

A Unified Model of Distress Risk Puzzles

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Abstract

We build a dynamic model to link two empirical patterns: the negative failure probability-return relation (Campbell, Hilscher, and Szilagyi, 2008) and the positive distress risk premium-return relation (Friewald, Wagner, and Zechner, 2014). We show analytically and quantitatively that (i) procyclical debt dynamics result in procyclical equity beta, which are negatively covarying with countercyclical market risk premium; (ii) the negative covariance generates low or negative stock returns and alphas among highly distressed firms; and (iii) firms with lower distress risk premiums endogenously choose higher leverage, so they are more likely to become distressed and earn negative returns. We provide strong empirical evidence to support our model predictions.

1 Introduction

Distress risk plays an important role in corporate financing choices and asset prices. Even though distress risk deters debt taking, empirical evidence on the *equity* distress risk premium in asset prices is mixed.¹ Recently, while Campbell et al. (2008) document a negative relation between failure probabilities and stock returns, Friewald et al. (2014) find a positive relation between distress risk premium (from credit default swaps) and stock returns. Moreover, firms with a high failure probability or a low distress risk premium have high equity beta but low and even *negative* stock returns on average. In this study, we develop a unified framework to explicitly link seemingly contradicting puzzles, i.e., the negative failure probability-return relation and the positive distress risk-return relation. In essence, endogenous debt financing and endogenously determined distress status over the business cycle connect and explain these empirical regularities.

Debt financing is procyclical in our model. After good shocks in the good states where the market risk premium is low, firms increase leverage (Goldstein, Ju, and Leland, 2001). In contrast, after negative shocks in the bad states where the market risk premium is high, firms decrease leverage via costly asset sales (Strebulaev, 2007). Because of real and financial frictions, firms more reluctantly reduce debt via asset sales and hence are more likely to fail in bad states. We empirically confirm that debt financing is positively correlated with the GDP but negatively associated with the expected market risk premium. While procyclical debt dynamics result in procyclical equity betas, negative covariance between procyclical beta and counter-cyclical market risk premium generates negative stock returns among highly distressed firms. Combined with the fact that distressed firms have high leverage and high equity beta, we produce simultaneously high unconditional CAPM beta and negative stock returns, as documented by Campbell et al. (2008).

Endogenous distress status helps understand the positive relation between the distress risk premium and stock returns, i.e., heterogeneity in the distress risk premium has a first order effect on the endogenous debt choice and therefore the firm's future financial status. That is, firms with a low exposure to distress risk (and therefore a low distress risk premium) choose higher leverage

¹Dichev (1998), Griffin and Lemmon (2002), Campbell, Hilscher, and Szilagyi (2008) and Avramov, Chordia, Jostova, and Philipov (2013) find a negative relation between various proxies of default likelihood and stock returns. Vassalou and Xing (2004) use the Merton (1974) model and document the positive relation between the default probability and stock returns. Garlappi and Yan (2011) show there is a hump-shaped relation between default probability and stock returns.

ex ante. When hit by a large market-wide shock, those firms are more likely to become distressed relative to their counterparts. In other words, firms with a low distress risk premium are more likely to be distressed and hence earn low or negative stock returns.²

To facilitate our understanding of endogenous debt financing and heterogeneous distress premium, we start with a simplified model. Its closed form solutions reveal the negative failure probability-return relation is due to the negative covariance of equity beta and market premium. Building on the standard Leland-type models, we explicitly model the endogenous financial distress, before liquidation. Following Andrade and Kaplan (1998), we define a firm as “distressed” when its cash flow level falls below its contractual interest payment. Higher coupon payments imply an earlier time of entering distress. Hence, the distress threshold is endogenously chosen in our model, because debt levels are endogenously chosen over the business cycle.³ To our knowledge, we are the first one to model the endogenous distress status in the class of dynamic capital structure/credit risk models, which has profound implications for the distress risk premium puzzle in Friewald et al. (2014).

In a fully fledged model, firms choose optimal financing policies over the business cycle, given a countercyclical market risk premium. The endogeneity of debt financing becomes more severe when the economy fluctuates between good and bad states. Equity holders are concerned about bad states even when they finance in the good states, because the economy may suddenly switch into the bad states, in which they will face a higher distress cost. Thus, they choose lower leverage (Hackbarth, Miao, and Morellec, 2006). Then, we calibrate this fully fledged model and demonstrate it can generate the sizable distress risk premium quantitatively. With the calibrated economies, we are able to assess the negative failure probability-return relation quantitatively. We apply Campbell et al.’s (2008) coefficient estimates from the actual data to our simulated data to construct failure probability. When sorting firms on the failure probability, highly distressed firms exhibit high leverage and default probability, but have negative returns and unconditional CAPM alphas.

To understand the positive relation between the implied distress risk premium and returns, we

²Glover (2014) shows this kind of endogeneity causes an underestimate in the liquidation costs. Building on Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010b), he uses the simulated method of moments (SMM) to estimate the expected cost of default across 2,505 firms without considering the expected cost of distress. He does not separate the distress cost from the liquidation because he needs to keep the model parsimonious for structural estimation. We explicitly model the endogenous financial distress.

³Distress is exogenous in prior studies. For example, Elkamhi, Ericsson, and Parsons (2012) are the first who explicitly study financial distress in a Leland-type model (Leland, 1994). They take the distress threshold as exogenous and calibrate the threshold to match the firm characteristic before and after the downgrades of credit rating, and find a small flow distress cost before liquidation substantially helps to explain the low financial leverage puzzle.

propose a simple procedure to imply the distress risk premium in spirit of Almeida and Philippon (2007).⁴ Motivated by our analytical solution for the simplified baseline model, we use as proxy of distress risk premium the log-difference between risk-neutral and actual default probability in our calibrated economies. In our calibration, we allow for heterogeneity in the distress risk premium across firms. We mimic standard empirical procedure, imply the distress risk premium from our simulated data, and form portfolios on the implied risk premium. Consistent with findings of Friewald et al. (2014), firms with a lower implied distress risk premium, on average, tend to have higher leverage ratios, higher expected default probabilities, and higher realized distressed frequencies.⁵

Goldstein et al. (2001) and Strebulaev (2007) build dynamic models of debt refinancing, and show that this class of models are able to produce dynamics of capital structure, consistent with several documented empirical patterns. Recent literature has introduced macroeconomic risk on corporate financing and investment decisions as well as credit risk. Hackbarth et al. (2006) were the first to introduce macroeconomic dynamics to dynamic capital structure/credit risk models. Along this line, Chen (2010) seeks to explain the observed credit spreads and leverage ratios, Bhamra et al. (2010b) focus on a levered equity premium and Bhamra, Kuehn, and Strebulaev (2010a) focus on the dynamics of leverage in an economy with macroeconomic risk.⁶

Our work also adds to the literature on how financial or real frictions affect asset prices.⁷ Gomes (2001) is the first that studies the effect of financial frictions on the asset prices in a dynamic model. Gomes and Schmid (2010) examine the interaction between investment and financing, and their implications for the levered equity risk. Gomes and Schmid (2016) develop a general equilibrium model and connect the prices of stocks and bonds with endogenous leverage and aggregate volatility. Gomes, Jermann, and Schmid (2016) study the debt overhang problem in a tractable general equilibrium model that allows the interaction between nominal long-term corporate debt, inflation,

⁴They show that the expected cost of default is larger than previously thought and use the log-difference between risk-neutral and physical probability to proxy for the distress premium.

⁵These authors show that the distress risk premium from CDS data and equity risk premium are positively correlated in the Merton (1974) model. Thus, physical or risk-neutral default probability alone is insufficient to correctly assess the distress risk premium. However, they do not explain why firms with a low distress risk premium have a high default probability and low credit rating, and why those firms have high betas but *negative* stock returns on average. We complement their point and explicitly establish an *ex post* negative relation between distress risk premium and default probability, because of the endogenous debt choice.

⁶Moreover, Chen, Collin-Dufresne, and Goldstein (2009), Arnold, Wagner, and Westermann (2013), and Chen, Cui, He, and Milbradt (2014) examine credit spreads in the framework of credit risk over the business cycles.

⁷Several papers study the implications of corporate investment on stock returns, such as Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), Zhang (2005) and Hackbarth and Johnson (2015). Recent studies that consider macroeconomic risk include Kuehn and Schmid (2014) and Ai and Kiku (2013).

and real aggregates. Kojien, Lustig, and Van Nieuwerburgh (2010) show that bond factors from different business cycle horizons are priced in the cross-section of stock returns.

Our paper relates to recent risk-based theories to explain the distress puzzles. A partial list includes George and Hwang (2010), O'Doherty (2012), and Boualam, Gomes, and Ward (2017). All the aforementioned theories appeal to the decline in the equity beta among the highly distressed firms. However, distressed stocks have high volatility and high unconditional equity betas in the data. Boualam et al. (2017) argue that measurement error in equity betas explains the distress risk puzzle. Nevertheless, their framework does not explicitly explain the *negative* returns in the highly distressed firms, but we do. Thus, our work differs in at least two perspectives. First, our model provides the first risk-based story for the negative return in the distressed firms. We illustrate the importance of the negative covariance between the equity beta and market premium in the closed form solution and in the calibrated economies, and verify its quantitative implications in the data. Second, we show that the default probability and distress risk premium are inherently connected. The endogenous connection between them allows us to explain the negative failure probability-return relation and the positive distress risk premium and stock return relation jointly.

The rest of the paper proceeds as follows. Section 2 derives the implication for a baseline (simple) model, whereas Section 3 develops the fully fledged model, which we calibrate in Section 4. Section 5 contains the main results by presenting the calibrated model's predictions. Section 6 provides empirical evidence in support of the calibrated model and its implications, namely we document empirically procyclical debt financing is negatively related to the countercyclical market risk premium, especially for more distressed firms in our setting, and we also document empirically a negative covariance between levered equity beta and market risk premium. Finally, Section 7 concludes.

2 Baseline Model and Its Implications for Asset Prices

We use a baseline model to illustrate the implications of optimal procyclical financing policy for the levered equity risk and returns in closed-form solutions.

2.1 Setup

The baseline model is partial equilibrium with pricing kernel, m_t , following the differential equation:

$$\frac{dm_t}{m_t} = -r dt - \theta d\hat{W}_t^m, \quad (1)$$

where r is the risk-free rate, θ is the market price of risk, and \hat{W}_t is a standard Brownian motion under the physical measure. The economy consists of a large number of firms whose financial status (w_t) can be healthy (H) or distressed (D), i.e., $w_t = H, D$. A solvent firm produces instantaneous cash flow X_t , evolving under the physical measure according to the differential equation:

$$\frac{dX_t}{X_t} = \hat{\mu}_{w_t} dt + \beta_{w_t} \sigma^m d\hat{W}_t^m + \sigma^{i,X} d\hat{W}_t^i, \quad (2)$$

where $\hat{\mu}_{w_t}$ is the expected growth rate, σ^m is the systematic volatility, $\sigma^{i,X}$ is the idiosyncratic volatility, \hat{W}_t^m and \hat{W}_t^i are standard Brownian motions, and $X_0 > 0$ at time $t = 0$. The total volatility of the cash flow growth rate is $\sigma_{w_t} = \sqrt{(\beta_{w_t} \sigma^m)^2 + (\sigma^{i,X})^2}$. We use $\hat{\cdot}$ to denote variables under the physical measure, and define $\hat{\mu}_{w_t} = \mu_{w_t} + \zeta_{w_t}$, where μ_{w_t} (under Q) is the risk-neutral counterpart of $\hat{\mu}_{w_t}$ (under P), and ζ_{w_t} is the asset risk premium, i.e., $\zeta_{w_t} = \beta_{w_t}(\theta \sigma^m) = \beta_{w_t} \zeta^m$.

According to Gordon's growth model, the asset value under the risk-neutral measure Q is:

$$A_{t,w_t} \equiv A(X_t, w_t) = \mathbb{E}^Q \left[\int_t^\infty X_\tau e^{-r\tau} d\tau \right] = \frac{X_t}{r - \mu_{w_t}}. \quad (3)$$

Because the asset value A_{t,w_t} is linear in X_t , it follows that

$$\frac{dA_{t,w_t}}{A_{t,w_t}} = \hat{\mu}_{w_t} dt + \beta_{w_t} \sigma^m d\hat{W}_t^m + \sigma^{i,X} d\hat{W}_t^i. \quad (4)$$

Hence, assets A_{t,w_t} and cash flows, X_t , share the same parameters of growth and volatility.

2.2 Time Line

To illustrate the time line of this dynamic model, Figure 1 plots possible paths a firm could take in one refinancing cycle. At time 0, the firm enters the market and finances its investments with a mix of equity and debt. The installed assets produce cash flows, X_t , that are characterized by a physical

growth rate, $\hat{\mu}_{w_t}$, and a total volatility parameter, σ_{w_t} . In observing its dynamic asset value, the firm makes financing and default decisions. Path 1 shows that, when its cash flows reach an upper threshold X_u , the firm decides to issue more debt to take advantage of tax benefits. Following Goldstein et al. (2001), we assume that the firm calls back its outstanding debt at par and issues a greater amount of debt to take advantage of tax benefits. In contrast, if cash flow X_t crosses the low threshold X_s along Path 2, it is insufficient to make debt payments, so the firm becomes distressed and incurs a distress cost η in the form of a depressed growth rate. Following Strebulaev (2007), the firm sells a fraction ω of assets to retire a fraction k of debt. A distressed firm might survive and rebound, leading to a subsequent debt restructuring at the *same* upper threshold X_u . As the additional distress cost lowers the firm's cash flow growth rate (see (8)), its cash flow may continue to deteriorate to the point at which equity holders are no longer willing to inject capital, and decide to go bankrupt at X_d , as shown in Path 4. Bankruptcy leads to immediate liquidation.

