Specifically, in response to shocks to total asset purchases by the FED, shocks to the issuance of commercial paper, shocks to the level of the credit spread, and shocks to the overall performance of private equity (TVPI), about the same number of funds experience an increase in bidding activity as the number of funds that experience a decrease in bidding activity.

Shocks to the slope of the yield curve, to GDP growth in times of monetary contraction, or to the cross-industry dispersion returns are followed mostly by a negative demand response, whereas the number of positive responses exceeds the negative responses across funds for all other demand state variables. For the case of GDP growth, the demands for all but one fund type respond positively.

The magnitude in the responses are also quite heterogeneous for most variables. A comparison of mean and median average effects conditional on their sign reveals that the responses tend to be skewed, i.e., whichever the sign, they are disproportionately large for a few fund types. For example, a one standard deviation increase in the slope of the yield curve is associated with a median increase by 0.15 bids per month for 8 fund types, but a mean of 0.21. Similarly, the average positive response to shocks to the return of the S&P 500 index in times of monetary contraction (0.15 over 15 fund types) exceeds the median (0.10) by 50%.

The heat map in Figure 5 looks closer into which fund types see a decrease or an increase in bidding for shocks to each state variable. Starting from the left, the figure shows the breakdown of this response by fund size, then by fund age, and by location. Each box corresponds to the expected change in the demand for all funds in the same size or age or location category given a positive (denoted by Δ) or a negative (∇) one sample standard deviation shock to the relevant state variable in each row. We simulate either negative or positive shocks to each variable so that liquidity worsens in all rows, e.g., we simulate a sale of the FED's assets but an increase in the slope of the yield curve. Rows (and variables) are sorted in descending order according to the overall demand response (in the 'Total' column).

Insert Figure 5 Here

Figure 5 shows that the increase in the quantity demanded following a shock to the Fontaine-Garcia funding liquidity premium reported in Table 3 is concentrated in Young funds; Old funds see a decrease in their demand. Asset sales by the Federal Reserve are also associated with opposing demand responses across different fund types: the increased bidding during monetary contractions is concentrated on Small or Medium and Old funds. A steepening of the yield curve predicts a sharp contraction in the demand for Large, and Very Large funds, as well as for Old and Middle-aged funds, but also predicts an increase in demand for Small and Young funds.

To summarize, the distribution of estimated demand responses to a tightening of aggregate liquidity and increases in funding costs is consistent with the capital call risk channel for selling PEF stakes under pressure: the *positive* liquidity-driven demand responses are concentrated in the Young funds, i.e., those where a larger proportion of committed capital remains to be called in the future. But this evidence is yet preliminary: the liquidity provision discount hypothesis requires not only an increase in the number of bids to funds exposed to liquidity shocks but also lower prices. Moreover, the sensitivities across funds to any given shock vary too, implying that we need to consider the effects on each fund type's demand of all shocks combined at any point in time, as we do in Section 5.

4.3.3 Demand responses by investor and fund types

We wish to understand whether different investor types respond differently to the same shocks to aggregate liquidity. For this analysis, we must reestimate the Poisson demand system using bids by each investor type at a time. We summarize the demand responses estimated by investor type using the heat map in Figure 6. This heat map is constructed similarly to Figure 5, but with each column corresponding to a different investor type.

Insert Figure 6 Here

First, the demand by SFs (left most column) is the most sensitive, in absolute terms, to shocks to any state variable. This result might be expected as SFs are the most frequent bidders as a group. However, it is surprising that the demand by AOs is at least as responsive to shocks to some state variables given that bids by AOs represent just over 20% of bids by SFs. That is, investors that are on average not active in this market may become the most active in some illiquidity states.

Second, shocks to some aggregate variables are associated with different and opposite bidding flows by different investors: an increase in the Fontaine-Garcia bond liquidity premium or the yield curve slope, both resulting in negative shocks to funding liquidity, is mainly explained by the response of AOs (more than 4 additional bids per month), who are not very active on average (8 bids per month). These results suggest that AOs – who are least constrained by the amount they can invest in the secondary market for PEF stakes – enter the market aggressively in illiquid times. Their bidding behavior is therefore consistent with that of a liquidity provider.

Third, there are shocks for which all investors' bidding react in the same direction, if with different magnitudes. The total number of bids by every investor type decreases in times of high GDP growth coupled with monetary contractions. From Figure 5, we see that this behavior affects all funds.

To sum up, we have shown in this section that the most salient feature of the impact of aggregate measures of liquidity on the total number of bids for PEF stakes is that the demand responses by fund type, and by investor type, are hugely heterogeneous. Young funds appear to experience increased bidding in response to liquidity shocks to many of the proposed proxies for liquidity. At the same time, some investors, namely asset owners, bid more frequently whereas others bid less so given the same shock, which is consistent with the former providing liquidity (submitting more bids) to the sellers of exposed funds.

5 Demand, bids and fund performance

We now explore whether liquidity-driven demand is associated with lower bid levels. This analysis is conducted at the individual bid level.

5.1 Model and identification

Each observed bid $b_{i,j,h,t}$ is for a fund j of type $i = 1,..,24$, by a bidder of type $h = \{\text{SF}, \text{FoF}, \text{AO}\}$, placed in month $t = 1, ..., 88$. The reduced form equation we estimate is as follows:

$$b_{i,j,h,t} = \alpha_i + \alpha_h + \alpha_t + \alpha_Y \mathbf{Y}_{j,h,t} + \alpha_D D_{i,t} + \epsilon_{j,t}, \tag{2}$$

where α_i, α_h, and α_t are fixed effects. $\mathbf{Y}_{j,h,t}$ is a vector of control variables containing fund size, fund age as of month t, and the number of bids submitted by bidder h in month t. $D_{i,t}$ is the demand for fund type i during month t (by all bidder types). The demand can be either the observed demand ($D = X$), or the total predicted demand ($D = \hat{\lambda}$), or the liquidity-driven demand ($D = \hat{\lambda}_Z$), i.e. the number of bids predicted by the liquidity variables, $\mathbf{Z}_t$, only.

The coefficient of interest is $\alpha_{\hat{\lambda}Z}$, which describes whether bids are on average higher or lower in times of high liquidity-driven demand, i.e., the slope of the demand function. If bidders provide liquidity, then $\alpha_{\hat{\lambda}_Z}$ should be negative because bidders are meeting the increase in supply, but at lower prices. Note that $\alpha_{\hat{\lambda}_Z}$ does not reflect private information since the predicted demand $\hat{\lambda}_{i,t}$ is a function only of variables that are publicly observable, and that it is correctly identified by OLS if the predicted demand variable, $\hat{\lambda}_Z$ is orthogonal to the bid equation error, $\epsilon_{j,t}$.

Omitted variables that influence individual bids may also influence demand. However, we use only the component of the demand that is a function of publicly observable variables. Moreover, we also control for the demand *residual*, which varies over time and across fund types and captures the unobservable demand shifters, such as the buyer's private information. Further, we control for time fixed effects. In addition, as shown in Section 4.3, the estimates of $\beta_{i,k}$ vary *significantly* across fund types, and the sign of the slope coefficients is different across fund types for most explanatory variables. So even if time-varying omitted variables were correlated with some variable in $\mathbf{Z}_t$, they would only bias the estimates of $\alpha_{\hat{\lambda}Z}$ if they related to bids of each fund type with the same sign and magnitude as each estimates of $\beta_{i,k}$. To sum up, as the mapping from $\mathbf{Z}_t$ onto $\hat{\lambda}$ is non-linear and varies for each i, it is unlikely that any time-varying omitted variables correlated with $\mathbf{Z}_t$ will be correlated with $\hat{\lambda}$.[22]

5.2 Bids and Demand: Results

Table 4 presents the estimates of the parameters in the regression model of equation (2). Our test for liquidity provision is that the coefficient of demand, $\alpha_{\hat{\lambda}_Z}$, is negative. Column (1) shows that the relationship between observed demand, i.e., $D = X$, and bids is negative. Next, in column (2), we use instead $D = \hat{\lambda}$, i.e. predicted demand, while controlling for the demand model residual. The coefficient is also negative and significant. In column (5) we set $D = \hat{\lambda}_Z$, so that we use the demand component predicted only by liquidity state variables. Again, the coefficient is negative and statistically significant.

These results support our hypothesis that demand flows to where there is selling pressure. We observe lower bids for those funds that experience higher liquidity-driven demand than normally in a given month. Note that our results do not contradict the usual view that, overall, bids drop because bidders are withdrawing from the market. However, we find that, *cross-sectionally*, the funds that experience the largest discount are those for which liquidity-driven demand increases.

Insert Table 4 Here

To interpret the magnitude of these coefficients, we report in Panel B the average implied difference between bids for a given change in the demand induced by shocks to aggregate liquidity as $\Delta E(\text{Bid}) \equiv \hat{\alpha}^{\lambda} \times \Delta\lambda$. We set $\Delta\lambda$ to one sample standard deviation in the liquidity-induced demand for each fund type. This statistic has a similar interpretation to 'Kyle's Lambda' (Kyle (1985)) because it measures the impact on the bid from a change in bidding volume. Panel B shows that a one sample standard deviation increase in the predicted demand for any given fund type is on average associated with a bid that is lower by 0.89% (column 2). Using only the demand component predicted by the liquidity variables,

[22] Note that our liquidity-driven demand variable, $\hat{\lambda}_{i,t}$ varies across fund type and time. Therefore, we can estimate its coefficient while additionally controlling for unobservable bid differences over time (month fixed effects) across funds (fund type fixed effects) that are unrelated to liquidity.

a one sample standard deviation change in liquidity-driven demand is associated with a large 2.17% discount (column 5).

This figure represents an average discount on bids and not necessarily on transaction prices, which could be higher if the seller finds a higher bid. The bidder does not know the other bids and may therefore submit a dominated bid. This estimated discount is therefore an upper bound to the discount expected by the seller from this channel. This upper bound notwithstanding, we note that we have 35 transactions in our data for which we are able to follow the negotiation process through time leading to an actual transaction. In those cases, we identify the last bid on the fund prior to the transaction. In 26 cases, the transaction price is identical to the bid, in 4 cases the transaction price is higher (but, on average, by less than 1 percentage point) and in 5 cases the transaction occurs at a lower value relative to the prior bid.