[Insert Figure 1 Here]

The firm pays dividends to equity holders and taxes to the government at an effective rate τ . The dividend received by equity holders is the entire cash flow X_t , net of coupon payments c_{w_t} to debt holders and tax payments, i.e., $d_t = (1 - \tau)(X_t - c_{w_t})$, where c_{w_t} equals c when the firm is healthy and becomes $c(1 - k)$ when the firm reduces debt by a fraction of k in distress.

2.3 Optimal Policies

Because we solve the dynamic by backward induction, we first show how to determine the firm's default policy. Then, we present its refinancing policy and how to determine its financial status.

Endogenous Default Policy

If the risk-taking action does not save the firm, equity holders choose the optimal bankruptcy threshold X_d to maximize their own equity value $E(X_t, w_t)$ as follows:⁸

$$\lim_{X_t \downarrow X_d} E'(X_t, D) = 0, \tag{5}$$

⁸Davydenko (2008) documents that the majority of *negative-net-worth* firms do not default for at least a year, and that equity holders of distressed firms renegotiate with debt holders and violate bond covenants.

where $E'(X_t, w_t)$ denotes the first-order partial derivative of the equity value function $E(X_t, w_t)$ with respect to X_t . Equation (5) is the smooth-pasting condition that allows equity holders to choose the optimal bankruptcy threshold by considering a tradeoff between the costs of keeping the firm alive and future tax shelter benefits (Leland, 1994).

Endogenous Refinancing Policies

At time 0, the firm chooses the optimal coupon, c and the timing of debt refinancing, X_u , *ex ante* to maximize the present value of the firm. The healthy firm's value for $w_t = H$ is the sum of the equity value and debt value, net of a proportional flotation cost ϕ , as follows:

$$\arg \max_{c, X_u} E(X_0, H) + (1 - \phi)D(X_0, H), \quad (6)$$

subject to equation (5) and $P = D(X_0, H)$.

Endogenous Distress Policy

The firm becomes distressed before liquidation. We model the financial distress explicitly. Following Andrade and Kaplan (1998), we assume that the firm becomes distressed once its cash flow X_t falls below its the coupon c . Different from that in Elkamhi, Ericsson, and Parson (2012) who assume an exogenous distress threshold, an distress threshold, X_s , is endogenous, because the coupon c is endogenously chosen. The greater the distress cost η , the less debt issued, and the smaller the coupon c . The smaller coupon implies a lower distress threshold. In other words, a firm with a high distress cost is less likely to become distressed if it optimally chooses less debt *ex ante*.

When becoming distressed at the threshold $X_s = c$, the firm sells a fraction ϖ of its assets and uses the proceeds to retire a fraction k of its debt. Following Strebulaev (2007) and Arnold, Hackbarth, and Puhan (2017), a given reduction k in debt implies an asset sale proportion ϖ via $kP = \varpi A(X_s)$ when X_t crosses X_s the first time from above. In addition, we assume, for tractability, that asset sales are fully reversable when X_t crosses X_s the first time from below.

Overall, as shown in Figure 1, the upper bound X_u and the lower bound X_d characterize the length of each refinancing cycle. The optimal X_s between X_u and X_d determines the relative weight of a high growth rate of the healthy firm and a low growth rate of the distressed firm within each refinancing cycle. The higher the threshold X_s , the earlier the firm suffers from the distress costs.

To summarize, we assume that the order of the optimal thresholds within each refinancing cycle is:

$$X_d < X_s < X_0 < X_u. \quad (7)$$

2.4 Distress Cost

Distress costs differ from liquidation costs incurred when bankruptcy implies immediate liquidation. For example, according to (Titman, 1984), firms start to lose reputation, capable workers, and customers and suppliers when they started entering distress, but well before officially filing for bankruptcy. Specifically, capable workers have incentives to seek a more stable position when they observe that their firm is sinking. Customers (Suppliers) are reluctant to buy (sell) products from (to) a troubled firm because they are worried about replacement of parts or services (payments).

Notably, while debt holders bear liquidation costs in bankruptcy, equity holders bear distress costs before bankruptcy, i.e., equity holders do not pay anything ex post because of limited liability when debt holders take over the assets and pay all the liquidation and bankruptcy costs. In contrast, equity holders of a distressed firm receive lower cash flows because the loss of productive workers decreases their firm's productivity and operating profits, while they have to service debt payments regardless of their firms' financial status. In other words, the cost of distress affects equity holders more directly, even though they also bear ex ante the relatively smaller value due to bankruptcy costs. Following Elkamhi et al. (2012), we assume a flow cost of the expected distress cost as follows:

$$\hat{\mu}_D = \hat{\mu}_H - \eta, \quad (8)$$

where $\hat{\mu}_D$ and $\hat{\mu}_H$ denote the growth rate of distressed (D) firms and healthy (H) firms, respectively. That is, distress causes a reduction of the growth rate in (2). The distress cost $\eta \geq 0$ is a deadweight loss due to the loss of reputation, customers, suppliers, and productive workers.

Distress costs are related to the market condition. Intuitively, the distress cost is greater in the bad aggregate states than in the good states. Hence, part of the cost is systematic. We assume the distress cost consists of the nonsystematic component, η^0 , and the systematic component, η^s :

$$\eta = \eta^0 + \eta^s = \eta^0 + (\beta_D - \beta_H)(\theta\sigma^m), \quad (9)$$

where β_D and β_H denotes the loading of distressed firms and healthy firms on the systematic risk $\theta\sigma^m$, respectively. We assume $\beta_D > \beta_H$. The excess loading $\beta_D - \beta_H$ on the market risk $\theta\sigma^m$ during the distress time indicates this component of distress is associated to the market risk premium. Given the same growth rate of $\hat{\mu}_D$, the high market risk premium implies a high discount rate and a low continuation value of the assets for the distressed firm. Therefore, those firms are less willing to keep their business and file for bankruptcy early, resulting in a high default probability.

2.5 Scaling Property of Optimal Policies

The geometric process in equation (2), debt retirement at par value, and proportional debt issuance costs ensure the scaling property (Goldstein et al., 2001), so the dynamic problem reduces to a static problem. The scaling property states that, given the state of the economy, the coupon, default, distress and restructuring thresholds as well as the value of debt and equity at the restructuring points are all homogeneous of degree one in cash flow. Notably, the firm at two adjacent restructuring points faces an identical problem, except that the cash flow levels are scaled by a constant; e.g., if cash flow has doubled, it is optimal to double default, distress and restructuring boundaries.

2.6 Asset Pricing Implications

To gain preliminary insights, we first simplify the baseline model and use a closed-form solution to illustrate the effect of procyclical financing on levered equity beta. Then, we study the interaction between levered equity risk and the countercyclical market risk premium in the conditional CAPM framework. Our discussion on the equity returns focuses on distressed firms (i.e., for $w_t = D$).

To illustrate the asset pricing implications for distressed firms, we further simplify the baseline model. In the simplified baseline model, the firm has no option to refinance its debt. It has a reversible option of entering the distress status and an option to go bankrupt. The following proposition provides semi-closed-form solutions for stock returns of the baseline model.

Proposition 1 *Outside of bankruptcy, $X_t > X_d$, the conditional excess return of equity r_{t,w_t}^{ex} is:*

$$r_{t,w_t}^{ex} = \mathbb{E}_t[r_{t,w_t}^E] - r = (\gamma_{t,w_t}\beta_{w_t})\zeta^m = \beta_{t,w_t}^E\zeta^m. \quad (10)$$

In distress, $w_t = D$ for $X_s > X_t > X_d$, and the firm's elasticity of equity to cash flow, $\gamma_{t,D}$, is:

$$\begin{aligned} \gamma_{t,D} &= \frac{\partial E_{t,D}/\partial X_t}{E_{t,D}/X_t} \tag{11} \\ &= 1 + \underbrace{\frac{\frac{c(1-k)}{r}(1-\tau)}{E_{t,D}}}_{\text{Leverage}} + \underbrace{\frac{(\frac{c(1-k)}{r} - A(X_d, D))(1-\tau)\mathcal{L}_t}{E_{t,D}}}_{\text{Put Option of Going Bankruptcy (-)}} + \underbrace{\frac{\left(E(X_s, H) - \left(A(X_s, D) - \frac{c(1-k)}{r}\right)(1-\tau)\right)\mathcal{H}_t}{E_{t,D}}}_{\text{Call Option of Rebounding (+)}} \end{aligned}$$

where

$$\mathcal{L}_t = \frac{(\omega_{D,1} - 1)X_t^{\omega_{D,1}}(X_s)^{\omega_{D,2}} - (\omega_{D,2} - 1)X_t^{\omega_{D,2}}(X_s)^{\omega_{D,1}}}{(X_s)^{\omega_{D,2}}(X_d)^{\omega_{D,1}} - (X_s)^{\omega_{D,1}}(X_d)^{\omega_{D,2}}},$$

and

$$\mathcal{H}_t = \frac{(\omega_{D,2} - 1)X_t^{\omega_{D,2}}(X_d)^{\omega_{D,1}} - (\omega_{D,1} - 1)X_t^{\omega_{D,1}}(X_d)^{\omega_{D,2}}}{(X_s)^{\omega_{D,2}}(X_d)^{\omega_{D,1}} - (X_s)^{\omega_{D,1}}(X_d)^{\omega_{D,2}}}.$$

Outside of distress, $w_t = H$ for $X_t > X_s$, and the firm's elasticity of equity to cash flow, $\gamma_{t,H}$, is:

$$\gamma_{t,H} = \frac{\partial E_{t,H}/\partial X_t}{E_{t,H}/X_t} = 1 + \underbrace{\frac{\frac{c}{r}(1-\tau)}{E_{t,H}}}_{\text{Leverage}} + \underbrace{(1 - \omega_{H,1})\frac{(A(X_s, H) - \frac{c}{r} - E(X_s, D))}{E_{t,H}}\left(\frac{X_t}{X_s}\right)^{\omega_{H,1}}(1-\tau)}_{\text{Put Option of Deleveraging (-)}} \tag{12}$$

Equation (10) shows that the expected excess return, r_{t,w_t}^e , is the product of the *systematic* risk premium, ζ , the sensitivity of stocks to underlying assets, γ_{t,w_t} , and the cash flow beta, β_{w_t} . The time-varying element for the expected excess stock return is then γ_{t,w_t} in equation (10). We denote by γ_{t,w_t} the “equity-cash flow elasticity” or the “equity-asset elasticity” because it measures the percentage change in the equity value in response to a percentage change in cash flows.

When the firm is distressed, the sensitivity, $\gamma_{t,D}$, consists of four components, as shown in equation (11). The first is the baseline sensitivity, which is normalized to one. The second is related to financial leverage, as the perpetual value of the reduced coupon payment $\frac{c(1-k)}{r}$ can be regarded as risk-free equivalent debt. The equity-cash flow sensitivity is positively associated with the financial leverage. The distressed firm has two options. The first one is the option of delaying bankruptcy, which decreases the equity-cash flow sensitivity. This option, which is essentially an American put option, protects equity holders from downside risk. Given limited liability, equity holders choose to go bankrupt only when the asset value $A(X_d, D)$ falls below the risk-free equivalent debt $c(1-k)/r$. Hence, $\frac{c(1-k)}{r} - A(X_d, D) > 0$. The second one is the option of rebounding. It

might be able to rebound and get out of the distress.

When the firm is healthy, the sensitivity, $\gamma_{t,H}$, has three components. Compared with the financial leverage component of the distressed firm, the leverage component of the healthy firm is greater because it has more debt in place. Everything else equal, the leveraged beta of a healthy firm is greater than that of a distressed firm. However, if the cash flow X_t of this healthy firm is declining, it has an American put option to deleverage. Therefore, this option helps to reduce the equity risk when the firm is approaching the distress. This put option is particularly valuable when the economy is in the bad state because the firm might be able to survive after its debt reduction.

Numerical Example

We use numerical examples to illustrate the impact of endogenous leverage on stock returns via comparative statics and generate three results.

2.6.1 First Implication

In the first case, we consider one firm that operates across two aggregate states. We exogenously change the market volatility, σ^m , from 0.1 to 0.12, and the market price of risk, θ , from 0.35 to 0.4. That is, the market risk premium, ζ^m , increases from 0.035 to 0.048. At time 0, $X_0 = 1$, the firm chooses a different level of debt P and coupon c , given the market volatility and market price of risk. The endogenous leverage determines the levered equity beta in the two separate states.

As shown in the legend of Panel A of Figure 2, the optimal coupon decreases from 0.578 in the good state to 0.404 in the bad state. This is consistent with our intuition that, anticipating the distress risk premium, the firm with a greater exposure to this risk chooses less debt (lower coupon payments). Consequently, the levered equity elasticity shifts down parallelly. This is consistent with equation (11): the lower the coupon, the lower the leverage effect, and the lower the equity elasticity.