Next, we decompose the partial correlation between bid levels and liquidity-driven demand by fund age, interacting the predicted demand with binary variables for Young funds, Middle-aged funds and Old funds (columns 3 and 6). The only effect that is economically and statistically strong across both specifications is that for Young Funds: bids are lower by 2.47 percentage points given a one standard deviation increase in the demand for Young funds that is predicted by aggregate liquidity variables. This result is consistent with capital call risk being the source of illiquidity discounts in the secondary market for private equity fund stakes. Indeed, young funds have the highest proportion of uncalled capital and, therefore, their LPs have to meet capital calls in the future. LPs would be most pressed to sell stakes in Young funds in times of low liquidity, and its precisely for these funds that we find the largest negative association between bid levels and liquidity-driven demand.[23]

In columns (4) and (7) we decompose the correlation between bids and predicted or liquidity-driven demands by fund size. The negative, statistically and economically significant correlation is only present for the Very Large funds. This result is consistent with our hypothesis that LPs under pressure to sell due to a sudden preference for cash will choose to

[23]Another explanation for this negative correlation could be the demand flows to relatively cheap funds when borrowing is expensive. While young funds may indeed by cheaper, there is no a reason *a priori* for why they should be more *discounted* with respect to NAV. Moreover, the inclusion of fund type fixed effects and fund age account for age-related value differences across funds, ruling out this alternative interpretation.

liquidate the known funds, with the least uncertain valuation and, therefore, sellable for the least discount. The fact that we do not find a correlation between liquidity-driven demand and bids for small, i.e., more opaque, funds is also inconsistent with the asymmetric information hypothesis. According to our estimates, a one sample-standard deviation increase in the liquidity-driven demand for Very Large funds is associated with an average discount of 2.26% of NAV (column 7, Panel B).

Appendix D. shows the result from decomposing further the negative correlation between bids and liquidity-driven demand by fund size and liquidity state. The negative correlation between bids and the demand for Young funds is strongest in months of relatively low or intermediate, but not high, levels of asset purchases by the Federal Reserve or volume of issuance of commercial paper, or when the yield curve slope or the Fontaine-Garcia funding liquidity premium are above their respective first tercile. For example, a one sample standard deviation increase in the Fontaine-Garcia index is associated with an additional bid discount of up to 3.62% of NAV when the Fontaine-Garcia funding liquidity premium is within the intertercile range.

The pattern for Very Large funds is similar, with negative correlations across all states but the differences between the implied liquidity discounts in low and high states of rebalancing risk are larger. Together with our previous findings, this result is consistent with our view that bids for Young funds are most sensitive to shocks to liquidity variables influencing capital call risk, whereas bids for Very Large funds are at a higher discounting risk when the preference for cash or in high states of preference for cash or the need to rebalance PE-overweighted portoflios are strongest.

5.3 Asymmetric Information and Bid discounts

While our results suggest that bidders are compensated for providing liquidity to funds that experience selling pressure, there is the potential that the low bids we observe reflect the bidders' response to an increase in adverse selection. This effect could happen if some shocks to liquidity also gave private signals to sellers. For example, increases in the VIX may also enhance the advantage of some LPs to value stakes more precisely. In such context, uninformed investors may still increase their bidding in response to selling orders by the potentially informed LPs, albeit at a discount to compensate for the possibility of adverse selection.

To address this concern, we follow Llorente et al. (2002) and note that although the liquidity and private information motivations for trading predict similar *contemporaneous* correlations between demand and bids, they have different *dynamic* patterns. If the initial higher demand and associated lower bids are due to negative private signals, then bids will remain low until NAVs have incorporated the information, suggesting a relation between lagged demand and bids. Moreover, higher demand would predict worse future (NAV-to-NAV) fund performance. In contrast, if the lower bids made following an increase in demand were instead due to liquidity provision, we would expect the effect on bids to revert as soon as liquidity is restored and in addition there will be no predictability of fund performance.

We test whether past demand flows induced by liquidity variables have a persistent effect on bids by regressing bids on lags of predicted demand up to four months. The estimates in column (1) of Table 5 show that bids are only negatively correlated with contemporaneous demand and not correlated at all with any lag up to the fourth. Columns (2) to (8) show the same lagged correlation structure for all size and age categories of funds. The possibility of private information effects captured in the demand predicted by liquidity variables, especially for the cases of Young or Very Large funds, is largely rejected. The only case where we observe a negative and significant correlation between bids and lagged demand are for Small funds (2 months lag), for which there is no contemporaneous correlation in the first place.

Insert Table 5 Here

Table 6 shows the estimates of the regression of the fund's returns, computed over one, two and three years from the date of the reference NAV, on the liquidity-driven demands, by fund age or size. Rejecting the private information motive for trading, columns (1) to (3) show that demand predicted by liquidity variables is uncorrelated with NAV-to-NAV fund returns at any horizon. The estimates are both statistically insignificant and economically very small.

Insert Table 6 Here

We do not find negative correlations either between liquidity-driven demand and NAV-to-NAV at any horizon for most fund sizes and ages, especially for Young or Very Large funds, i.e., the fund types for which liquidity-driven demand flows are negatively correlated with bids and are therefore candidates to verify the pure liquidity discount hypothesis.

Interestingly, we do find predictability of the two- or three-year NAV-to-NAV Small funds' performance, as these returns are negatively and significantly correlated with liquidity-driven demand for Small funds (columns 8 and 9). It is therefore plausible that investors who bid for Small funds, whose valuation is typically more uncertain than larger funds, are indeed privately informed that their current NAVs overstate their true value. However, previous results suggest that liquidity-driven demand for Small funds is not necessarily associated with lower bids: the 1% discount (column 7 of Table 4, Panel B) is not significantly different from 0.

5.4 Who provides liquidity?

Table 7 explores the relation between bids placed (column 1) or fund future performance (columns 2 to 4) with liquidity-driven demand by type of bidder. Column (1) shows that it is precisely the liquidity-driven demand by investors that are not specialized in secondary acquisitions, which we refer to as asset owners (AOs), that correlates negatively with their bids. On average, AOs bids are lower by almost 1 percentage point when their liquidity-driven demand is higher by one sample standard deviation (Panel B, column 1). The liquidity-driven demand by AOs is not correlated with the funds' future performance (columns 2 to 4), suggesting that the bid reduction is a compensation for providing liquidity and not adverse selection.

We find no evidence that either secondary funds (SFs) or funds-of-funds (FoFs) submit lower bids when increasing their number of bids in response to liquidity shocks. In other words, AOs appear to be the liquidity providers in the secondary market for PEF stakes. One interpretation is that, unlike SFs, AOs are not constrained to invest exclusively in secondary market acquisitions, but can diversify across asset classes. Therefore, they have the flexibility to increase their activity in this market when capital call funding, cash preference or portfolio rebalancing risks put pressure on other constrained investors (SFs and FoFs). Indeed, the role of SFs and FoFs resembles that of the specialists investors in Mitchell et al. (2007), which are slow to raise arbitrage capital when liquidity is low, sustaining lower asset prices.

Insert Table 7 Here

Note that the increases in liquidity-driven demand by FoFs and SFs are correlated with a negative NAV performance of between 3.2% to 4.2% over two to three years for the former (Panel B, columns 3 and 4) and 2.9% in the next year (column 2) for the latter. Surprisingly, their bids are *not* lower on average, which suggests the possibility that these investors overpay.

Results in Appendix D. confirm the the role of AOs as traditional liquidity providers: they require steeper discounts systematically in states where capital call risk is highest or when cash preferences or portfolio rebalancing needs are strongest. FoFs and SFs are constrained to bid for PE stakes by their annual buying targets and as a result, their ability to provide liquidity to LPs pressed to sell is limited. While they appear to post discounted bids for overvalued funds, this behavior is not systematically predicted by liquidity variables.

6 Conclusions

We show that aggregate liquidity variables are important determinants of illiquidity in the secondary market for private equity. Our results suggest that young funds and large funds are the most vulnerable to liquidity shocks. The estimated liquidity discount from the channel we identify on these funds is over 2 percentage points. We argue that the discount associated with young funds is likely to be caused by the risk of funding future capital calls. The discount associated with large funds may be caused by these being the go to asset for rebalancing purposes.

We also show that asset owners, a class of of investors that trade the least on average in our sample, become more active participants in response to liquidity shocks, suggesting that they are acting as liquidity providers. In contrast, institutions specialized in purchasing PE stakes in the secondary market, so called secondary funds, do not systemically provide liquidity, most likely due to constrains imposed on their investment targets specific to the PE class and the secondary market.

Our evidence is consistent with recent theoretical literature (e.g. Lagos and Rocheteau (2009) and Hugonnier et al. (2014)) that predicts that investors take the role of liquidity providers in the absence of designated market makers and that when they do so they may not be the investor that most values the asset. Future research could aim to precisely quantify the expected liquidity discount of private equity stakes using a theory of the optimal timing of sales by LPs, and data of the demand, supply and execution of deals.

References

Acharya, V. V. and L. H. Pedersen (2005). Asset pricing with liquidity risk. *Journal of Financial Economics 77*, 375–410.

Albuquerque, R. and E. Schroth (2015, August). The value of control and the costs of illiquidity. *Journal of Finance 70*(4), 1405–1455.

Ang, A., D. Papanikolaou, and M. M. Westerfield (2014). Portfolio choice with illiquid assets. *Management Science 60*, 2737–2761.

Bollen, N. P. B. and B. A. Sensoy (2016). How much for a haircut? Illiquidity, secondary markets, and the value of private equity, Fisher College of Business Working Paper.

Bongaerts, D., F. De Jong, and J. Driessen (2011). Derivative pricing with liquidity risk: Theory and evidence from the credit default swap market. *The Journal of Finance 66*, 203–240.

Brunnermeier, M. K. and L. H. Pedersen (2009). Market liquidity and funding liquidity. *Review of Financial Studies 22*, 2201–2238.

Chang, B. and S. Zhang (2015). Endogenous Market Making and Network Formation, Working Paper.

Crain, N. and K. Law (2017). The Bright Side of Fair Value Accounting: Evidence from Private Company Valuation, Working Paper.

Driessen, J. (2005). Is default event risk priced in corporate bonds? *Review of Financial Studies 18*, 165–195.

Duffie, D., N. Gârleanu, and L. H. Pedersen (2005). Over-the-counter markets. *Econometrica 73*, 1815–1847.

Fontaine, J.-S. and R. Garcia (2012). Bond liquidity premia. *Review of Financial Studies 25*, 1207–1254.