[Insert Figure 2 Here]

The endogenous leverage effect sustains when the firm is off the optimal financing points. The difference in the equity-cash flow sensitivity is more evident when the firm becomes more distressed (or X_t becomes smaller). Specifically, the difference in the distressed area where $X_t < X_s = 0.404$ is much stronger than that in the healthy area where $X_t > X_s = 0.578$. Moreover, a further decrease occurs when $X_s = 0.404$ and 0.578 due to the debt reduction in distress. In other words,

for the same increase in the market risk premium, the equity-cash flow sensitivities (or equity beta) of distressed firms decrease more than those of healthy firms. Therefore, distressed firms are significantly more sensitive to changes in the market risk premium.

The following result summarizes the effect of procyclical debt financing on levered equity beta.

Result 1 *Distressed firms have high levered equity betas, which negatively covary with the market risk premium.*

2.6.2 Second Implication

The following proposition heuristically shows that the expected excess stock return differs across the two states $s_t \in \{G, B\}$ and the two levels of financial status, $w_t \in \{H, D\}$.

Proposition 2 *For a firm that operates in the two aggregate states s_t , the conditional expected excess equity return of equity, r_{s_t, w_t}^{ex} is*

$$r_{s_t, w_t}^{ex} = \mathbb{E}_t[r_{s_t, w_t}^E] - r = (\gamma_{s_t, w_t} \beta_{s_t, w_t}) \zeta_{s_t}^m = \beta_{s_t, w_t}^E \zeta_{s_t}^m \quad (13)$$

where $\gamma_{s_t, w_t} = \frac{\partial E_{s_t, w_t}}{\partial X_t} \frac{X_t}{E_{s_t, w_t}}$, which measures the sensitivity of equity to the cash flow X_t , and $\beta_{s_t, w_t}^E = \gamma_{s_t, w_t} \beta_{s_t, w_t}$ is the equity beta in the aggregate states s_t and financial status w_t .

Different from the constant market risk premium in equation (10) in the baseline model, the market risk premium $\zeta_{s_t}^m = \theta_{s_t} \sigma_{s_t}^m$, is countercyclical because the market price of risk $\theta_B > \theta_G$ and the market volatility $\sigma_B^m > \sigma_G^m$ (see e.g., Bhamra et al. (2010b) and Chen et al. (2009)).

It follows that the unconditional expected excess return of a distressed firm is:⁹

$$r_{s_t, D}^{ex} = \mathbb{E} \beta_{s_t, D}^E \mathbb{E} \zeta_{s_t}^m dt + \underbrace{\mathbf{cov}(\beta_{s_t, D}^E, \zeta_{s_t}^m)}_{\leq 0} dt, \quad (14)$$

where $\beta_{s_t, D}^E$ is the equity beta, $\zeta_{s_t}^m$ is the expected market risk premium, and $\mathbf{cov}(\beta_{s_t, D}^E, \zeta_{s_t}^m)$ is the covariance between the equity beta and market risk premium. We have demonstrated in our first result that in distressed firms, the levered equity beta and the market risk premium covary

⁹This is in the same spirit as Jagannathan and Wang (1996). They argue that the covariance between the market beta and the expected market risk premium plays an important role in the conditional CAPM.

negatively, i.e., $\mathbf{cov}(\beta_{s_t,D}^E, \zeta_{s_t}^m) < 0$. Therefore, the negative covariance results in a reduction in the unconditional expected equity return for the portfolio of distressed firms. When the negative covariance dominates the first component, our model generates negative stock returns for distressed firms. Our paper is the first that provides a risk-based story for the negative returns of distressed firms.

Next, we discuss the negative alphas of distressed firms in the conditional CAPM. Lewellen and Nagel (2006) show that, if the conditional CAPM holds, the unconditional alpha α^u is

$$\alpha^u \approx \underbrace{\mathbf{cov}(\beta_t^E, \zeta_{s_t}^m)dt}_{\leq 0} - \frac{\mathbf{E}[\zeta_{s_t}^m]}{(\mathbf{E}[\sigma_t^m])^2 \mathbf{cov}(\beta_t^E, (\sigma_t^m)^2)} < 0, \quad (15)$$

where σ_t^m is the time-varying market volatility.¹⁰ Recall that $\mathbf{cov}(\beta_{s_t,D}^E, \zeta_{s_t}^m) < 0$ in equation (14). This negative covariance generates a negative unconditional alpha α_D^u and helps us to understand the negative unconditional alpha for distressed firms.

The following result summarizes our discussion on the effect of the covariance between levered equity beta and market risk premium on the stock returns and the unconditional CAPM alphas.

Result 2 *The negative covariance between equity beta and the market risk premium causes low and negative returns as well as negative CAPM alphas in highly distressed firms.*

2.6.3 Third Implication

To illustrate the effect of heterogeneous distress risk on the endogenous distress status and resulting stock returns, we consider two firms with different exposure to market risk, i.e., $\beta_D = 1$ and 1.5, when they are distressed. However, both firms have the same $\beta_H = 1$ when they are healthy.

As shown in Panel B, compared with Firm 2, Firm 1 with a low cash flow beta β_D chooses a high leverage and coupon. After the debt is in place, both firms become distressed when their cash flow X_t level falls below the coupon level c , respectively. If the asset sales do not save them, they decide to declare bankruptcy at the threshold X_d . It is evident that the greater the debt, the earlier the firm becomes distressed (X_s), and the earlier the bankruptcy and liquidation (X_d). Hence, the cash flow beta β_D and distress risk premium determines the optimal level of debt, which in turn determine the distressed status and stock returns.

¹⁰Specifically, Lewellen and Nagel (2006) demonstrate that the third item $\frac{\mathbf{E}[r_t^m]}{(\mathbf{E}[\sigma_t^m])^2 \mathbf{cov}(\beta_t^E, (r_t^m - \mathbf{E}[r_t^m])^2)}$ is trivial.

Result 3 *Firms with a low distress risk premium choose more debt and are more likely to become distressed, resulting in high betas but low and negative stock returns.*

In summary, we derive the closed-form solution and use comparative statics to demonstrate that the countercyclical market risk premium results in procyclical optional leverage. The negative covariance between them causes low and negative stock returns among distressed firms. Then, we show that firms with a low distress risk premium choose high debt, which results in a high likelihood of distress and receive low and negative stock returns.

Notably, the comparative static analysis in the simple model assumes the aggregate states switch with probability one when we exogenously change the market volatility and market risk premium. In reality, the aggregate states do not switch with a probability of one. In the next section, we develop a fully-fledged model to quantitatively assess our comparative statics results via calibration.

3 Fully-Fledged Model

We build a dynamic capital structure model that endogenizes a firm's financing, distress/deleveraging, and default decisions in an environment with time-varying macroeconomic risk. Our model is built on the recent development of credit risk models, including Chen (2010), Bhamra et al. (2010a,b).

Considering an economy with business-cycle fluctuations, and without loss of generality, we assume the economy has two aggregate states, i.e., $s_t = \{G, B\}$ for good (G) and bad (B) states, respectively. The state s_t follows a continuous-time Markov chain as follows:

$$\begin{bmatrix} 1 - \hat{\lambda}_B & \hat{\lambda}_B \\ \hat{\lambda}_G & 1 - \hat{\lambda}_G \end{bmatrix} \quad (16)$$

where $\hat{\lambda}_{s_t} \in (0, 1)$ is the rate of leaving the current state of s_t for another state. The probability of switching states, s_t , within a small interval Δt is approximately $\hat{\lambda}_{s_t} \Delta t$. While the long-run duration of the economy in the bad state is $\hat{\lambda}_G / (\hat{\lambda}_G + \hat{\lambda}_B)$, the duration of the economy in the good state is $\hat{\lambda}_B / (\hat{\lambda}_G + \hat{\lambda}_B)$. Recall that we use $\hat{\cdot}$ to denote the physical measure throughout the paper.

An exogenous pricing kernel is specified as follows:

$$\frac{dm_t}{m_t} = -r_{s_t} dt - \theta_{s_t} dW_t^m + (e^{\kappa_{s_t}} - 1) dM_t, \quad (17)$$

where r_{s_t} is the risk-free rate, θ_{s_t} is the market price of risk of small shocks, κ_{s_t} is the relative jump size of the stochastic discount factor, W_t^A is a standard Brownian motion, and M_t is a compensated Poisson process with intensity $\hat{\lambda}_{s_t}$ that follows the Markov chain specified in equation (16). κ_{s_t} determines the market price of large shocks in the aggregate economy: $\kappa_B = -\kappa_G$ and $\kappa_G > 0$.

3.1 Firm

A representative firm operates in one of two aggregate states s_t . In each state, its firm-specific financial status (w_t) can be healthy (H) or distressed (D), i.e., $w_t = H, D$. Before it goes bankrupt, the firm produces instantaneous cash flows X_t governed by the following stochastic process:

$$\frac{dX_t}{X_t} = \hat{\mu}_{s_t, w_t} dt + \beta_{s_t, w_t} \sigma_{s_t}^m d\hat{W}_t^m + \sigma_{s_t}^{i, X} d\hat{W}_t^i, \quad (18)$$

where $\hat{\mu}_{s_t, w_t} = \mu_{s_t, w_t} + \beta_{s_t, w_t} \zeta_{s_t}^m$ is the physical growth rate, $\zeta_{s_t}^m = \theta_{s_t} \sigma_{s_t}^m$ is the state-varying market risk premium, β_{s_t, w_t} is the firm's cash flow beta, $\sigma_{s_t}^{i, X}$ is the idiosyncratic cash flow volatility, and \hat{W}_t^i is a standard Brownian motion. Similar to the baseline model, the growth rate in the healthy condition is greater than its counterpart in the distressed condition by a rate of η_{s_t} as follows:

$$\hat{\mu}_{s_t, D} = \hat{\mu}_{s_t, H} - \eta_{s_t}, \quad (19)$$

where η_{s_t} is the distress cost in the state, s_t . The total volatility of cash flow rates $\sigma_{s_t, w_t} = \sqrt{(\beta_{s_t, w_t} \sigma_{s_t}^m)^2 + (\sigma_{s_t}^{i, X})^2}$.

3.2 Financing and Default Decisions

At time 0, the firm finances its investments with a mix of equity and debt. We assume the debt issued at the initial state s_0 is perpetual with fixed coupon payments $c(s_0)$ and the issuance cost is a constant fraction ϕ of the amount of issued debt. The coupon payment is fixed until equity holders choose to default or restructure. The firm is operating between the good and bad state s_t , but both its default and restructuring decisions will depend on the initial aggregate state s_0 it enters and the coupon payment $c(s_0)$ it promises to pay, regardless of the current state s_t .

Following Goldstein et al. (2001), we assume that when restructuring its debt, a firm can

only adjust debt levels upward. When cash flow increases to a high threshold $X_u(s_t; s_0)$ at the aggregate state s_t , the firm first calls the outstanding debt and then issues new debt with a new coupon payment $c(s_t)$. When cash flow X_t declines to a low threshold $X_s(s_t; s_0)$ at either state s_t , the firm is entering distress and the growth rate $\hat{\mu}_{s_t,H}$ declines to $\hat{\mu}_{s_t,D}$ by the flow distress cost, η_{s_t} .

When cash flows cannot cover the coupon payments, the firm may be able to issue equity to cover the shortfalls. When equity holders are no longer willing to inject more capital, they decide to go bankrupt at $X_d(s_t; s_0)$. Bankruptcy leads to immediate liquidation at a cost of α_{s_t} . While debt holders take over the firm and pay the liquidation costs, equity holders receive nothing.

3.3 Distress and Deleveraging Threshold

Same as in the baseline model, we assume that the firm enters distress when cash flow X_t falls below its required coupon payment $c(s_0)$ issued at the initial state s_0 , i.e., $X_s(s_t; s_0) = c(s_0)$, regardless of the current state s_t the firm is in. In the two-state model the firm is more precautionary in its debt policies. That is, even if the firm enters at the good state, $s_0 = G$, it issues less debt than in the single-state baseline model, because it anticipates to carry the debt and make the contractual coupon payment in the future bad state $s_t = B$.

Similar to the baseline model, firms sell asset to retire their debt. However, the fraction of debt reduction in the fully fledged model, $k(s_t; s_0)$, depends on the initial state where debt is issued and the current state where debt is retired. This fraction is as follows:

$$k(s_t; s_0)P(s_0) = \varpi(s_t)A(X_s(s_t; s_0)). \quad (20)$$

For firms issuing debt in the good state but selling the asset in the bad states, the fraction of debt they can retire $k(B; G)$ is low because $P(G)$ is high but the market value of assets $A(X_s(B; s_0))$ is low. According to Maksimovic and Phillips (2001), Yang (2008) and Arnold et al. (2017), the asset sales is procyclical, i.e., $\varpi_G > \varpi_B$. The procyclical asset sale is intuitive. On the demand side, firms are generally more financially constrained and have less financial slack to acquire new assets in the bad states. On the supply side, asset values in the bad state are undervalued compared with the good state, so that firms are less willing to sell the assets.

Similar to equation (9) of the baseline model, the distress cost (reduced growth rate) has non-

systematic $\eta_{s_t}^0$ and systematic components $\eta_{s_t}^s$ in the two states as follows:

$$\eta_{s_t} = \eta_{s_t}^0 + \eta_{s_t}^s = \eta_{s_t}^0 + (\beta_{s_t,D} - \beta_{s_t,H})(\theta_{s_t} \sigma_{s_t}^m). \quad (21)$$

Almeida and Philippon (2007) document the distress cost is countercyclical (i.e., low in good aggregate states, but high in bad aggregate states). This is intuitive. For example, it is more difficult to sell assets in the bad state than in the good state. Based on their result, we assume that the distress cost in the good state η_G has only the non-systematic component, η_G^0 , while the distress cost in the bad state has the two non-systematic and systemic components. The systematic distress component is related to the market risk premium of the aggregate bad state.

3.4 Firm's Problems

The firm makes optimal financing and default decisions to maximize equity value. Specifically, it chooses optimal bankruptcy and restructuring timing, as well as the optimal coupon. When the firm is distressed ($w_t = D$) in either aggregate state s_t , equity holders choose the optimal bankruptcy timing $X_d(s_t; s_0)$. By making a tradeoff between the costs of keeping the firm alive and the tax benefits, equity holders choose their optimal bankruptcy thresholds $X_d(s_t; s_0)$ (Leland, 1994).

Let us denote $E(X_t, s_t, w_t; s_0)$ and $E'(X_t, s_t, w_t; s_0)$ as the equity value function and its first derivative in the aggregate state s_t and in the financial condition w_t , conditional on the initial state s_0 , respectively. We have the following smooth-pasting conditions in both states for distressed firms:

$$\lim_{X_t \downarrow X_d(B; s_0)} E'(X_t, B, D; s_0) = 0, \quad (22)$$

$$\lim_{X_t \downarrow X_d(G; s_0)} E'(X_t, G, D; s_0) = 0. \quad (23)$$

At time 0, when the firm in an initial aggregate economic state, s_0 is healthy (i.e., $w_t = H$), equity holders choose the optimal coupon $\mathbf{c}(s_0)$, debt $\mathbf{P}(s_0)$ and the optimal threshold of restructuring $\mathbf{X}_u(s_0)$ to maximize the firm value (Goldstein et al., 2001), where the vectors $\mathbf{c}(s_0) = \{c(B), c(G)\}$, $\mathbf{P}(s_0) = \{P(B), P(G)\}$, and $\mathbf{X}_u(s_0) = \{X_u(B; s_0), X_u(G; s_0)\}$, respectively. In choosing its capital structure, the firm makes a tradeoff between tax benefits and the expected cost of default, as well

as the expected cost of distress, as follows:

$$\max_{c(s_0), \mathbf{P}(s_0), \mathbf{X}_u(s_0)} E(X_0, s_0, H; s_0) + (1 - \phi)D(X_0, s_0, H; s_0). \quad (24)$$

subject to equations (22), (23), and $P(s_0) = D(X_0, s_0, H; s_0)$.

where $D(X_t, s_t, w_t; s_0)$ denotes the debt value function in the aggregate state s_t and in the financial condition w_t , conditional on the initial state s_0 .

3.5 Scaling Property

We have discussed the scaling property in the baseline model of a single aggregate state. Building on extensions by Chen (2010) and Bhamra et al. (2010b) to different aggregate states, we also consider an endogenous distress threshold. Our structural model preserves the scaling property across two aggregate states, because it is particularly useful when we calibrate the model. We do not have to solve for the optimal policies whenever the firms refinance their debt and increase their equity size repeatedly. Across two initial states, due to the homogeneity, the optimal thresholds are proportional to the coupons issued at the initial states as follow:

$$\frac{X_d(s_t; G)}{X_d(s_t; B)} = \frac{X_s(s_t; G)}{X_s(s_t; B)} = \frac{X_u(s_t; G)}{X_u(s_t; B)} = \frac{c(G)}{c(B)}. \quad (25)$$

Given an initial state s_0 , we impose the following order of thresholds:

$$X_d(G; s_0) < X_d(B; s_0) < X_s(G; s_0) = X_s(B; s_0) < X_0 < X_u(G; s_0) < X_u(B; s_0). \quad (26)$$

Figure 3 illustrates the order of the optimal thresholds in both states. It is intuitive that the firm goes bankruptcy earlier in the bad state than they are in the good state. That is, $X_d(G; s_0) < X_d(B; s_0)$. With the reasonable parameter values, we assume that the firm refinances debt earlier in the good state than in the bad state, $X_u(G; s_0) < X_u(B; s_0)$. As we explain for the distress thresholds, we assume they are the same in both current states and are endogenously determined by the initial coupon, i.e., $X_s(G; s_0) = X_s(B; s_0) = c(s_0)$.

[Insert Figure 3 Here]

4 Calibration

4.1 Data

We obtain accounting information from quarterly Compustat industrial data. Due to the availability of quarterly Computstat data, our sample period ranges from January 1975 to December 2014. We restrict the sample to firm-quarter observations with non-missing values for operating income and total assets, with positive total assets. We include common stocks listed on the NYSE, AMEX, and NASDAQ with CRSP share code 10 or 11. We exclude firms from the financial and utility sectors. Debt is the sum of current liabilities (Compustat item LCTQ) and long term debt (item DLTTQ). Market leverage is the ratio of book value of debt to the sum of debt and equity (PRCC*CSHO). We winsorize the outliers at the top and bottom two percentiles.

4.2 Parameters

To begin, we set commonly used parameters to predetermined values similar to prior studies. The parameter values are listed in Table 1, and are largely based on the literature (Bhamra et al. (2010b), Bhamra et al. (2010a), Chen et al. (2009) and Chen et al. (2014)). Starting with the macroeconomic variables, we set the risk free rate $r_G = r_B = 4\%$ in both aggregate states to abstract away from any term structure effects. The market volatility $\sigma_{s_t}^m$ is 0.1 and 0.12 in the good and bad states, respectively. The countercyclical market price of risk θ_{s_t} is 0.22 and 0.38 in the good and bad states, respectively. The transition intensities of the Markov chain are chosen to match average duration of NBER-dated expansions and recessions, i.e., $\hat{\lambda}_G = 0.5$ and $\hat{\lambda}_B = 0.1$, which gives the average durations 10 years for expansions and 2 years for recessions over the business cycle. We set $\kappa_G = 1/\kappa_B = 1.5$, which implies the risk-neutral probability of switching from the good state to the bad state is 1.5 times as high as the actual probability. The rest of macroeconomic parameter values are standard.

[Insert Table 1 Here]

At time 0, firms are healthy and in the good state $s_t = G$ with an initial cash flow X_0 of 1. For the firm-level parameters, we draw the parameter values of the expected growth rate and volatility of cash flows from the literature and the data. The growth rate of a healthy firm is $\hat{\mu}_{s_t,H}$

= 0.06 and -0.01 for the good and bad states, respectively. Following the estimates of countercyclical idiosyncratic cash flow volatility, we set $\sigma_G^{i,X}$ to 0.2 and $\sigma_B^{i,X}$ to 0.15. Following Yang (2008), we assume asset sales are procyclical and set ϖ_{s_t} to 0.038 and 0.043 in the bad and good states. We set debt issuance cost ϕ_{s_t} to 1%. The effective tax rate across both states is 0.15.

Notably, our study differentiates distress costs from liquidation costs. Most prior models require large liquidation costs to match low observed leverage. That is, the liquidation cost ranges from 0.3 to 0.45 in the capital structure literature. To focus on the distress cost, we assume a much smaller liquidation cost of 10%. We also assume that the distress cost has a non-systematic component $\eta_{s_t}^0$ and a systematic component $\eta_{s_t}^s$. We provide empirical evidence to support our assumption that the distress risk premium is the main force that drives the cross-sectional variation in financial leverage in subsection 6.2. Recently, Elkamhi et al. (2012) assume the cost is non-systematic and find a small cost of 1-2% helps to generate low leverage ratios. Based on their results, we assume the non-systematic distress cost η_0 equals 1.5% in both aggregate states. There is no systematic distress cost in the good state. Hence, $\eta_G = 0.015$.

We consider five different values of the systematic distress risk premium η_B^s , which is proportional to the systematic distress risk premium $\zeta_B^m = \theta_B \sigma_B^m$ in the bad state. The different distress risk premium is only the *ex ante* heterogeneity in our calibration. The increase in the cash flow beta $\beta_{B,D}$ suggests an increase in the distress cost η_B . That is,

$$\eta_G = \eta_G^0 = 0.015; \tag{27}$$

$$\eta_B = \eta_B^0 + \eta_B^s = \eta_B^0 + \theta_B \sigma_B^m (\beta_{B,D} - \beta_{B,H}) = 0.015 + 0.380 * 0.12 (\beta_{B,D} - \beta_{B,H}). \tag{28}$$

Because of computational difficulties, we limit our analysis to five different values of distress risk premium loading $\beta_{B,D}$, which ranges from 1.2 to 2 with increments of 0.2.¹¹ We calibrate the range of $\beta_{B,D}$ to match the interquartile of the financial leverage.

¹¹The five different values can be regarded as the industry fixed effect. In other words, one can think of five industries on the website of Kenneth French.

4.3 Calibration Procedure

To generate the model-implied moments, we simulate the model and generate 100 panels at the quarterly frequency for a period of 150 years. The first 100 years of observations are discarded to reduce the dependence on initial values. In each panel there are 4000 identical firms *ex ante* with the same initial parameters. At time 0, healthy firms $w_t = H$ enter the market in the initial state $s_0 = G$. In observing the dynamics of their cash flows X_t , they take action whenever X_t crosses the refinancing threshold X_u , or falls below the default threshold X_d . When a firm terminates at the bankruptcy threshold X_d , debt holders take over the firm and a new firm emerges immediately, for which cash flow parameters, including growth rate and volatility, are reset to the initial levels.

At the refinancing threshold, all policies variables scale up. First, given our parameterization, we solve for four pairs of optimal policies on $c(s_t; s_0)$, $P(s_t; s_0)$, $X_u(s_t; s_0)$ and $X_d(s_t; s_0)$, for the initial state $s_0 = G$. Then, we apply the scaling property across two initial states in equation (25) to obtain new policies for $s_0 = B$. With these optimal policies, we calculate equity value E_t and debt values D_t along cash flow paths X_t according to the value functions in different four regions.

4.4 Failure Probability

We compute failure probability as in the third column of Table 4 of Campbell et al. (2008):

$$\begin{aligned}
 F\text{-Probability} = & -9.164 - 20.264 * NIMTAAVG + 1.416 * TLMTA - 7.129 * EXRETAVG \\
 & + 1.411 * SIGMA - 0.045 * RSIZE + 0.075 * MB - 0.058 * PRICE - 2.132 * CASHMTA.
 \end{aligned} \tag{29}$$

$NIMTAAVG$ is the moving average of the net income

$$NIMTAAVG_{t-1,t-12} = \frac{1 - \phi^3}{1 - \phi^{12}} (NIMTA_{t-1,t-3} + \dots + \phi^9 NIMTA_{t-10,t-12}), \tag{30}$$

and $EXRETAVG$ is the moving average of the relative excess returns

$$EXRETAVG_{t-1,t-12} = \frac{1 - \phi}{1 - \phi^{12}} (EXRET_{t-1} + \dots + \phi^{11} EXRET_{t-12}), \tag{31}$$

where $NIMTA$ is net income divided by the sum of market equity and total liabilities; $EXRET$ is monthly log excess return on each firm's equity relative to market excess return ($EXRET = \log(1 + R_{it}) - \log(1 + R_{mkt,t})$); $TLMTA$ is the ratio of total liabilities divided by the sum of market equity and total liabilities; $SIGMA$ is the volatility of stock returns; $RSIZE$ is the relative size measured as the log ratio of the firm's market equity to that of the total market; $CASHMTA$ is the ratio of cash and short-term investments divided by the sum of market equity and total liabilities; MB is the market-to-book equity; and $PRICE$ is the log price per share. When applying equation (29) to our simulated data, we do not include $CASHMTA$ because we do not model cash holdings. Nor do we include $Price$ because we do not have the number of the shares in the model. These two items have no significant effect in our results because their estimated coefficients and t -statistics from Campbell et al. (2008) are relatively small. However, we include both of them when we use the actual data.

4.5 Proxy of distress risk premium

Inspired by Almeida and Philippon (2007), we propose to proxy the distress risk premium using the logarithmic difference between the risk-neutral and objective default probability.

4.5.1 Theoretical Justification

We motivate our proxy of the distress risk premium in Appendix B.2.2. The proposition connects the instantaneous stock return with distress risk premium explicitly.

Proposition 3 *If the risk-neutral rate $\mu \rightarrow r$, the firm's instantaneous excess stock return r_{t,w_t}^{ex} at time t for $X_t \geq X_d$ is:*

$$r_{t,w_t}^{ex} = \gamma_{t,w_t} \left(\zeta_{w_t} dt + \sigma_{w_t} d\hat{W}_t \right) = \gamma_{t,w_t} \left[\left(\frac{\log(\pi_t) - \log(\hat{\pi}_t)}{\log(X_t) - \log(X_d)} + 1 \right) \frac{\sigma_{w_t}^2}{2} dt + \sigma_{w_t} d\hat{W}_t \right], \quad (32)$$

where $\pi_t \equiv \left(\frac{X_t}{X_d}\right)^\omega$ and $\hat{\pi}_t \equiv \left(\frac{X_t}{X_d}\right)^{\hat{\omega}}$ is the risk-neutral and physical default probability, respectively. The exponents $\omega_i < 0$ and $\hat{\omega}_i < 0$ are defined in the Appendix.