Franzoni, F., E. Nowak, and L. Phalippou (2012). Private Equity Performance and Liquidity Risk. *Journal of Finance 67*, 2341–2373.

Gârleanu, N. (2009). Portfolio choice and pricing in illiquid markets. *Journal of Economic Theory 144*, 532 – 564.

Hasbrouck, J. (1991). Measuring the Information Content of Stock Trades. *The Journal of Finance 46*, 179–207.

Hege, U. and A. Nuti (2011). The Private Equity Secondaries Market During the Financial Crisis and the "Valuation Gap". *Journal of Private Equity 14*, 42–54.

Hugonnier, J., B. Lester, and P.-O. Weill (2014). Heterogeneity in decentralized asset markets. NBER Working Paper 20746.

Jurek, J. W. and E. Stafford (2015). The cost of capital for alternative investments. *The Journal of Finance 70*, 2185–2226.

Kleymenova, A., E. Talmor, and F. P. Vasvari (2012). Liquidity in the secondaries private equity market. Working paper, London Business School.

Koijen, R. and M. Yogo (2017, September). An equilibrium model of institutional demand and asset prices. NBER Working Paper 21749.

Kyle, A. (1985). Continuous auctions and insider trading. *Econometrica 53*(6), 1315–35.

Lagos, R. and G. Rocheteau (2009). Liquidity in Asset Markets With Search Frictions. *Econometrica 77*, 403–426.

Lerner, J. and A. Schoar (2004). The illiquidity puzzle: theory and evidence from private equity. *Journal of Financial Economics 72*, 3–40.

Llorente, G., R. Michaely, G. Saar, and J. Wang (2002). Dynamic volume-return relation of individual stocks. *The Review of Financial Studies 15*(4), 1005–1047.

Longstaff, F. A. (2009). Portfolio claustrophobia: Asset pricing in markets with illiquid assets. *American Economic Review 99*, 1119–44.

Mitchell, M., L. H. Pedersen, and T. Pulvino (2007, May). Slow moving capital. *American Economic Review 97*(2), 215–220.

Nadauld, T. D., B. A. Sensoy, K. Vorkink, and M. S. Weisbach (2017). The liquidity cost of private equity investments : Evidence from secondary market transactions. NBER Working Paper 22404.

Pasquariello, P. (2014). Financial Market Dislocations. *The Review of Financial Studies 27*(6), 1868–1914.

Sagi, J. S. (2017). Asset-level risk and return in real estate investments, Working Paper.

Shleifer, A. (1986). Do demand curves for stocks slope down? *Journal of Finance 41*(3), 579–590.

Sorensen, M., N. Wang, and J. Yang (2014). Valuing private equity. *Review of Financial Studies 27*, 1977–2021.

Yuan, K. (2005). Asymmetric price movements and borrowing constraints: A rational expectations equilibrium model of crises, contagion, and confusion. *Journal of Finance 60*(1), 379–412.

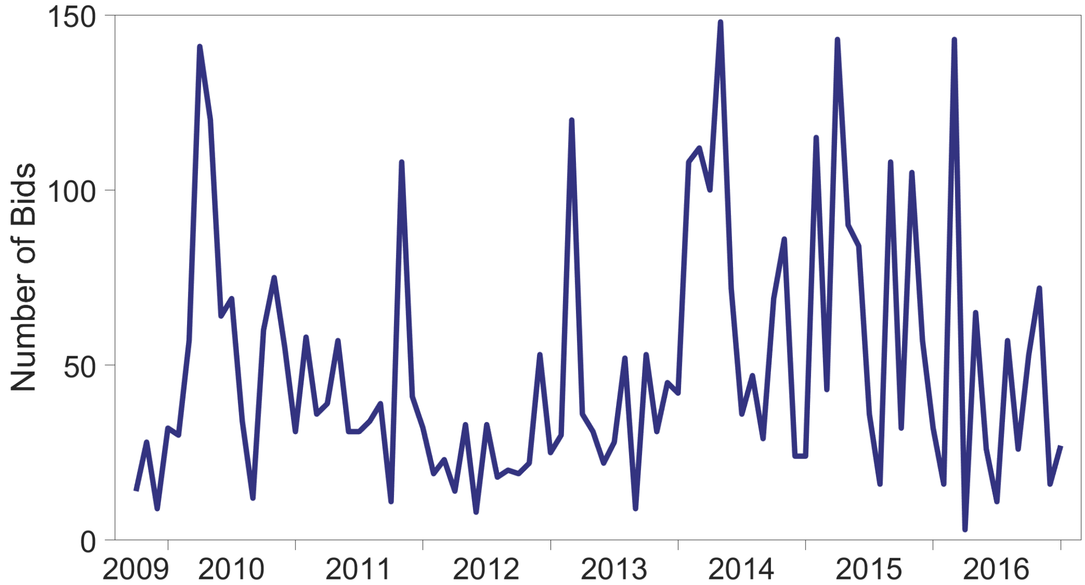

Figure 1: Total Number of Bids per Month in the Secondary Market for Limited Partnership Stakes. This figure plots the total number of bids submitted each month to a global sell-side broker of Private Equity stakes based in London, between September 2009 and December 2016.

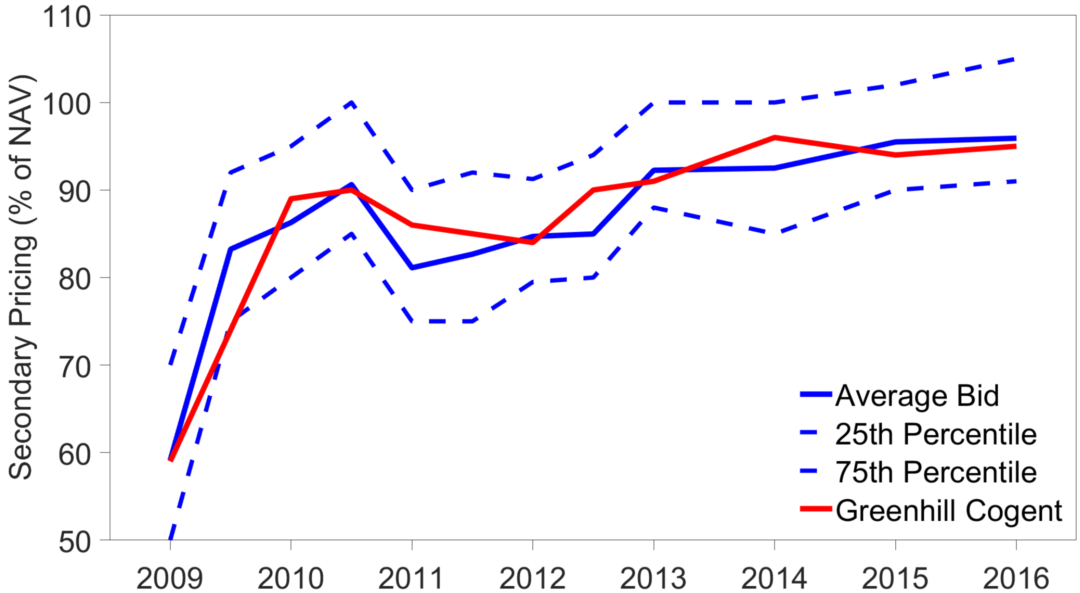

Figure 2: The Average Bid over Time. This figure plots the average High Bid over all the funds in our sample compared to the average High Bid reported by Greenhill Cogent 2016. Submitted bids are expressed as a percentage of the fund's latest available NAV. If a bid is submitted as a range, the 'High bid' is the maximum value in the range. Greenhill Cogent reports High Bids at a semi-annual frequency from 2010 to 2013 and annually in 2009, 2014, 2015, and 2016. The corresponding average High Bids for our sample are calculated at the same frequencies as in Greenhill Cogent's. The data includes all bids submitted between September 2009 and December 2016 to a global sell-side broker of Private Equity stakes based in London.

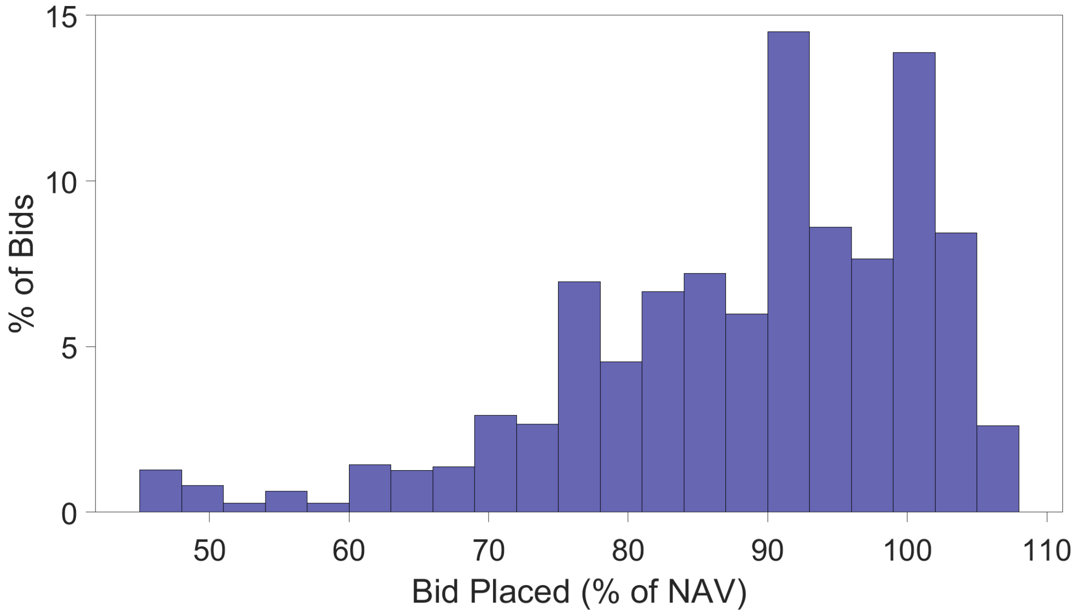

Figure 3: The Distribution of Bids. This Figure shows the histogram of bids levels, submitted each month to a global sell-side broker of Private Equity stakes based in London, between September 2009 and December 2016. Submitted bids are expressed as a percentage of the fund's latest available NAV.