Under the simplified assumption, we explicitly show that the risk premium of the underlying cash flows, ζ_{w_t} , is given by $[(\log(\pi_t) - \log(\hat{\pi}_t))/(\log(X_t) - \log(X_d)) + 1]\sigma_{w_t}^2/2$ and is indeed related to the default risk. Because the risk-free rate is lower than the objective growth rate $\hat{\mu}$ by the risk

premium ζ_{w_t} , the risk-neutral probability of default exceeds its actual counterpart for a risk-averse agent, i.e., $\log(\pi_t) - \log(\hat{\pi}_t) \geq 0$. Combined with the condition of $X_t \geq X_d$, the model implies a positive risk premium ζ_{w_t} . Hence, we use $\log(\pi_t) - \log(\hat{\pi}_t)$ to proxy the distress risk premium.

Although the risk-neutral and physical probabilities of default are positively correlated, the difference, $\log(\pi_t) - \log(\hat{\pi}_t)$, can be negatively related to $\hat{\pi}_t$ or π_t . In other words, a high physical or risk-neutral probability of default does not guarantee a high risk premium, ζ_{w_t} , as the two probabilities can be negatively correlated.

4.5.2 Procedure for implied distress risk premium

With the insight from this simple model, we use $\log(\pi_t) - \log(\hat{\pi}_t)$ to proxy for the asset risk premium when we calibrate our fully fledged model. Friewald et al. (2014) demonstrate the risk premium is the difference between the inverse function of the risk-neutral default probability and the physical probability in the framework of Merton (1974) and back out the risk premium from CDS. Compared with the procedure of Friewald et al. (2014), our proxy is easy to implement.

Based on our discussion of equation (32), we can proxy for the asset risk premium ζ_t by calculating the log-difference of default probabilities $\log(\pi_t) - \log(\hat{\pi}_t)$. Following the empirical literature, we apply the KMV procedure to the simulated data. Denoting $\mathbf{N}(\cdot)$ the cumulative probability function of a standard normal distribution, the one-year objective default probability is given by:

$$\hat{\pi}_t = \mathbf{N}(-DD_t), \quad (33)$$

in which \hat{DD}_t denotes the distance to default as follow

$$\hat{DD}_t = \frac{\log[P_t/(E_t + P_t)] + (\hat{\mu} - \sigma^2/2)T}{\sigma\sqrt{T}}, \quad (34)$$

where $\hat{\mu}$ is the actual asset growth rate, σ is the annual asset volatility, E_t is the equity value, P_t is the book value of debt, and T is the time to maturity. We assume the maturity $T = 1$ as in the empirical procedure. In the standard KMV procedure, the actual growth rate is obtained by using the observed equity value and equity volatility to back out the asset value and then use the asset value to calculate the asset growth rate. Because we observe asset values in our simulated data, we

follow the empirical KMV procedure and use the past 12 observations to calculate the recent asset growth rate. Similarly, we are able to calculate the risk-neutral default probability $\hat{\pi}_t$ by replacing the actual growth rate $\hat{\mu}$ with the risk-neutral growth rate μ , which we know in the simulations.

When simulating the model, we know the distress risk premium parameters *ex ante*, but, in the empirical tests, we do not. Friewald et al. (2014) infer the distress risk premium from CDS *ex post*. We follow them and infer the distress risk premium from the simulated data *ex post* by “pretending” that we do not know the distress risk premium to assess the empirical procedure.

5 Calibration Results

We start with presenting optimal policies, aggregate moments, and then the characteristics of firms sorted on failure probability and implied distress risk premium. Lastly, we present results on cross-sectional returns generated from our model.

5.1 Optimal Policies

We allow the heterogeneity in the distress risk premium as in Table 1 and assume all the firms are born in the good state, $s_0 = G$. Table 2 reports optimal policies for five different types of firms with different distress risk premium loading $\beta_{B,D}$. First, consistent with earlier findings, firms borrow more in the good state and the coupon in the good state $c(G)$ is greater than in the bad state $c(B)$. Second, compared with that of $c(B)$, the variation in $c(G)$ s is larger, ranging from 0.34 to 0.42. This variation implies that, because firm anticipate a greater distress risk premium in the bad state due to a higher loading $\beta_{B,D}$, they precautionarily select lower debt coupons. Consequently, this variation in coupons translates into variation in par values, $P(s_t)$, and quasi-market leverage ratios, $QML(s_t)$, at time zero. As shown in Panel B, initial leverage, $QML(G)$, declines from 0.28 to 0.23 when exposure to systematic distress risk premium rises.

The variation in the distress risk premium affects refinancing, default and distress thresholds (see, e.g., third to the last column in Panel A). First, the distress threshold is endogenously determined. Regardless of which state the firm is actually in, the distress threshold $X_s(s_t; G)$ is the initial coupon $c(G)$ if the firms issue debt in good states. Hence, even if the firms are in the good state initially, they choose a lower level of debt and coupon payments, because they anticipate dif-

difficulties of deleveraging in the future and high distress cost in bad states. Second, the refinancing threshold $X_u(B; G)$ is larger than that of $X_u(G; G)$. This is consistent with our intuition, because firms generally are more reluctant to increase their debt when they are in the bad state than when they are in the good state. Third, distressed firms default earlier in bad state than in good state, i.e., $X_d(B; G) > X_d(G; G)$, and $X_d(B; G)$ is declining faster than $X_d(G; G)$, because their continuation values are low in bad state and decrease faster with the distress risk premium.

5.2 Calibrated Aggregate Moments

Table 3 reports the moments generated from the calibrated model. The model-generated moments are averaged across 100 simulated economies. The average market risk premium is 0.05 on average. Its 5th and 9th percentiles across the simulated panels range from 0.02 to 0.08. The equity Sharpe ratio is 0.31, close to the 0.329 generated in Gomes and Schmid (2016), and 25% of the 100 simulated economies has a Sharpe ratio above 0.38. The averaged sample mean of the quarterly quasi-market leverages, QML_t is 0.30, and the averaged standard deviation of financial leverage is 0.11. Both of them are close to the data. As explained before, the interquartile of the leverage is largely determined by the range of the distress risk premium loading $\beta_{B,D}$. Because its average across all the simulated panel is 0.15, close to 0.16 from the data, indicating that the range of $\beta_{B,D}$ we choose is reasonable.¹² Lastly, the average of the sample mean of interest coverage is 2.69, slightly greater than the one in data. Overall, all of them are largely close to the data.

[Insert Table 3 Here]

In Panel B, we report the probabilities. The probability of going into bankruptcy (or filing for Chapter 7) at the X_d is 0.16%, and the probability of default on the coupon payments at the distress threshold X_s (or filing for Chapter 11) is 1.94%. Roughly, the total rate of bankruptcy (of combining the two events) is 2.10% on average, which is consistent with Campbell et al. (2008).¹³ After the firms become distressed, they delever via asset sales and manage to stay in business.

¹²As noted fore, we assume the five different loadings of distress risk premium because of the computational difficulties. They can be regarded as the industry fixed effect, instead of the firm fixed effect. Because our calibration does not have the firm fixed effect, we follow Strebulaev and Whited (2012) and demean the time series of financial leverage of each firm, before we calculate the interquartile of leverage each quarter in the actual data.

¹³It is worth noting that in reality the firms that default on the coupon payment do not necessarily file for Chapter 11, although they are officially distressed. Hence, the total bankruptcy rate of 2.10% is slightly higher than the average probability of failure in Campbell et al. (2008). They classify the firms into the “failure” status when a firm files for Chapter 7 or 11, is delisted for financial reason, or receives a D rating.

Notably, the fraction of firms remaining distressed is 5.01% on average. Finally, the probability of rebounding is 1.79% per year, and the probability of refinancing is 4.74% per year. Overall, our model matches aggregate moments with those of the data reasonably well.

5.3 Visual Inspection: Procyclical Debt Financing over the Business Cycle

To begin, we study the time-variation of debt financing across business cycles. Figure 4 plots one sample path of debt refinancing over time for a typical economy. The economy consists of 4,000 firms. Gray areas in the figure correspond to periods when the economy is in the bad states.

[Insert Figure 4 Here]

Three observations are worth noting about Panel A. First, firms actively refinance their debt upward during good macroeconomic times and are relatively inactive during the bad times. For example, the probability of refinancing before the end of year 19 is close to 10% and then substantially declines to nearly zero in year 19 and 20. Second, after the economy switches out of the bad states, firms start to increase their debt again. In years 21 and 33 when the economy switches from the bad to the good state, the once-troubled firms embrace the new opportunities and immediately increase their debt. This causes the spikes in the refinancing probability at the starts of the expansion period. Third, the longer time the economy stays in the expansion, the more firms increase their debt, i.e., the probability of upward refinancing peaks at 0.14 after the economy gets out the short recession in year 53 and then stays in the good state afterwards.

In contrast, in the same economy, Panel B shows that the firms often sell their assets to reduce their debt burden in bad states. Consider the same period of year 19 to 20 when the probability of upward refinancing is nearly zero, the probability of deleveraging increases substantially to about 0.08. A similar contrast occurs during the long period of year 30 to 36. Second, this second long-lasting recession causes the highest probability of deleveraging in this economy. That is, 12% of the firms sell their assets to retire their debt in year 33. This indicates that the chronic distress forces the troubled firms to retire debt. Lastly, the deleveraging helps reduce the probability of liquidation, although they share the same pattern over the business cycle.

Consistent with, e.g., Covas and Den Haan (2011), we obtain procyclical debt financing over the business cycle at the aggregate level. Debt's procyclicality has profound implications for equity risk.

5.4 Impacts of Distress Risk Premium Heterogeneity on Distress and Default

Having visually confirmed the procyclical debt financing at the aggregate level in simulated data, we now examine the impact of distress risk premium heterogeneity on the distress and default events by running probit regressions on 100 simulated samples to provide some statistics. Each economy contains 100 years of records on annual refinancing and deleveraging activities for 4000 firms. For each data set, we estimate a pooled probit regression for each of the following three horizons: one, two, and five years. We regress indicators of distress on an intercept, the indicator of low distress risk premium $\mathbf{1}_{\beta_{B,D}=L}$, and quasi-market leverage QML_t . The low distress risk premium indicator determines the financial leverage at the (re)financing points, which is not path dependent, while quasi-market leverage QML_t incorporates market and firm-specific information and is path-dependent. This exercise allows us to examine if the effect of the ex-ante distress risk premium is sustainable.

[Insert Table 4 Here]

We report the results in Table 4. Panel A shows that when the prediction horizon is one year, a low distress risk premium is likely to significantly cause financial distress, as shown by the coefficient of 0.143 (t -statistic = 10.208). This confirms our conjecture that those firms choose a high leverage ex ante, which in turn increases the future likelihood of becoming distressed. This effect fades away when the prediction horizon increases to five years. The second column shows that the quasi market leverage has a very significant effect on the potential distress, with a coefficient of 4.350 (t -statistic = 89.730), because this path-dependent measure provides more “updated” information on the firm’s operating performance. More important, in the third column where we include both of the two variables, the estimated coefficient of the low distress risk premium indicator declines to 0.033, but is still statistically significant, with a t -statistic of 1.848. This indicates that the ex-ante risk premium has a sustainable effect on the firm’s endogenous distress status.

Panel B exhibits the results for predicating default events. First, for the prediction horizon of one year, the coefficient of the low distress premium indicator is 0.106 (t -statistic = 2.410) in the first column, and substantially decreases to 0.009 (t -statistic = -0.022) when the horizon increases to five years. Overall, the effect of the distress risk premium on the default events is much smaller relative to its effect on the distress events. Second, the predictive power of quasi-market leverage for the default events remains significant and is slightly stronger for the distress events, throughout all

the horizons. Lastly, when we include two variables together across all the three predictive horizons, the low distress risk premium indicator is dominated by the quasi-market leverage and becomes statistically insignificant. This is not surprising, because we allow the firms to reduce their debt once they are distressed. Hence, the distress risk premium that determines the optimal leverage at the (re)financing points has relatively low predictive power, compared with the QML that has “updated” the information of the adjusted level of debt and the firms’ operating performance.

[Insert Figure 5 Here]

Furthermore, Figure 5 illustrates the impact of distress risk premium heterogeneity over years. For a typical economy, Panel A plots the time series of distress probability for two types of firms, i.e., one with a low distress risk premium ($\beta_{B,D} = 1.2$ or type 1) and another one with a high distress risk premium ($\beta_{B,D} = 2$ or type 5). Each type consists of 800 firms. Observe that the distress probabilities of type 1 firms is consistently greater than that of type 5 firms across business cycles. In contrast, the default (or liquidation) probabilities in Panel B do not exhibit any significant differences between the low- and high- distress risk premium firms.

In sum, because it determines optimal leverage, the standalone distress risk premium has predictive power for endogenous distress events, but not for default events. So, if we sort firms based on this variable, these distress risk premium sorts will affect cross-sectional stock returns significantly.