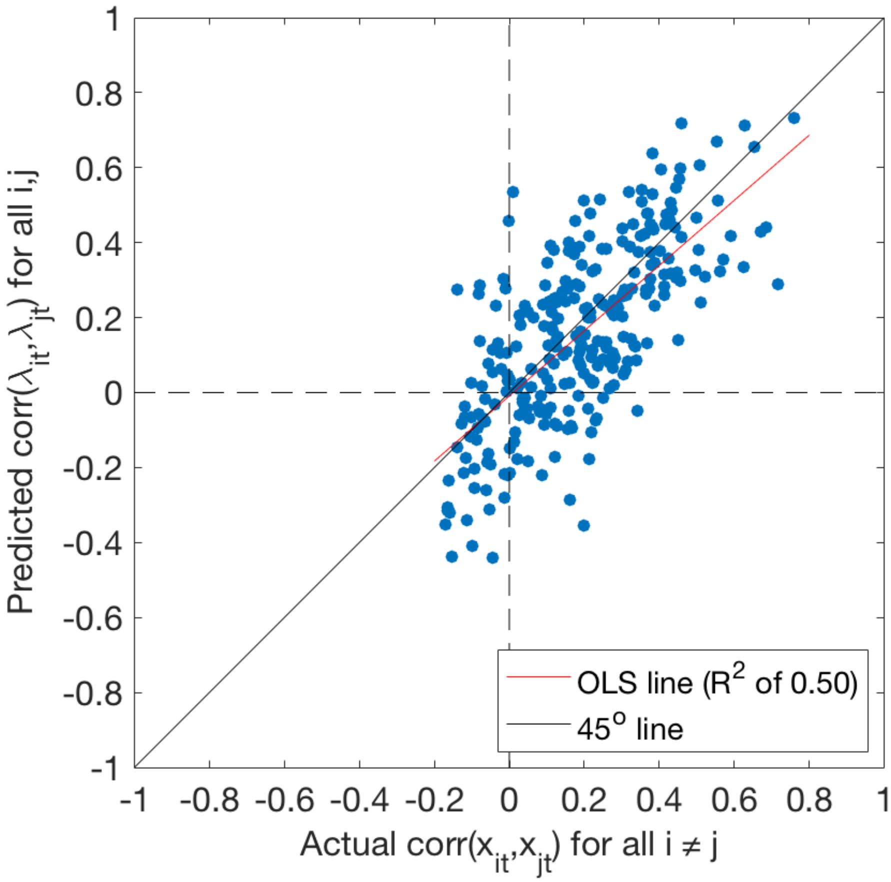

Figure 4: Actual and Predicted Demand Correlations. This figure compares the actual to the predicted monthly time series of the total number of bids (demand) for each fund type i in the data. The number of bids per month for each type of fund, $X_{i,t}$, is predicted using an estimated Poisson distribution, where the mean number of bids per month, $\lambda_{i,t}$, is a function of time-varying covariates capturing the state of growth, investment opportunities and aggregate liquidity. The data includes all bids submitted between September 2009 and December 2016 to a global sell-side broker of Private Equity stakes based in London.

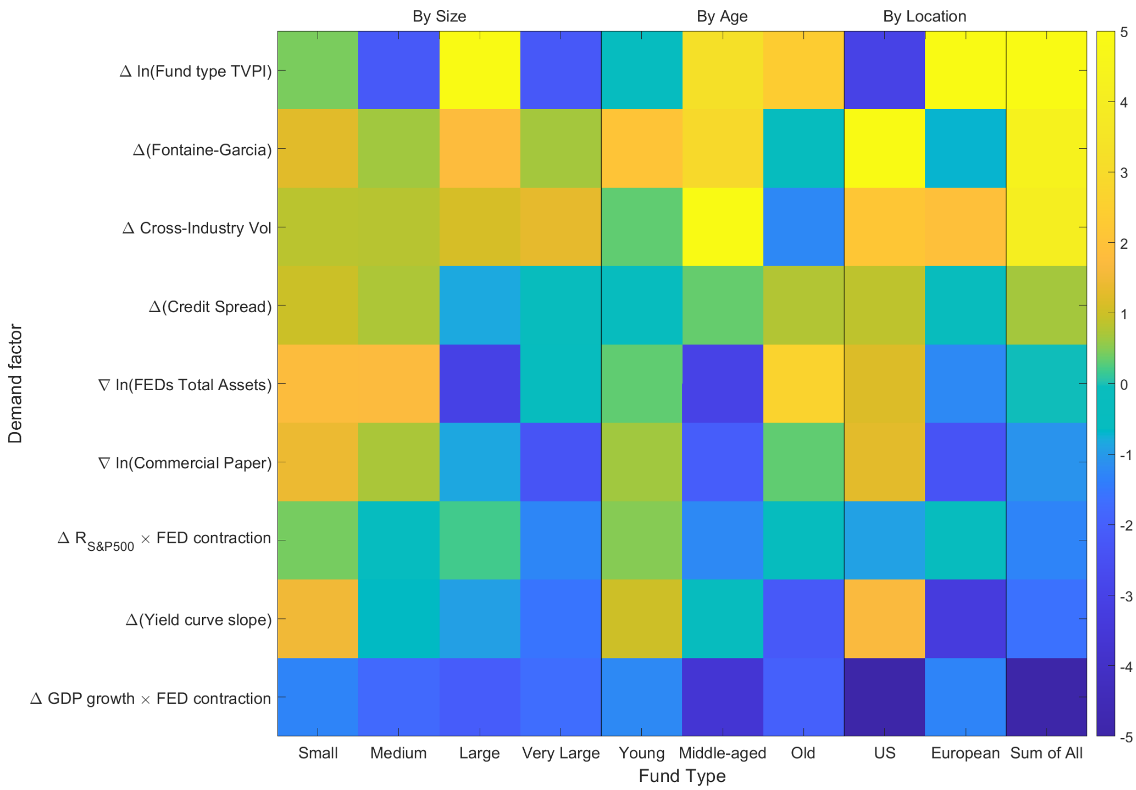

Figure 5: Heat Map of the Demand Response to Aggregate Liquidity Shocks, by Fund Type. This map shows the predicted changes in the number of bids per month for all fund types in response to a one sample standard deviation change to each of the explanatory variables of the demand model (in the 'Sum of All' column at the far right). Positive or negative shocks to the explanatory variable are denoted by Δ and ∇, respectively. Demand responses are broken down by fund size (Small, Medium, Large, Very large), age (Young, Middle-aged, Old) or the location of the fund's investmen focus (US, Europe). The demand response is predicted using the estimates of the model of bid arrivals per month per fund type, which is assumed to have a Poisson distribution with a mean number of bids conditional on the demand explanatory variables. The data includes all bids submitted between September 2009 and December 2016 to a global sell-side broker of Private Equity stakes based in London.

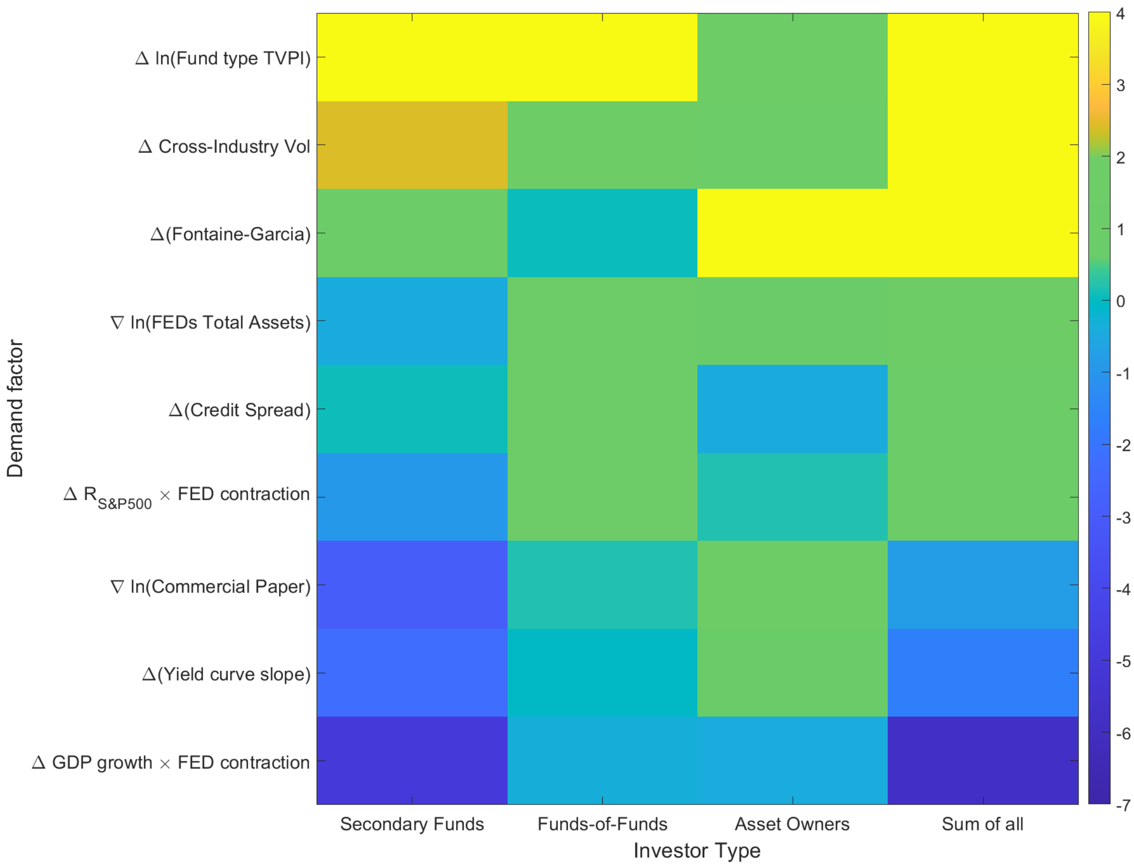

Figure 6: Heat Map of the Demand Response to Aggregate Liquidity Shocks, by Investor Type. This map shows the predicted average change in the total number of bids per month for each type of fund (according to their size, age, or location) by each type of bidder (Secondary Funds, Other Funds-of-funds, and Other Asset Owners) in response to a one sample standard deviation increase for some demand explanatory variable. The average change is predicted using the estimates of the model of bid arrivals per month per investor type per fund type, which is assumed to have a Poisson distribution with a mean number of bids conditional on the demand explanatory variables. The data includes all bids submitted between September 2009 and December 2016 to a global sell-side broker of Private Equity stakes based in London.