5.5 Characteristics of Distressed Firms

We report cross-sectional averages of key portfolio characteristics for our simulated data panels, including leverage, objective default probability of Merton, deleveraging frequency, and fraction of distressed firms in Table 5. The one-year default probability of Merton (1974) is calculated using equation (34), and the fraction of distressed firms is the ratio of the number of distressed firms to the total number of firms each period. As in Andrade and Kaplan (1998), we classify a firm as “distressed” if its cash flow falls below the coupon payment or if it defaults on coupon payments.

[Insert Table 5 Here]

In panel A, we sort the firms based on the failure probability of Campbell et al. (2008) and calculate the cross-sectional averages of the above characteristics. Three observations are worth

noting. First, failure probability rises with leverage. It is not surprising because, by construction, leverage is an input for calculating failure probability as in equation (29). Consequently, higher leverage produces higher expected, one-year default probabilities, which confirms that failure probability and Merton’s default probability are highly correlated.¹⁴ Second, we confirm the predictive power of both measures using the realized fraction of distressed firms. That is, the fraction of distressed firms is increasing with failure probability, particularly in bad states. Third, distressed firms decrease leverage. The frequency of deleveraging increases with failure probability as well, particularly in bad states. Thus, distressed firms are more sensitive to aggregate shocks and reduce their financial risk more aggressively, confirming our first result that levered betas are more negatively correlated with aggregate market premiums in distressed firms compared to healthy firms.

In Panel B, we sort firms based on our proposed proxy for the implied distressed risk premium. First, financial leverage monotonically declines from 0.37 to 0.26 with our distress risk premium proxy. This confirms our intuition that firms with a higher distress risk premium choose lower leverage *ex ante*, and also confirms that our proxy captures the distress risk premium well. Second, the expected one-year default probability decreases with the implied distress risk premium. This observation is in line with the results by Friewald et al. (2014). In their Tables II to VII, firms with a low distress risk premium actually have a low credit rating. Third, the realized fraction of distressed firms is slightly U-shaped. The low leverage in distressed firms is likely due to the deleveraging activities. This intuition is reinforced by U-shaped deleveraging frequencies, particularly in bad states. Taken together, we find firms with a low distress risk premium choose more debt. When the market is hit by a large, negative shock, highly leveraged firms are more likely to become distressed.

Moreover, observe that the difference in QML is 0.30 in Panel A of Table 5 and 0.11 in Panel B of Table 5, which are much larger than the spread 0.05 in QML in Table 2. At non-refinancing points, the default probability spread is larger, because firms do not adjust their debt frequently (Strebulaev, 2007), while a large negative shock might hit firms with the highest leverage.

In sum, the calibrated results consistently confirm that firms with lower asset risk premium (and therefore lower discount rate) endogenously choose higher leverage and are more likely to become distressed in the future. The endogenous distress status is crucial for explaining the empirical

¹⁴The Merton default probability is slightly U-shaped. The relatively high default probability in the firms with a low leverage and a low failure probability is likely due to the deleveraging in the distressed firms.

findings that firms with a low distress risk premium earn negative stock returns on average.

5.6 Equity Returns

Table 6 reports the average of annualized excess returns, $r_{i,t}^{ex}$, of value-weighted portfolios. Panel A reports the value-weighted returns of portfolios sorted on the probability of failure. It is evident that the excess return monotonically decreases from 10.38% to -7.27% by 17.66%. In addition, the alphas from the unconditional CAPM follow the same pattern but display a more significant decline. This confirms our second result that the negative returns in the highly distressed firms are due to the negative covariance between the equity beta and the market returns. Moreover, the unconditional betas rise from 0.89 to 1.37 by 0.48, confirming our first result that distressed firms have high leverage and high levered equity risk. To the best of our knowledge, this is the first paper that can generate simultaneously increasing unconditional equity betas and decreasing average returns, which match the surprising results by Campbell et al. (2008).

[Insert Table 6 Here]

Panel B reports results for portfolios sorted on the implied distress risk premium. In contrast to those in Panel A, the excess returns monotonically increase from -4.59% to 11.97% by 16.57%, in line with the pattern in Friewald et al. (2014).¹⁵ So do the unconditional alphas. The negative return in the firms with a low implied asset risk premium can be explained by the endogeneity in the firms' financial status, as discussed in our third result. That is, firms with a low distress risk premium issue more debt and have a high default probability. After becoming distressed, they have a negative stock return because of the negative covariance between their equity beta and market risk premium.

In short, we provide an internally coherent framework that connects two seemingly contradicting puzzles in our calibrated dynamic model. It is the endogenous debt financing that produces the negative failure probability-return relation as well as negative returns in distressed firms. Moreover, it is the endogenous distress status that causes the firm with a low distress risk premium to have a high leverage and high default probability but negative stock returns.

¹⁵Our results are slightly different from theirs in that we have positive returns for firms with a high distressed risk premium, because we calibrate the model to large sample of Compustat firms, while their data limited to a much smaller sample of firms that have CDS prices.

6 Empirical Results

We first provide empirical evidence for the procyclical debt financing and for its negative relation to the countercyclical market risk premium, which should be increasingly negative for more distressed firms in our setting. Second, we document empirically a negative covariance between levered equity beta and market risk premium consistent with our setting.

6.1 Procyclical Debt Financing

The channel of our model is the procyclical debt financing. We adopt the specification of Covas and Den Haan (2011) to examine the quarterly debt financing. Using the annual Compustat, Covas and Den Haan (2011) document that small-sized firms are more likely to engage with procyclical financing than big firms, by regressing debt changes on the HP-filtered GDP. For the purpose of this study, we replace the HP-filtered GDP with the expected market risk premium and divide the firms based on the failure probability of Campbell et al. (2008) instead of asset size.¹⁶

We obtain the expected market risk premium and credit market factors from Haddad, Loualiche, and Plosser (2017).¹⁷ They use the predicted expected excess equity returns to proxy for the aggregate risk premium. This market risk premium is a long-term estimate, because they use a three-year horizon to predict the market risk premium, which is suitable for our work because we model the perpetual debt. That is, they regress excess equity returns on the dividend-price ratio, D/P , consumption-wealth ratio, cay , and the three-month T-bill yield, $T - Bill$, to predict excess returns. Their regression yields the expected market risk premium as follows:

$$E_t(R_{M,t+1}^e) = -0.76 + 2.89 * (D/P)_t + 2.54 * cay_t - 0.97 * (T - Bill)_t, \quad (35)$$

where $R_{M,t+1}^e$ is the annualized return of the value-weighted market portfolio over the next three years in excess of the current three-month T-bill yield. The dividend-price ratio $(D/P)_t$ is constructed using CRSP data on monthly returns and the variable cay_t is an empirical proxy for the log consumption-wealth ratio (Lettau and Ludvigson, 2001). Interest rates are constant maturity

¹⁶We complement their study by using quarterly Compustat industrial data. We follow them and construct our sample. We require the quarterly observations from the Compustat industrial data have non-missing and positive sales and total assets, and the firms have a fiscal year end of 12. We winsorize the data at the top and bottom 2 percentiles.

¹⁷We download the data from the website of Erik Loualiche.

rates according to the Federal Reserve’s H.15 release.

We employ a panel regression as follows:

$$\begin{aligned} \frac{\Delta D_{i,t}}{A_{i,t-1}} = & \alpha_{0,i} + \sum_{j=1}^7 I_{i,t}(j) \{ \alpha_{j,t} + \alpha_{j,Y} Y_t + \alpha_{j,CF} \left(\frac{CF_{i,t-1}}{A_{i,t-2}} - \frac{\overline{CF_{j,t-1}}}{\overline{A_{j,t-2}}} \right) \right. \\ & \left. + \alpha_{j,Q} (Q_{i,t-1} - \overline{Q_{j,t-1}}) + \alpha_{j,S} (Size_{i,t-1} - \overline{Size_{j,t-1}}) \right\} + u_{i,t}, \end{aligned} \quad (36)$$

where $\Delta D_{i,t}$ is the change in the long term debt (compustat item DLTTY) scaled by total assets (compustat item ATQ) of last quarter¹⁸, $I_{i,t}(j)$ is an indicator function that takes on a value equal to 1 if firm i is a member of seven groups j . The cyclical measure, Y_t , includes HP-filtered GDP, expected market risk premium and the decomposed expected market risk premium. Control variables include lagged cash flows, $CF_{i,t-1}$, logarithm of book asset, $Size_{i,t-1}$, and Tobin’s Q, $Q_{i,t-1}$, relative to their averages of the other firms in the same group. In addition to controlling for firm fixed effect, we add a linear time trend as explanatory variable and include quarterly dummy variables to account for seasonality. When constructing the firm group, we follow Covas and Den Haan (2011) and classify firms into seven categories based on their failure probability (Campbell et al., 2008) at the end of last quarter. The seven groups are in the increasing order with the failure probability but with uneven percentile brackets: [0, 25], [25, 50], [50, 75], [75, 90], [90, 95], [95, 99], and [99, 100]. This uneven division is also similar to Table VI of Campbell et al. (2008).

Table 7 reports the estimated coefficients for the key variable $I_{i,t}(j)Y_t$. Note that a higher value of $j = 1, \dots, 7$ indicates that the firms are in a group with a higher failure probability. Panel A shows that the debt financing is procyclical across all the seven groups, because debt financing is positively correlated with the HP-filtered GDP, and the positive coefficients are increasing with the failure probability. For the firms with an extremely high failure probability (i.e, in the bracket of [99, 100]), the estimated coefficient is 0.13 (t -statistic = 2.17), which is much greater than 0.02 (t -statistic = 1.53) for firms in the bracket of [0, 25].

Notably, to examine whether the market risk premium drives the procyclical debt financing, we replace the GDP variable with the countercyclical market risk premium in Panel B. Indeed, the

¹⁸We convert both the year-to-date item DLTTY to the quarterly frequency. Our results are similar when we use include the short term debt (Compustat item DLCY). We mainly report the results using the long term debt because we model the perpetual debt and the expected market premium we employed is predicted from a three-year long horizon.

estimated coefficients of the expected market risk premium $I_{i,t}(j)MRP_t$ are all negative. With the increasing failure probability, the negative coefficient increases monotonically in absolute term from -0.04 (t -statistic = -5.26) to -0.15 (t -statistic = 6.12), confirming that the increase in expected market risk premium prevents firms from debt financing over the business cycle, and that distressed firms are more sensitive to the countercyclical market risk premium.

In sum, we provide direct evidence to support the calibrated model’s result that the countercyclical market risk premium negatively impacts debt financing, particularly in distressed firms.

6.2 Impact of the Distress Risk Premium on Procyclical Debt Financing

Our calibration assumes financial leverage largely varies due to the heterogeneous loadings on the aggregate distress risk premium (see equation (28)). To ensure this assumption is largely consistent with the data, we decompose the expected risk premium into two components, the distressed risk premium (DRP) and non-distress risk premium (NDRP) components in the two specifications.

In the first decomposition, we regress the expected market risk premium $E_t(R_{M,t+1}^e)$ on the *HY* Spread, which is a composite of the Merrill Lynch high-yield bond indices less the yield on the three-month T-bill (Axelson, Jenkinson, Strömberg, and Weisbach, 2013). Because this high bond yield is largely due to the default risk premium, we use the fitted value to proxy for the aggregate distress risk premium and the residuals for the non-distress risk premium. Haddad et al. (2017) also point out the correlation coefficient between the expected market risk premium and the *HY* spread is 0.55, implying a large fraction of this market risk premium is due to the aggregate default risk premium. Panel C demonstrates the contrast for the effects on the debt financing from two components. First, all the coefficients of the distressed component $I_{i,t}(j)DRP_t$ are all negative and greater in absolute terms than those in Panel B. Second, the contrast in the estimated coefficients between firms with a lowest failure probability and those with highest probability increases. That is, the difference increases from 0.11 ($0.15 - 0.04$) in Panel B to 0.15 ($0.24 - 0.09$) in Panel C. Lastly, in a sharp contrast, the estimated coefficients of the non-distressed component $I_{i,t}(j)NDRP_t$ all become weakly positive or negative.

In the second decomposition, we regress $E_t(R_{M,t+1}^e)$ on the *HY* Spread, and the other two credit market factors from Haddad et al. (2017), including *EBITDA* Spread and *GZ* Spread. The *EBITDA* Spread is the median *EBITDA/EV* ratio for Compustat firms minus the yield on the

composite high-yield bond index (Kaplan and Stromberg, 2009), and *GZ Spread* is a measure of the excess premium in corporate bonds (Gilchrist and Zakrajšek, 2012). Panel D report results very similar to those in Panel C, with a relatively weaker magnitude.

Therefore, by decomposing the expected market risk premium into the distressed and non-distressed components, we provide further support for the assumption in our calibration exercise that it is the distress risk premium, instead of the total market risk premium, that drives the cross-sectional variation in financial leverage.

6.3 Cross-Sectional Equity Beta and Returns

Having demonstrated the strong procyclicality of debt financing, we now proceed to test its implications for stock returns. Combined with a countercyclical market risk premium, it generates negative expected stock returns and negative CAPM alphas. We provide empirical evidence to confirm the second result of the model: the negative covariance between the levered equity beta and the market risk premium generates a low or negative return and CAPM alphas in distressed firms.

We follow Lewellen and Nagel (2006) and use the realized excess stock market return r_t^m to proxy for the market risk premium λ_{s_t} , and the monthly CAPM beta to proxy for the time-varying market beta β_t^E . The monthly CAPM beta is obtained by regressing daily returns on daily excess market returns. We also use the procedure of Lewellen and Nagel (2006) to mitigate microstructure noises. Empirically, the unconditional expected stock excess return is:

$$\mathbf{E}[r_t^{ex}] = \mathbf{E}[r_t^E] - r = \mathbf{E}[\beta_t^E r_t^m] = \mathbf{E}[\beta_t^E] \mathbf{E}[r_t^m] + \mathbf{cov}(\beta_t^E, r_t^m). \quad (37)$$

and the unconditional CAPM alpha is

$$\alpha^u \approx \mathbf{cov}(\beta_t^E, r_t^m) dt - \frac{\mathbf{E}[r_t^m]}{(\mathbf{E}[\sigma_t^m])^2 \mathbf{cov}(\beta_t^E, (\sigma_t^m)^2)}. \quad (38)$$

Table 8 reports the results. Panel A presents the average excess return in percent, $r_{i,t}^{ex}$, and unconditional CAPM alpha, α^u , for the value-weighted portfolios, sorted on the prior month's failure probability (Campbell et al., 2008).¹⁹ The difference in the excess return is -11.75% with a

¹⁹NIMTA is net income (COMPUSTAT quarterly item NIQ) divided by the sum of market equity (PRCCQ x CSHOQ) and total liabilities (Compustat item LTQ); $EXRET = \log(1 + R_{it}) - \log(1 + R_{S\&P500,t})$ is the monthly

t-statistic of -2.14 , and the unconditional CAPM alpha is -15.58% .

Next, we turn to the conditional CAPM. Using $\beta_{i,t}^E$ from the data and following the procedure of Lewellen and Nagel (2006), we calculate the excess return and the unconditional alpha. The first row of Panel B presents the value-weighted average of the conditional market equity betas, $\beta_{i,t}^E$. We report the value-weighted average of the time-varying CAPM beta $\beta_{i,t}^E$ from the data in the first row. They are obtained from the regression of daily returns on the daily excess market returns, month by month, and are adjusted by the procedure of Lewellen and Nagel (2006). The difference between equity betas is small with a value of 0.12 . The next two rows report the two components of the excess return, $\mathbf{E}[\beta_{i,t}^E]\mathbf{E}[r_t^m]$ and $\mathbf{cov}(\beta_{i,t}^E, r_t^m)$, respectively. Without the covariance, $\mathbf{E}[\beta_{i,t}^E]\mathbf{E}[r_t^m]$ is simply the unconditional CAPM alpha. The spread in $\mathbf{E}[\beta_{i,t}^E]\mathbf{E}[r_t^m]$ in the second row is 0.64% per year.

The covariance, $\mathbf{cov}(\beta_{i,t}^E, r_t^m)$, in the third row decreases monotonically from 0.27% to -5.33% , a total fall of 5.61% . Even though the equity beta, $\beta_{i,t}^E$, implies a small positive spread, the covariance term dominates 0.79% to -7.06% , a drop of -7.84% , which is slightly more than 50% of the 15.68% in the data. It is worth noting that the monotonically decreasing α^u is mainly due to the $\mathbf{cov}(\beta_{i,t}^E, r_t^m)$ because the second item in the α^u formula is very small. Hence, the negative covariance between levered equity beta and market risk premium helps to explain previously puzzling negative return patterns of distressed firms. Moreover, spreads in the model-implied excess returns and unconditional alphas account for about 50% of their empirical counterparts.

In short, our empirical evidence supports our analytical and quantitative model results that the negative covariance between market risk premium and equity returns generates the negative stock returns for the highly distressed firms, which helps to produce the negative relation between the failure probability and stock returns.

7 Concluding Remarks

In a unified, dynamic model, we establish that optimal debt dynamics and, as a result, endogenous distress status over the business cycle can explain simultaneously negative returns of a firm in

log excess return on each firm's equity relative to the S&P 500 index; TLMTA is the ratio of total liabilities divided by the sum of market equity and total liabilities; SIGMA is the volatility of daily stock return over the past three months; RSIZE is the relative size measured as the log ratio of its market equity to that of the S&P 500 index; CASHMTA is the ratio of cash and short-term investments (CHEQ) divided by the sum of market equity and total liabilities; MB is the ratio of market equity to book equity (CEQ); PRICE is the log price per share.

distress, which has a high failure probability, but that also exhibits a low distress risk premium.

While a basic extension of Merton's (1974) framework can rationalize the empirical finding on the positive relation between distress risk premium and stock returns, what remains puzzling in Friewald et al.'s (2014) setting is (i) why firms with a low distress risk premium have low credit ratings, and are highly likely to default, and (ii) why highly distressed firms earn negative returns, which drives the positive distress risk premium-returns relation. In other words, the negative average return of distressed firms remained unexplained, because a homogeneous, positive distress risk premium can not produce negative expected or realized returns for any of the portfolio sorts.

We build a simplified model to derive closed form solutions that reveal three main results. First, distressed firms with high failure probabilities are more sensitive to business cycle conditions and reduce their debt more aggressively during economic downturns. Second, endogenous debt financing induces distressed firms' levered equity betas to covary negatively with countercyclical market risk premiums. Importantly, the negative covariance effect generates low and even negative equity returns among distressed firms. Moreover, we establish empirical results that are consistent with our dynamic model's predictions. Third, firms that have low exposure to distress risk choose higher debt levels and hence are more likely to fail in bad macroeconomic states. Fourth, our calibrated model demonstrates quantitatively that firms with a lower implied distressed premiums select higher leverage and display higher default probabilities, but earn lower and even negative returns.

Finally, we provide empirical evidence in support of our setting, namely we document empirically procyclical debt financing is negatively related to the countercyclical market risk premium, especially for more distressed firms in our setting, and we also document empirically a negative covariance between levered equity beta and market risk premium.

References

- Ai, H., and D. Kiku. 2013. Growth to value: Option exercise and the cross section of equity returns. *Journal of Financial Economics* 107:325–349.
- Almeida, H., and T. Philippon. 2007. The Risk-Adjusted Cost of Financial Distress. *Journal of Finance* 62:2557–2586.
- Andrade, G., and S. N. Kaplan. 1998. How Costly Is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions That Became Distressed. *Journal of Finance* 53:1443–1493.
- Arnold, M., D. Hackbarth, and T. Puhan. 2017. Financing asset sales and business cycles. *Review of Finance* 22:243–277.
- Arnold, M., A. F. Wagner, and R. Westermann. 2013. Growth options, macroeconomic conditions, and the cross section of credit risk. *Journal of Financial Economics* 107:350–385.
- Avramov, D., T. Chordia, G. Jostova, and A. Philipov. 2013. Anomalies and financial distress. *Journal of Financial Economics* 108:139–159.
- Axelson, U., T. Jenkinson, P. Strömberg, and M. S. Weisbach. 2013. Borrow cheap, buy high? The determinants of leverage and pricing in buyouts. *The Journal of Finance* 68:2223–2267.
- Berk, J. B., R. C. Green, and V. Naik. 1999. Optimal Investment, Growth Options and Security Returns. *Journal of Finance* 54:1553 – 1607.
- Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev. 2010a. The aggregate dynamics of capital structure and macroeconomic risk. *Review of Financial Studies* 23:4187–4241.
- Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev. 2010b. The Levered Equity Risk Premium and Credit Spreads: A Unified Framework. *Review of Financial Studies* 23:645–703.
- Boualal, Y., J. F. Gomes, and C. Ward. 2017. Understanding the behavior of distressed stocks .
- Campbell, J., J. Hilscher, and J. Szilagyi. 2008. In Search of Distress Risk. *Journal of Finance* 63:2899–2939.
- Carlson, M., A. Fisher, and R. Giammarino. 2004. Corporate Investment and Asset Price Dynamics: Implications for the Cross-Section of Returns. *Journal of Finance* 59:2577–2603.
- Chen, H. 2010. Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure. *Journal of Finance* 65:2171–2212.
- Chen, H., R. Cui, Z. He, and K. Milbradt. 2014. Quantifying liquidity and default risks of corporate bonds over the business cycle. Tech. rep., National Bureau of Economic Research.
- Chen, L., P. Collin-Dufresne, and R. S. Goldstein. 2009. On the relation between the credit spread puzzle and the equity premium puzzle. *Review of Financial Studies* 22:3367–3409.
- Covas, F., and W. J. Den Haan. 2011. The cyclical behavior of debt and equity finance. *American Economic Review* 101:877–99.
- Dichev, I. D. 1998. Is the Risk of Bankruptcy a Systematic Risk? *Journal of Finance* 53:1131–1147.

- Elkamhi, R., J. Ericsson, and C. A. Parsons. 2012. The cost and timing of financial distress. *Journal of Financial Economics* 105:62–81.
- Friewald, N., C. Wagner, and J. Zechner. 2014. The cross-section of credit risk premia and equity returns. *Journal of Finance* 69:2419–2469.
- Garlappi, L., and H. Yan. 2011. Financial Distress and the Cross Section of Equity Returns. *Journal of Finance* 66:789 – 822.
- George, T. J., and C.-Y. Hwang. 2010. A resolution of the distress risk and leverage puzzles in the cross section of stock returns. *Journal of Financial Economics* 96:56–79.
- Gilchrist, S., and E. Zakrajšek. 2012. Credit spreads and business cycle fluctuations. *American Economic Review* 102:1692–1720.
- Glover, B. 2014. The Expected Cost of Default. *Journal of Financial Economics, forthcoming* .
- Goldstein, R., N. Ju, and H. Leland. 2001. An EBIT-Based Model of Dynamic Capital Structure. *Journal of Business* 74:483–512.
- Gomes, J., U. Jermann, and L. Schmid. 2016. Sticky leverage. *American Economic Review* 106:3800–3828.
- Gomes, J. F. 2001. Financing Investment. *The American Economic Review* 91:1263–1285.
- Gomes, J. F., and L. Schmid. 2010. Levered Returns. *Journal of Finance* 65:467–494.
- Gomes, J. F., and L. Schmid. 2016. Equilibrium asset pricing with leverage and default. *Working Paper* .
- Griffin, J. M., and M. L. Lemmon. 2002. Book-to-Market Equity, Distress Risk, and Stock Returns. *Journal of Finance* 57:2317–2336.
- Hackbarth, D., and T. C. Johnson. 2015. Real Options and Risk Dynamics. *Review of Economic Studies* 82:1449–1482.
- Hackbarth, D., J. Miao, and E. Morellec. 2006. Capital Structure, Credit Risk, and Macroeconomic Conditions. *Journal of Financial Economics* 82:519–550.
- Haddad, V., E. Loualiche, and M. Plosser. 2017. Buyout activity: The impact of aggregate discount rates. *The Journal of Finance* 72:371–414.
- Harrison, J. M. 1985. *Brownian motion and stochastic flow systems*. Wiley New York.
- Jagannathan, R., and Z. Wang. 1996. The Conditional CAPM and the Cross-Section of Expected Returns. *Journal of Finance* 51:3–53.
- Kaplan, S. N., and P. Stromberg. 2009. Leveraged buyouts and private equity. *Journal of Economic Perspectives* 23:121–46.
- Koijen, R. S., H. Lustig, and S. Van Nieuwerburgh. 2010. The cross-section and time-series of stock and bond returns. Tech. rep., National Bureau of Economic Research.
- Kuehn, L.-A., and L. Schmid. 2014. Investment-Based Corporate Bond Pricing. *Journal of Finance* 69:2741–2776.

- Leland, H. E. 1994. Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. *Journal of Finance* 49:1213–1252.
- Lettau, M., and S. Ludvigson. 2001. Consumption, aggregate wealth, and expected stock returns. *the Journal of Finance* 56:815–849.
- Lewellen, J., and S. Nagel. 2006. The conditional CAPM does not explain asset-pricing anomalies. *Journal of Financial Economics* 82:289–314.
- Maksimovic, V., and G. Phillips. 2001. The market for corporate assets: Who engages in mergers and asset sales and are there efficiency gains? *Journal of Finance* 56:2019–2065.
- Merton, R. C. 1974. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance* 29:449–470.
- O’Doherty, M. S. 2012. On the conditional risk and performance of financially distressed stocks. *Management Science* 58:1502–1520.
- Strebulaev, I. A. 2007. Do Tests of Capital Structure Theory Mean What They Say? *Journal of Finance* 62:1747–1787.
- Strebulaev, I. A., and T. M. Whited. 2012. Dynamic models and structural estimation in corporate finance. *Foundations and Trends® in Finance* 6:1–163.
- Titman, S. 1984. The effect of capital structure on a firm’s liquidation decision. *Journal of Financial Economics* 13:137–151.
- Vassalou, M., and Y. Xing. 2004. Default Risk in Equity Returns. *Journal of Finance* 59:831–868.
- Yang, L. 2008. The real determinants of asset sales. *Journal of Finance* 63:2231–2262.
- Zhang, L. 2005. The Value Premium. *Journal of Finance* 60:67–103.