Table 1: Descriptive Statistics

The table reports the mean, median and standard deviation (S.D.) of the characteristics of the targeted funds and the bids submitted to a global sell-side broker of Private Equity stakes based in London, between September of 2009 and December 2016. There are 4,365 bids for 497 funds over 88 months. Bids are expressed as a percentage of the referenced NAV. The category Asset Owners includes all investors that are neither specialized secondary funds nor other funds-of-funds.

Panel A: Full Sample

Bids by:	All Bidders			Funds-of-Funds			Secondary Funds			Asset Owners		
	Mean	Median	S.D.	Mean	Median	S.D.	Mean	Median	S.D.	Mean	Median	S.D.
Bid Placed (% of NAV)	88.25	90.00	13.61	82.53	85.00	16.42	87.61	90.00	13.19	93.91	97.50	12.12
Proportion of US Funds	0.31	0.00	0.46	0.31	0.00	0.40	0.33	0.00	0.47	0.20	0.00	0.40
Fund Size ($ billions)	4.64	3.50	4.86	3.48	1.64	4.17	5.06	3.74	4.92	3.84	1.65	4.69
Fund Age (years)	6.63	7.00	2.71	6.46	6.00	2.83	6.82	7.00	2.62	5.88	6.00	2.91
Number of Observations		4,365			348			3,291			726	
Number of Unique Bidders		144			38			54			52	

Panel B: Subsample of bids for funds that can be matched to the Preqin cash flow data base

Bids by:	All Bidders			Funds-of-Funds			Secondary Funds			Asset Owners		
	Mean	Median	S.D.	Mean	Median	S.D.	Mean	Median	S.D.	Mean	Median	S.D.
Bid Placed (% of NAV)	88.57	90.50	12.97	84.04	87.25	15.70	88.20	90.00	12.52	93.03	95.25	12.63
Proportion of US Funds	0.37	0.00	0.48	0.36	0.00	0.48	0.39	0.00	0.49	0.27	0.00	0.44
Fund Size ($ billions)	6.15	5.00	5.00	4.77	3.76	4.53	6.37	5.18	5.00	5.63	4.29	5.09
Fund Age (years)	6.49	7.00	2.60	6.20	6.00	2.87	6.66	7.00	2.54	5.71	6.00	2.67
Fund PME at time of bid	1.18	1.14	0.33	1.20	1.15	0.38	1.18	1.14	0.32	1.16	1.11	0.32
Number of Observations		3,093			230			2,427			436	
Number of Unique Bidders		136			36			53			47	

Table 2: Summary of Data for the Demand Model

Panel A of this table presents summary statistics of the total number of bids received each month by a global sell-side broker of Private Equity stakes based in London, between September of 2009 and December 2016 for all funds classified into 24 different types according to their size, age, and location of their investment focus. Panel B summarizes the economy-wide variables used as explanatory variables in the demand model, which are also recorded at a monthly frequency. All variables are defined in Appendix A..

Panel A: Total number of bids per month for all funds of each type

	Mean	S.D.	Skewness	min	Q1	Median	Q3	Max
All funds	2.19	3.65	2.62	0.00	0.00	1.00	3.00	25.00
By fund size:								
Small	1.78	2.81	2.22	0.00	0.00	1.00	2.00	16.00
Medium	1.33	2.44	2.17	0.00	0.00	0.00	2.00	14.00
Large	3.04	4.40	2.31	0.00	0.00	1.00	4.00	25.00
Very large	2.09	3.64	2.60	0.00	0.00	0.00	2.00	25.00
By fund age:								
Young	1.92	3.89	2.74	0.00	0.00	0.00	1.00	19.00
Middle-aged	2.32	3.91	2.78	0.00	0.00	1.00	3.00	25.00
Old	2.06	3.10	1.99	0.00	0.00	1.00	3.00	17.00
By location of the fund's investment focus:								
Europe	2.15	3.39	2.38	0.00	0.00	1.00	3.00	20.00
US	2.38	4.24	2.73	0.00	0.00	1.00	2.00	25.00

Panel B: Economy wide variables used as explanatory variables ($\mathbf{Z}_t$)

	Mean	S.D.	Skewness	min	Q1	Median	Q3	Max
Δ ln(FED's Total Assets)	0.01	0.01	0.69	−0.01	0.00	0.00	0.02	0.05
Δ ln(Commercial Paper)	0.00	0.05	0.20	−0.13	−0.03	0.00	0.03	0.13
Δ(Fontaine-Garcia)	0.00	0.00	−0.93	−0.01	0.00	0.00	0.00	0.01
Δ(Yield curve slope)	−0.03	0.18	−0.39	−0.68	−0.15	0.00	0.09	0.53
Δ(Credit Spread)	0.00	0.08	0.74	−0.24	−0.05	−0.02	0.04	0.26
Cross-Industry Volatility	0.04	0.01	0.93	0.02	0.03	0.04	0.05	0.07
Δ ln(Private Equity's TVPI)	0.05	0.67	0.82	−1.75	−0.39	−0.09	0.47	3.09
GDP growth	0.02	0.01	−1.96	−0.04	0.02	0.02	0.02	0.04
$R_{\text{S\&P 500}}$	0.01	0.04	0.16	−0.08	−0.01	0.01	0.03	0.11
VIX	0.18	0.05	1.32	0.11	0.14	0.17	0.21	0.43

Table 3: Estimates of the Demand Model

This table summarizes the goodness-of-fit measures (Panel A) and the Maximum Likelihood estimates (Panel B) of the Poisson models of the demand for LP stakes for each of the 24 fund types in the data. The mean number of bids for all funds of type i in month t is given by

$$\lambda_{it} = \exp(\tau t + \mathbf{Z}_t' \boldsymbol{\beta}_i^Z + \mathbf{W}_{it}' \boldsymbol{\beta}_i^W + \sum_{j=1}^{J} 1\{i=j\}\gamma_j) \ \forall i, t,$$

where the variables in $\mathbf{Z}_t$ measure the state of aggregate liquidity or aggregate investment opportunities. The sample includes all bids received each month by a global sell-side broker of Private Equity stakes based in London, between September of 2009 and December 2016. The estimated economic effects of each variable correspond to a change in the average number of bids given a one sample standard deviation change in the explanatory variable. The Wald statistic is for the null hypothesis that all parameters for each fund type demand are zero. The Binomial deviance statistic is for the null hypothesis that the difference between the actual number of bids and the predicted average number of bids each month is zero under the asumed conditional Poisson distribution. All variables are defined in Appendix A..

Panel A: Summary of Goodness-of-fit measures

	Mean	Median		Mean	Median
Wald statistic	542.10	365.11	Binomial deviance	67.74	64.48
p-value	0.00	0.00	p-value	0.20	0.01
Pseudo R^2	0.30	0.26	$\text{corr}(X_{i,t}, \hat{\lambda}_{i,t})$	0.53	0.51

Panel B: Summary of economic effects implied by the parameter estimates associated to aggregate liquidity and investment opportunities variables ($\mathbf{Z}_t$)

	Statistically significant positive effects				Statistically significant negative effects			
Variable	#	Mean	S.D.	Median	#	Mean	S.D.	Median
$\Delta \ln$(FED's Total Assets)	13	0.30	0.30	0.20	11	−0.24	0.22	−0.14
$\Delta \ln$(Commercial Paper)	12	0.24	0.21	0.20	12	−0.13	0.17	−0.05
Δ(Fontaine-Garcia)	15	0.25	0.20	0.23	9	−0.19	0.27	−0.05
Δ(Yield curve slope)	8	0.21	0.14	0.15	16	−0.23	0.29	−0.10
Δ(Credit Spread)	13	0.14	0.13	0.11	11	−0.16	0.14	−0.13
GDP growth $\times \mathbf{1}$\{FED contraction\}	9	0.17	0.16	0.09	15	−0.28	0.25	−0.18
$R_{\text{S\&P 500}} \times \mathbf{1}$\{FED contraction\}	15	0.15	0.15	0.10	9	−0.15	0.14	−0.08
Cross-Industry Volatility	3	0.07	0.08	0.04	21	−0.29	0.21	−0.27
$\Delta \ln$(Private Equity's TVPI)	11	0.12	0.08	0.13	13	−0.18	0.14	−0.20
GDP growth	23	0.52	0.48	0.39	1	−0.05	0.00	−0.05
$R_{\text{S\&P 500}}$	15	0.28	0.27	0.20	9	−0.17	0.16	−0.11
VIX	16	0.34	0.33	0.20	8	−0.50	0.48	−0.29

Table 4: The Relation between Bid Levels and Demand

This table presents estimates of the regressions of the bid placed on different measures of demand (total number of bids per type fund, i, per month, t) predicted by the demand model of Table 3. Bids are expressed as a percentage of the referenced NAV. Demand by fund type per month is either the observed demand ($X_{i,t}$), the demand predicted using all of the model's explanatory variables ($\hat{\lambda}_{i,t}$, columns 2 to 4) or the demand predicted using only aggregate liquidity variables ($\hat{\lambda}^{\mathbf{Z}}_{i,t}$, columns 5 to 7). Control variables for all regressions include the exact (log of) size and age of the bidded fund, the (log of) the number of bids by the bidder. For all columns but (1), the controls include also the residuals of the demand model. All regressions include also month, bidder and fund type fixed effects. Standard errors (in parentheses under each estimate) are clustered at the fund type level. Estimates followed by the symbols ***, **, or * are statistically significant at the 1%, 5%, or 10% levels, respectively. Please refer to Appendix A. for a definition of all the variables.

Panel A: The dependent variable is the bid placed, as a % of reference NAV

		Predicted demand $\hat{\lambda}_{i,t}$			Liqudity-driven demand $\hat{\lambda}^{\mathbf{Z}}_{i,t}$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Observed demand ($X_{i,t}$)	-0.126^{*}						
	(0.067)						
Predicted demand		-0.346^{***}			-0.152^{*}		
		(0.124)			(0.088)		
Predicted demand for:							
Young funds			-1.483^{***}			-0.316^{***}	
			(0.421)			(0.101)	
Mid-aged funds			-0.379^{**}			0.009	
			(0.167)			(0.133)	
Old funds			-0.259			0.229	
			(0.175)			(0.345)	
Predicted demand for:							
Small funds				0.371			-0.114
				(0.340)			(0.086)
Medium funds				-0.210			0.533^{*}
				(0.387)			(0.316)
Large funds				-0.297			0.039
				(0.278)			(0.151)
Very large funds				-0.576^{***}			-0.463^{***}
				(0.157)			(0.159)
Observations	4,365	4,365	4,365	4,365	4,365	4,365	4,365
Adjusted R^2	0.436	0.351	0.353	0.352	0.350	0.351	0.353

(*Continues*)

Table 4 – *Continued*

Panel B: Economic significance of slope coefficients ($\hat{\alpha}$) in Panel A

			$\Delta E(\text{Bid }\%) \equiv \hat{\alpha}^{\lambda} \times \text{Std.Dev.}(\hat{\lambda})$				
			Predicted demand $\hat{\lambda}_{i,t}$		Liquidity-driven demand $\hat{\lambda}^{\mathbf{Z}}_{i,t}$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
All funds	-0.459^{*}	-0.893^{***}			-2.169^{*}		
	(0.245)	(0.321)			(1.256)		
Young funds			-2.414^{***}			-2.467^{***}	
			(0.684)			(0.790)	
Mid-aged Funds			-0.886^{**}			0.032	
			(0.390)			(0.468)	
Old Funds			-0.642			0.401	
			(0.432)			(0.605)	
Small Funds				0.877			-0.941
				(0.805)			(0.713)
Medium Funds				-0.464			1.103^{*}
				(0.855)			(0.655)
Large Funds				-0.670			0.253
				(0.627)			(0.970)
Very large Funds				-1.452^{***}			-2.261^{***}
				(0.396)			(0.778)

Table 5: Bid Levels and Liquidity-driven Demand over Time

This table presents estimates of the regressions of the bid placed on different measures of demand (total number of bids per type fund, i, per month, t), and their lags, predicted by the demand model of Table 3. Bids are expressed as a percentage of the referenced NAV. Demand by fund type per month is predicted using aggregate liquidity variables only ($\hat{\lambda}^{\mathbf{Z}}_{i,t}$). Control variables for all regressions include the exact (log of) size and age of the bidded fund, the (log of) the number of bids by the bidder, and the residuals of the demand model. All regressions include also month, bidder and fund type fixed effects. Standard errors (in parentheses under each estimate) are clustered at the fund type level. Estimates followed by the symbols ***, **, or * are statistically significant at the 1%, 5%, or 10% levels, respectively. Please refer to Appendix A. for a definition of all the variables.

	Fund Type							
Lags of liquidity-driven demand ($\hat{\lambda}^{\mathbf{Z}}$)	All (1)	Young (2)	Mid-aged (3)	Old (4)	Small (5)	Medium (6)	Large (7)	Very large (8)
$\hat{\lambda}^{\mathbf{Z}}_{i,t}$	-0.057^{***}	-1.722^{***}	-0.043	0.520	0.431	-1.394	0.150	-0.302^{***}
	(0.018)	(0.494)	(0.158)	(0.389)	(0.410)	(1.018)	(0.107)	(0.100)
$\hat{\lambda}^{\mathbf{Z}}_{i,t-1}$	-0.025	-0.351	0.202	-0.864	0.672	0.228	0.120	-0.155
	(0.123)	(1.837)	(1.600)	(1.617)	(0.883)	(0.655)	(0.174)	(0.233)
$\hat{\lambda}^{\mathbf{Z}}_{i,t-2}$	-0.083	0.794	-0.562	0.717	-0.901^{***}	-0.234	0.262	-0.273
	(0.221)	(1.347)	(1.121)	(2.095)	(0.212)	(0.732)	(0.254)	(0.219)
$\hat{\lambda}^{\mathbf{Z}}_{i,t-3}$	-0.139	0.107	-0.328	0.198	-0.092	0.869	-0.241	0.055
	(0.150)	(1.366)	(0.889)	(1.092)	(0.183)	(0.749)	(0.155)	(0.177)
$\hat{\lambda}^{\mathbf{Z}}_{i,t-4}$	0.029	2.237^{**}	-0.754	1.206	-0.047	-0.098	-0.053	0.110
	(0.090)	(0.920)	(0.663)	(1.275)	(0.250)	(1.062)	(0.060)	(0.214)
Observations	3,618	3,618	3,618	3,618	3,618	3,618	3,618	3,618
Adjusted R^2	0.322	0.325	0.321	0.322	0.323	0.321	0.321	0.323

Table 6: The Relation between Fund Returns and Demand

This table presents estimates of the regressions of the fund performance following each bid on different measures of demand (total number of bids per type fund, i, per month, t) predicted by the demand model of Table 3. The dependent variable is the fund's NAV-to-NAV public market equivalent performance T years from the NAV at the end of the quarter a bid was placed, referred to as $t = 0$, defined as $\frac{\mathrm{NAV}_T \times \frac{I_0}{I_T} + \sum D_t \times \frac{I_0}{I_t}}{\mathrm{NAV}_0 + \sum C_t \times \frac{I_0}{I_t}}$, where D_t are distributions made and C_t are capital calls issued at time t, and I_t is a market index value, set to the S&P 500 for US funds, the FTSE 250 for UK funds, and the STOXX Europe 600 for all other European funds. Demand by fund type per month ($\hat{\lambda}^{\mathbf{Z}}_{i,t}$) is predicted using aggregate liquidity variables only. Control variables for all regressions include the exact (log of) size and age of the bidded fund, and the (log of) the number of bids by the bidder. All regressions include also month, bidder and fund type fixed effects. The table reports the economic significance implied by each estimated coefficient, i.e., the implied change in the fund's annualized return between times t and T associated with a one sample standard deviation change in its liquidity-driven demand. Their standard errors (in parentheses under each estimate) are clustered at the fund type level. Estimates followed by the symbols ***, **, or * are statistically significant at the 1%, 5%, or 10% levels, respectively. Please refer to Appendix A. for a definition of all the variables.

	Returns horizon: $T =$								
Liquidity-driven	1 year	2 years	3 years	1 year	2 years	3 years	1 year	2 years	3 years
demand ($\hat{\lambda}^{\mathbf{Z}}$)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
For all funds	0.006	−0.001	−0.008						
	(0.009)	(0.010)	(0.012)						
By fund age:									
Young				0.002	−0.017	−0.004			
				(0.013)	(0.020)	(0.018)			
Mid-aged				0.011	0.017	0.001			
				(0.013)	(0.020)	(0.018)			
Old				−0.001	−0.012	−0.033*			
				(0.013)	(0.017)	(0.020)			
By fund size:									
Small							−0.058	−0.192***	−0.142***
							(0.062)	(0.055)	(0.055)
Medium							0.069*	−0.010	0.060
							(0.040)	(0.035)	(0.039)
Large							−0.023**	−0.007	−0.017
							(0.011)	(0.017)	(0.017)
Very large							0.012	0.008	0.002
							(0.010)	(0.011)	(0.015)
Observations	2,652	2,121	1,482	2,652	2,121	1,481	2,652	2,121	1,481
Adjusted R^2	0.326	0.407	0.383	0.326	0.408	0.383	0.335	0.411	0.389

Table 7: The Relation between Bid Levels and Demand, by Type of Bidder

This table presents estimates of the regressions of the bid placed or of the fund performance following each bid on different measures of demand (total number of bids per type fund, i, per month, t) predicted by the demand model of Table 3. Bids are expressed as a percentage of the referenced NAV (column 1). In columns 2 to 4, the dependent variable is the fund's NAV-to-NAV public market equivalent performance from the NAV at the end of the quarter a bid was placed, referred to as $t = 0$, defined as $\frac{\text{NAV}_T \times \frac{I_0}{I_T} + \sum D_t \times \frac{I_0}{I_t}}{\text{NAV}_0 + \sum C_t \times \frac{I_0}{I_t}}$, where D_t are distributions made and C_t are capital calls issued at time t, and I_t is a market index value, set to the S&P 500 for US funds, the FTSE 250 for UK funds, and the STOXX Europe 600 for all other European funds. Demand by fund type per month ($\hat{\lambda}^{\mathbf{Z}}_{i,t}$) is predicted using aggregate liquidity variables only. The category Asset Owners includes all investors that are neither specialized secondary funds nor other funds-of-funds. Control variables for all regressions include the exact (log of) size and age of the bidded fund, and the (log of) the number of bids by the bidder. The bid regression (column 1) also includes the residuals of the demand model. All regressions include also month, bidder and fund type fixed effects. Standard errors (in parentheses under each estimate) are clustered at the fund type level. Estimates followed by the symbols ***, **, or * are statistically significant at the 1%, 5%, or 10% levels, respectively. Please refer to Appendix A. for a definition of all the variables.

Panel A: Estimated regression coefficients

Liquidity-driven demand ($\hat{\lambda}^{\mathbf{Z}}$)	Bid as a % of Reference NAV	Returns horizon: $T =$ 1 year	2 years	3 years
	(1)	(2)	(3)	(4)
By bidder type :				
Secondary funds	0.018	−0.014***	−0.003	−0.001
	(0.206)	(0.004)	(0.006)	(0.007)
Funds-of-funds	−0.454	0.014	−0.105**	−0.137**
	(1.304)	(0.036)	(0.045)	(0.067)
Asset Owners	−1.638**	0.010	0.032	0.026
	(0.687)	(0.020)	(0.023)	(0.024)
Observations	4,365	2,652	2,121	1,481
Adjusted R^2	0.352	0.330	0.410	0.386

Panel B: Economic significance of coefficients ($\hat{\alpha}$) in Panel A, calculated as $\hat{\alpha}^{\lambda} \times \text{Std.Dev}(\hat{\lambda}^{\mathbf{Z}}_{i,t})$

	ΔE(Bid %)	ΔE(Fund Performance)		
Secondary funds	0.037	−0.029***	−0.006	−0.002
	(0.415)	(0.009)	(0.013)	(0.016)
Funds-of-funds	−0.146	0.004	−0.032**	−0.042**
	(0.419)	(0.011)	(0.014)	(0.021)
Asset Owners	−0.970**	0.005	0.017	0.014
	(0.407)	(0.011)	(0.012)	(0.013)

Appendix A. Variables in the data set

Table A. Variable Definitions

Variable name	Description
1. Dependent variables	
Bid (% of NAV)	The bid placed for a particular fund, expressed as a percentage of NAV.
Return of Fund (T years NAV-to-NAV)	The return of the fund is defined as the public market equivalent of the fund between the reported NAV at the end of the quarter the bid was placed, and the reported NAV T years later. The first NAV is accounted for as an initial investment and the final NAV as a distribution. Formally, the calculation is $\frac{\mathrm{NAV}_T \times \frac{I_0}{I_T} + \sum D_t \times \frac{I_0}{I_t}}{\mathrm{NAV}_0 + \sum C_t \times \frac{I_0}{I_t}}$, where D_t are distributions made and C_t are capital calls issued at time t, and I_t is a market index value, set to the S&P 500 for US funds, the FTSE 250 for UK funds, and the STOXX Europe 600 for all other European funds.
2. Bidder-specific variables	
Bidder is a Fund-of-Funds	Takes the value of 1 if the bid was made by a fund-of-funds, and 0 if it was made by a secondary fund or an asset owner.
Bidder is a Secondary Fund	Takes the value of 1 if the bid was made by a secondary fund, and 0 if it was made by a fund-of-funds or an asset owner.
Bidder is an Asset Owner	Takes the value of 1 if the bid was made by an asset owner, and 0 if it was made by a fund-of-funds or a secondary fund. The types of LPs included in the asset owner category are: insurance companies, banks, asset managers, government agencies, pension funds, foundations, endowments and others.

(*Continues*)

Table A.—*Continued*

Variable name	**Description**
Number of Bids made by the Bidder (log)	The logarithm of the total number of bids placed by the bidder in the last 10 days (including the date of the bid).
3. Fund-specific variables	
Age	Number of years since the fund's inception.
Young Fund	Takes the value of 1 if the fund Age$\leq$ 3 years.
Mid-aged Fund	Takes the value of 1 if the fund is between 4 and 7 years old, and 0 otherwise.
Old Fund	Takes the value of 1 if the fund Age$\geq$ 8 years, and 0 otherwise.
Fund Size (log)	The logarithm of the fund size as reported in Preqin.
Small Fund	Takes the value of 1 if the fund size is less than \$0.5 billion, and 0 otherwise.
Medium Fund	Takes the value of 1 if the fund size is more than \$0.5 billion but less than \$1.5 billion, and 0 otherwise.
Large Fund	Takes the value of 1 if the fund size is more than \$1.5 billion but less than \$5 billion, and 0 otherwise.
Very Large Fund	Takes the value of 1 if the fund size is more than \$5 billion, and 0 otherwise, corresponding to the 90th percentile.
European Fund	Takes the value of 1 if the fund is a European fund, and 0 otherwise.

(*Continues*)

Table A.—*Continued*

Variable name	Description
US Fund	Takes the value of 1 if the fund is a US fund, and 0 otherwise.
4. Liquidity variables	
4.1 Capital call risk variables	
Δln(FED's Total Assets)	Logarithm of the total value of the Federal Reserve System's Assets (in monthly first differences; source: Federal Reserve Board).
1{FED contraction}	Takes the value of 1 when the monthly change in the FED's total assets is negative, and 0 otherwise.
Δln(Commercial Paper)	Logarithm of the total value of outstanding financial commercial paper (in monthly first differences; source: Federal Reserve Board).
Δ(Fontaine-Garcia)	The Fontaine and Garcia (2012) funding liquidity premium, obtained as the bond premium age factor in an arbitrage-free term structure model (in monthly first differences; source: Jean-Sebastien Fontaine's website).
Δ(Yield Curve Slope)	Difference between the 10-year and 3-month yields on US Treasury bills (in monthly first differences; source: Federal Reserve Board).
Δ(Credit Spread)	Difference between the yields on BAA- and AAA-rated corporate bonds (in monthly first differences; source: Federal Reserve Board).
4.2 Preference for cash variables	
GDP growth $\times 1\{$FED contraction$\}$	The product of GDP growth and $\times 1\{$FED contraction$\}$.
$R_{S\&P500} \times 1\{$FED contraction$\}$	The product of $R_{S\&P500}$ and 1{FED contraction}.

(*Continues*)

Table A.—*Continued*

Variable name	Description
4.3 Portfolio rebalancing variables	
Cross-Industry Volatility	Calculated each month as the cross-sectional standard deviation of the Fama and French 49 industry portfolio returns.
Δln(Private Equity's TVPI)	The difference of the logarithm of the mean 6-month total value to paid-in (TVPI) multiple of buyout funds in the Preqin cash flow database. The 6-month TVPI is calculated by accounting for the six month old NAV as an initial investment, and the current NAV as a distribution. Formally, it is defined as $\frac{\text{NAV}_t+\sum_{s=t-5}^{t} D_s}{\text{NAV}_{t-6}+\sum_{s=t-5}^{t} C_s}$. As NAVs are reported quarterly, we use interpolated NAVs for months in between reporting months.
5. Other control variables	
GDP growth	Real growth in GDP for the OECD area, measured over the same quarter in the previous year (source: OECD).
$R_{S\&P500}$	Monthly returns, including all distributions, on a value-weighted S&P 500 market portfolio (excluding American Depository Receipts (ADRs); source: CRSP).
VIX	The CBOE volatility index.

Appendix B. Fund classification procedure

To inform us of how to classify fund types for our empirical demand model we examine the likelihood of receiving a bid as a function of observable characteristics. We consider fund size, age, and location and run a logit model on the likelihood that a fund receives a bid in a given quarter using the subsample of funds that we can match with Preqin data. We include the 375 funds in our sample that are covered by Preqin, as well as 285 comparable funds in the Preqin database which do not show up in our sample.[24] We define an indicator variable taking the value of 1 if a fund receives a bid in a given quarter, and 0 otherwise. Table B. - Panel A presents logit regressions that characterize the likelihood of receiving a bid within a given quarter. The three models we run differ only in the set of fixed effects that are included.

Consistent with Nadauld et al. (2017) we find that larger funds are more likely to receive bids. The effect of fund age is non-linear: it has an inverted U-shape where young and old funds are less likely to receive a bid. We find that US funds are less likely to receive bids. This is likely due to two reasons: our data provider is London based and Preqin has a better coverage of the US market and is thus more likely to have coverage of US funds that did not receive any bids.

We re-run the logit with dummy variables representing different size, age and performance categories to identify potential breakpoints to use for fund classifications. Results are reported in Panel B. For fund age, we define our age dummies for funds above a given age. This allows us to observe the points where there are significant changes in demand. We observe a breakpoint at the age of 3 (consistent with the assumption in Nadauld et al. (2017) that funds below age 3 are special, and less likely to be targeted). There is a jump in demand at age 6, and it turns negative at age 8 with a relatively large jump at age 10 (the typical liquidation age). To keep it simple we define three age categories: young funds (three years old or less), middle aged funds (between 4 and 7 years old), and old funds (eight years old and above). For fund size we observe a significant jump for funds that are above the 30th

[24] A fund is comparable if it is a Buyout fund focusing on Europe or the US and is of a vintage of 2000 or later. A given fund-quarter is included if the fund is no older than 12 years old and the NAV is at least 10 % of committed capital. This ensures that the set of funds that we compare resembles funds that received bids in our data.

percentile, and then it keeps on increasing from the 60th percentile onwards. We therefore decide to create a small fund category corresponding to the bottom tercile, one mid-size category corresponding to mid tercile, a large category that goes to the 90th percentile and a very large category for funds beyond the 90th percentile. This corresponds to cutoffs at \$0.5 bn, \$1.5 bn and \$5 bn, respectively.

This results in us forming 24 groups by assigning each fund to one of four size categories, one of three age categories and whether it is a US or European fund.

Table B. Bid probabilities and fund characteristics

This table presents the estimates of a logit model of the probability that a given private equity fund receives a bid in a given quarter. The dependent takes the value of 1 in any quarter that the given fund received at least one bid, or zero otherwise. The data includes all 375 funds receiving a bid between September 2009 and December 2016 through a global sell-side broker of Private Equity stakes based in London, as well as 285 comparable funds in Preqin that are not included in our proprietary data sets, of a vintage later than 2000, and with a NAV of at least 10 % of committed capital. The 'Fraction in Sample' is the fraction of fund-quarter observations in which we observe a bid. The coefficients show the change in this probability for an infinitesimal (discrete) change in each continuous (binary) variable. The marginal effects are evaluated at the variables sample means. The Z-Statistics are reported in parenthesis below. The model is estimated including a constant and controlling for the number of funds in the fund family, whether the fund is a low reputation fund, and for the fund's past performance, measured as the PME to date. Independent variables are winsorized at the 1% level. Estimates followed by the symbols ***, **, or * are statistically significant at the 1%, 5%, or 10% levels, respectively.

Panel A: Bid probabilities and Fund Age and Size as continuous variables

	(1)	(2)	(3)
Fund Size (log)	0.052***	0.043***	0.042***
	(0.002)	(0.002)	(0.002)
Fund Age	0.033***	0.029***	0.025***
	(0.003)	(0.002)	(0.005)
Fund Age Squared	−0.002***	−0.002***	−0.002***
	(0.000)	(0.000)	(0.000)
US Fund	−0.084***	−0.072***	−0.07***
	(0.003)	(0.003)	(0.003)
Quarter Fixed Effects	No	Yes	Yes
Vintage Fixed Effects	No	No	Yes
Fraction in Sample	0.119	0.119	0.119
Number of Observations	11,810	11,810	11,810
Pseudo R^2	0.325	0.378	0.383

(*Continues*)

Table B.—*Continued*

Panel B: Bid probabilities and Age and Size categories

	(1)	(2)	(3)
Fund Age $\geq$ 1 years	0.025*	0.019*	0.017
	(0.018)	(0.015)	(0.014)
Fund Age $\geq$2 years	0.024***	0.019***	0.016***
	(0.009)	(0.008)	(0.007)
Fund Age $\geq$3 years	0.017***	0.018***	0.017***
	(0.007)	(0.006)	(0.006)
Fund Age $\geq$4 years	0.009	0.012**	0.009*
	(0.006)	(0.005)	(0.005)
Fund Age $\geq$5 years	0.007	0.006	0.002
	(0.006)	(0.005)	(0.005)
Fund Age $\geq$6 years	0.017***	0.012**	0.007
	(0.006)	(0.005)	(0.005)
Fund Age $\geq$7 years	0.006	0.001	0.000
	(0.005)	(0.005)	(0.004)
Fund Age $\geq$8 years	−0.012**	−0.012***	−0.013***
	(0.006)	(0.005)	(0.005)
Fund Age $\geq$9 years	−0.005	−0.004	−0.005
	(0.006)	(0.005)	(0.005)
Fund Age $\geq$ 10 years	−0.022***	−0.015***	−0.016***
	(0.008)	(0.006)	(0.006)
Fund Size $\geq$20th percentile	0.032***	0.026***	0.025***
	(0.014)	(0.011)	(0.011)

(*Continues*)

Table B. *−Continued*

	(1)	(2)	(3)
Fund Size ≥30th percentile	0.025**	0.020**	0.020**
	(0.011)	(0.009)	(0.009)
Fund Size ≥40th percentile	0.009	0.009	0.010
	(0.009)	(0.007)	(0.007)
Fund Size ≥50th percentile	0.000	−0.001	−0.001
	(0.008)	(0.007)	(0.007)
Fund Size ≥60th percentile	0.027***	0.023***	0.022***
	(0.007)	(0.006)	(0.006)
Fund Size ≥70th percentile	0.027***	0.024***	0.023***
	(0.006)	(0.005)	(0.005)
Fund Size ≥80th percentile	0.035***	0.031***	0.029***
	(0.005)	(0.004)	(0.004)
Fund Size ≥90th percentile	0.046***	0.042***	0.041***
	(0.004)	(0.004)	(0.004)
US Fund	−0.076***	−0.068***	−0.065***
	(0.004)	(0.004)	(0.003)
Quarter Fixed Effects	No	Yes	Yes
Vintage Fixed Effects	No	No	Yes
Fraction in Sample	0.119	0.119	0.119
Number of Observations	11,810	11,810	11,810
Pseudo R^2	0.321	0.366	0.373

Appendix C. Indentification of Demand Parameters

This Appendix discusses which features of our data drive the estimates of the full set of parameters in the Demand model across all fund types. To clarify, consider a simple specification where the average number of arrivals depends only on $\mathbf{Z}_t$. Using a first-order approximation of $\lambda_{i,t}$ around a mean arrival rate of one bid per month (where $\mathbf{Z}_t\boldsymbol{\beta}_i = 0$ and $\exp(\mathbf{Z}_t\boldsymbol{\beta}_i) = 1$), then the solution to the first-order condition of the above problem yields

$$\hat{\boldsymbol{\beta}}_i^{\text{MLE}_0} = \left(\frac{1}{T}\sum_{t=1}^{T} Z_t' Z_t\right)^{-1} \left(\frac{1}{T}\sum_{t=1}^{T} Z_t'(x_{it}-1)\right),$$

where the k-th element of this vector is

$$\hat{\beta}_{ik}^{\text{MLE}_0} = \left(\frac{1}{T}\sum_{t=1}^{T} Z_t' Z_t\right)^{-1}_{k,k} \left(\frac{1}{T}\sum_{t=1}^{T} Z_{kt}(x_{it}-1)\right),$$

and $Z_{k,t}$ is the k-th variable in Z_t. In this case, the MLE estimator is the OLS projection of $\mathbf{Z}_t$ on the mean-corrected $x_{i,t}$. Therefore, the MLE produces positive loadings, $\beta_{i,k}$, for aggregate-wide variables in $\mathbf{Z}_t$ that take high values whenever the number of bids is high relative to the sample mean. In short, the parameter estimates of this model are obtained by the same data moments used by linear estimators, e.g., OLS.

Appendix D. Additional Results

Table D.1: The Relation between Bid Levels and Demand, conditional on Aggregate Liquidity

This table presents estimates of the regressions of the bid placed on the liquidity-driven demand (predicted total number of bids per type fund, i, per month, t) for Young funds (Panel A) or Very Large funds (Panel B). Demand by fund type per month is predicted using all of the aggregate liquidity variables only ($\hat{\lambda}^{\mathbf{Z}}_{i,t}$) and interacted with binary indicators of whether each given liquidity variable z_t is below the first tercile (Lower) above the second tercile (Upper) or in between (Middle). Each row of each panel corresponds to the regression where demand is interacted with the tercile indicators of each liquidity variable. Control variables for all regressions include the exact (log of) size and age of the bidded fund, the (log of) the number of bids by the bidder, and the residuals of the demand model. All regressions include also month, bidder and fund type fixed effects. Standard errors (in parentheses under each estimate) are clustered at the fund type level. Estimates followed by the symbols ***, **, or * are statistically significant at the 1%, 5%, or 10% levels, respectively. Please refer to Appendix A. for a definition of all the variables.

	Panel A: Demand for Young funds			Panel B: Demand for Very Large Funds		
Liquidity variable (z_t) tercile:	Lower	Middle	Upper	Lower	Middle	Upper
Δ ln(FED's Total Assets)	-1.897^{**}	-2.736^{***}	-1.834	-2.047^{**}	-3.048^{***}	-1.628^{**}
	(0.933)	(0.768)	(1.216)	(1.014)	(1.077)	(0.687)
Δ ln(Commercial Paper)	-3.118^{***}	-2.074^{**}	-1.615	-2.331^{*}	-3.057^{***}	-1.400
	(0.921)	(0.835)	(1.034)	(1.348)	(1.103)	(0.874)
Δ(Fontaine-Garcia)	-1.555	-3.618^{***}	-1.813^{**}	-2.016^{***}	-2.830^{**}	-2.285^{**}
	(0.968)	(0.962)	(0.843)	(0.735)	(1.347)	(0.952)
Δ(Yield curve slope)	-1.845^{**}	-3.137^{***}	-2.857^{***}	-0.996	-4.092^{***}	-2.713^{**}
	(0.819)	(1.001)	(0.917)	(0.776)	(1.139)	(1.224)
Δ(Credit Spread)	-2.823^{***}	-3.271^{***}	-1.570^{**}	-3.497^{***}	-3.452^{***}	-0.495
	(1.068)	(0.946)	(0.757)	(0.708)	(1.268)	(0.688)
$R_{\text{S\&P 500}} \times \mathbf{1}\{\text{FED contraction}\}$	-1.599	-2.687^{***}	-2.138^{**}	-1.896	-2.552^{***}	-2.007^{*}
	(1.246)	(0.807)	(0.942)	(1.322)	(0.874)	(1.124)
Cross-Industry Volatility	-1.725^{**}	-1.533^{*}	-3.038^{***}	-1.303^{*}	-2.043^{**}	-3.269^{**}
	(0.729)	(0.884)	(0.938)	(0.681)	(0.857)	(1.293)
Δ ln(Private Equity's TVPI)	-2.024^{**}	-2.237^{*}	-2.867^{***}	-1.874^{*}	-1.549	-3.441^{***}
	(0.815)	(1.245)	(0.884)	(1.006)	(0.966)	(1.112)

Table D.2: The Relation between Bid Levels and Demand by Investor Type, conditional on Aggregate Liquidity

This table presents estimates of the regressions of the bid placed on different measures of demand (total number of bids per type fund, i, per month, t) by Funds-of-funds (FoFs), Secondary funds (SFs) and Other Asset Owners (AOs) predicted by the demand model of Table 3. Bids are expressed as a percentage of the referenced NAV. Demand by fund type per month is predicted using all of the aggregate liquidity variables only ($\hat{\lambda}^{\mathbf{Z}}_{i,t}$) and interacted with binary indicators of whether each given liquidity variable z_t is below the first tercile (Lower) above the second tercile (Upper) or in between (Middle). Each row corresponds to the regression where demand is interacted with the tercile indicators of each liquidity variable. The category Asset Owners includes all investors that are neither specialized secondary funds nor other funds-of-funds. Control variables for all regressions include the exact (log of) size and age of the bidded fund, the (log of) the number of bids by the bidder, and the residuals of the demand model. All regressions include also month, bidder and fund type fixed effects. Standard errors (in parentheses under each estimate) are clustered at the fund type level. Estimates followed by the symbols ***, **, or * are statistically significant at the 1%, 5%, or 10% levels, respectively. Please refer to Appendix A. for a definition of all the variables.

	Panel A: Demand by Secondary funds			Panel B: Demand by Funds-of-funds			Panel C: Demand by Asset Owners		
Liquidity variable (z_t) tercile:	Lower	Middle	Upper	Lower	Middle	Upper	Lower	Middle	Upper
$\Delta\ln$(FED's Total Assets)	−0.995*	−0.916**	0.574	−1.927***	0.057	0.282	−1.347**	−1.112**	−0.194
	(0.539)	(0.440)	(0.532)	(0.673)	(0.390)	(0.531)	(0.565)	(0.511)	(0.585)
$\Delta\ln$(Commercial Paper)	−0.225	−0.389	−0.231	−0.031	−0.678	0.074	−0.944*	−0.545	−1.149**
	(0.540)	(0.492)	(0.515)	(0.519)	(0.493)	(0.490)	(0.530)	(0.496)	(0.574)
Δ(Fontaine-Garcia)	0.618	−0.187	−1.233***	−0.31	−0.201	−0.153	−0.094	−1.558***	−0.495
	(0.497)	(0.542)	(0.443)	(0.464)	(0.521)	(0.457)	(0.587)	(0.500)	(0.463)
Δ(Yield curve slope)	−1.120**	0.187	0.218	−0.111	−0.345	−0.205	−0.785	−1.366***	−0.347
	(0.456)	(0.457)	(0.618)	(0.439)	(0.495)	(0.530)	(0.517)	(0.498)	(0.570)
Δ(Credit Spread)	−0.157	0.083	−0.755*	−0.813	−0.180	0.040	−0.271	−1.189**	−0.885
	(0.529)	(0.583)	(0.399)	(0.695)	(0.467)	(0.400)	(0.585)	(0.465)	(0.578)
$R_{\text{S\&P 500}} \times \mathbf{1}\{\text{FED contraction}\}$	0.129	0.319	−1.000**	−0.211	−0.970**	0.279	−0.822	−0.560	−1.119**
	(0.528)	(0.516)	(0.461)	(0.434)	(0.474)	(0.540)	(0.509)	(0.475)	(0.568)
Cross-Industry Volatility	−1.140**	0.217	0.241	1.161**	−0.672	−0.626	−0.890	−0.602	−1.002**
	(0.500)	(0.438)	(0.505)	(0.540)	(0.431)	(0.486)	(0.599)	(0.486)	(0.511)
$\Delta\ln$(Private Equity's TVPI)	−0.625	−0.136	−1.140*	−0.973	0.145	−2.876***	−0.215	−0.663	−2.847***
	(0.644)	(0.406)	(0.626)	(0.757)	(0.350)	(1.090)	(0.652)	(0.415)	(0.780)