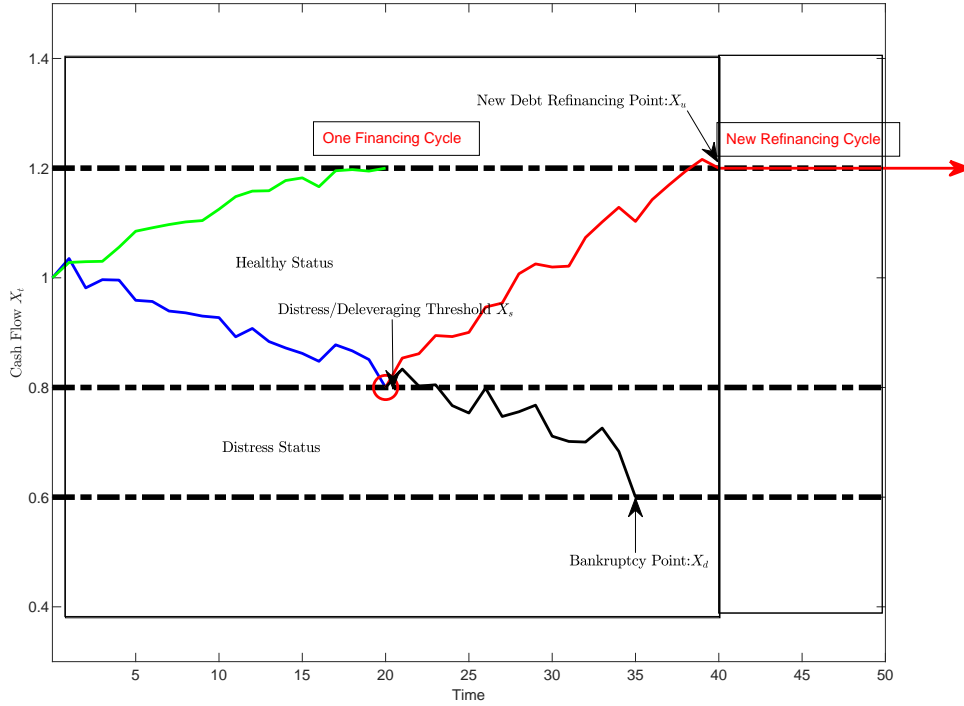


Figure 1. Dynamic Paths

This figure plots four possible paths that a firm could take within one refinancing cycle, which can be repeated infinitely. In observing its dynamic asset value, the firm makes financing and default decisions. Path 1 (green line) shows that, when its cash flows reach an upper threshold X_u , the firm decides to issue more debt to take advantage of tax benefits. In contrast, if the cash flows X_t decline to a low threshold X_s along Path 2 (blue line), the firm becomes distressed. It sells its asset to retire its debt. After the corrective action, this distressed firm might survive and rebound, leading to a subsequent debt restructuring at the *same* upper threshold X_u , as shown in Path 3 (red line). In contrast, the additional distress cost induces a more severe cash flow shortfall through a depressed growth rate (see (8)). If the firm continues to deteriorate, equity holders will no longer be willing to inject more capital, and decide to go bankrupt at X_d , as shown in Path 4 (black line). Bankruptcy leads to immediate liquidation.

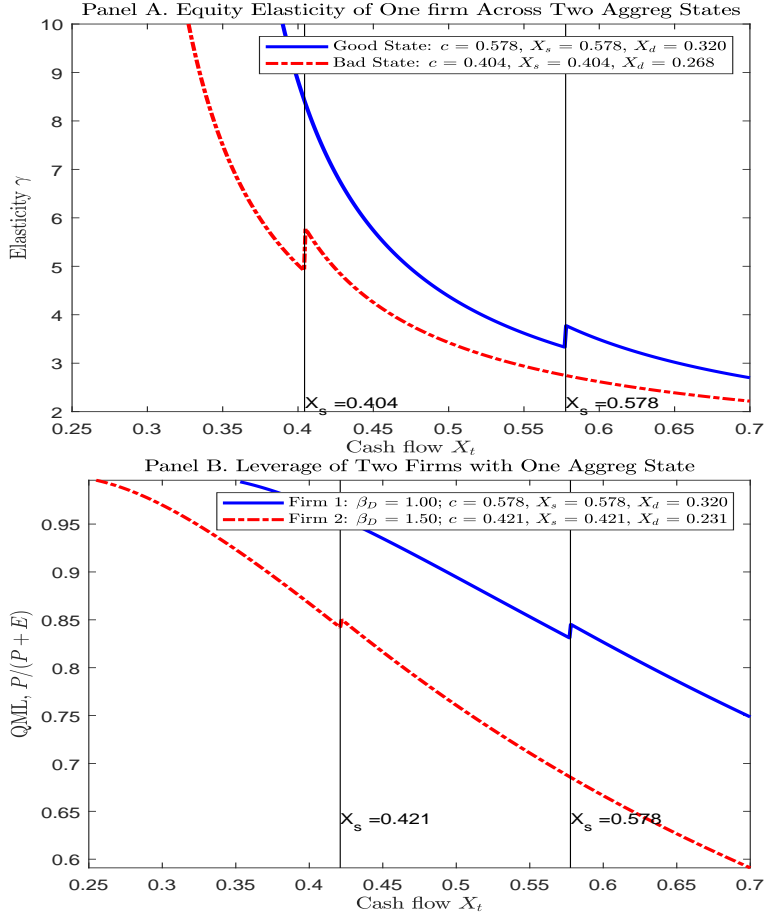


Figure 2. Equity-Cash Flow Sensitivity and Leverages in the Baseline Model

This figure plots the equity-cash flow sensitivity in Panel A and the financial leverage in Panel B against cash flows X_t for two cases, with optimal policies as shown in the legend. We set nominal Interest rate r to 0.04, market price of risk θ to 0.35, systematic volatility σ^m to 0.1, debt issue cost ϕ to 0.01, liquidation cost α to 0.3, asset sales w to 0.1, effective tax rate τ to 0.20, cash flow growth rate $\hat{\mu}_H = \hat{\mu}_D$ to 0.04, cash flow beta $\beta_H = \beta_D$ to 1, and idiosyncratic volatility $\sigma^{i,X}$ to 0.1. For both cases, at time 0, $X_0 = 1$, they choose different level of debt P and coupon c . They sell the asset by a fraction ϖ and retire its debt by a fraction k at the threshold of X_s . Finally, they liquidate their firms at the threshold of X_d if the corrective action does not save their firms. In Panel A, we consider one firm is operating across two aggregate states. We exogenously change the market volatility from 0.1 to 0.12. and the market price of risk from 0.35 to 0.40. The lower (higher) values of market volatility and market price are for the good (bad) state as in the literature. In Panel B, we consider two firms with different cash flow risk exposure to the market risk, i.e., $\beta_D = 1$ and 1.5, respectively, when they are distressed. However, both of them have the same $\beta_H = 1$ when they are healthy. They choose different levels of debt and coupons, because of different risk exposures.

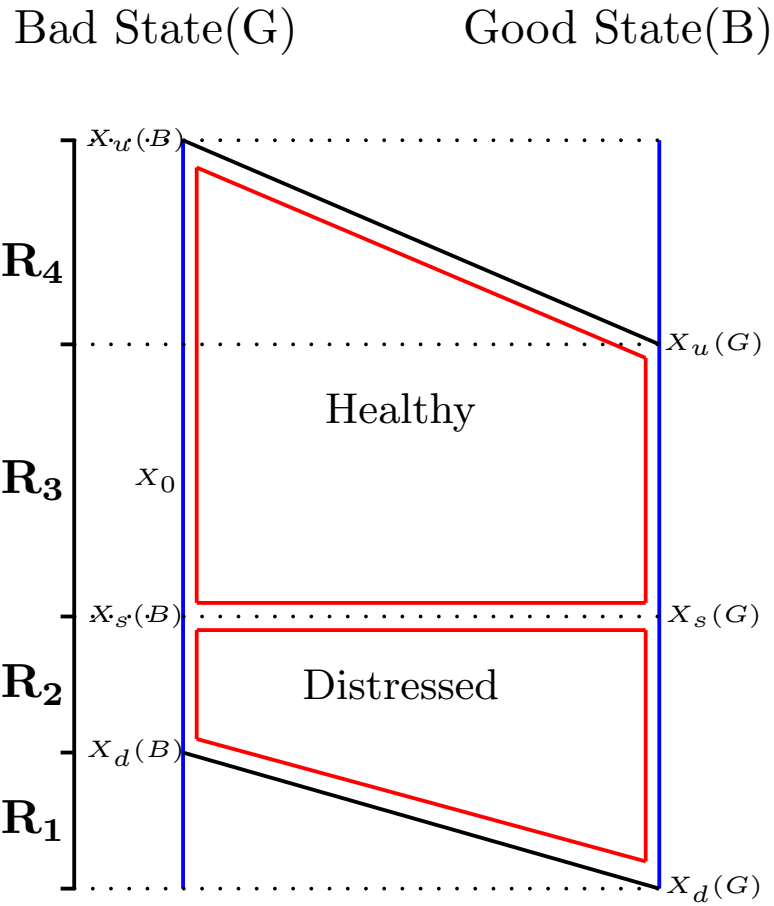


Figure 3. Optimal Thresholds in Two States

This figure plots the optimal default thresholds $X_d(s_t; s_0)$, distress threshold $X_s(s_t; s_0)$ and refinancing threshold $X_u(s_t; s_0)$ across the two states over four regions of cash flow. $X_s(s_t; s_0)$ are the same in both states and equals to the initial coupon $c(s_0)$.

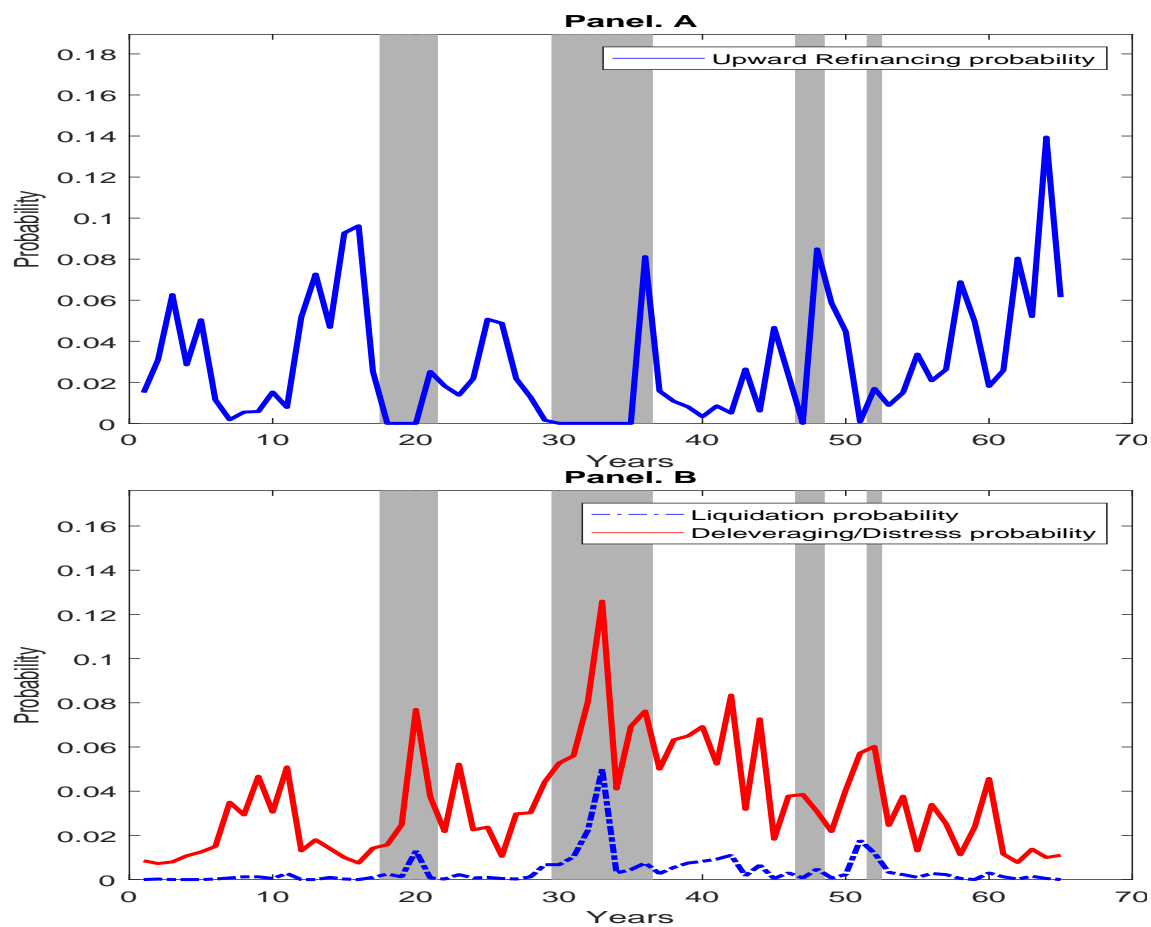


Figure 4. Time Series of Aggregate Probabilities

This figure plots over years for a typical economy the time series of the upward refinancing probability in Panel A and the time series of the distress/deleveraging and liquidation probabilities in Panel B. The economy consists of 4,000 firms. Both probabilities are cumulative over one year. Gray areas show times when the economy is in the bad state, i.e., $s_t = B$.

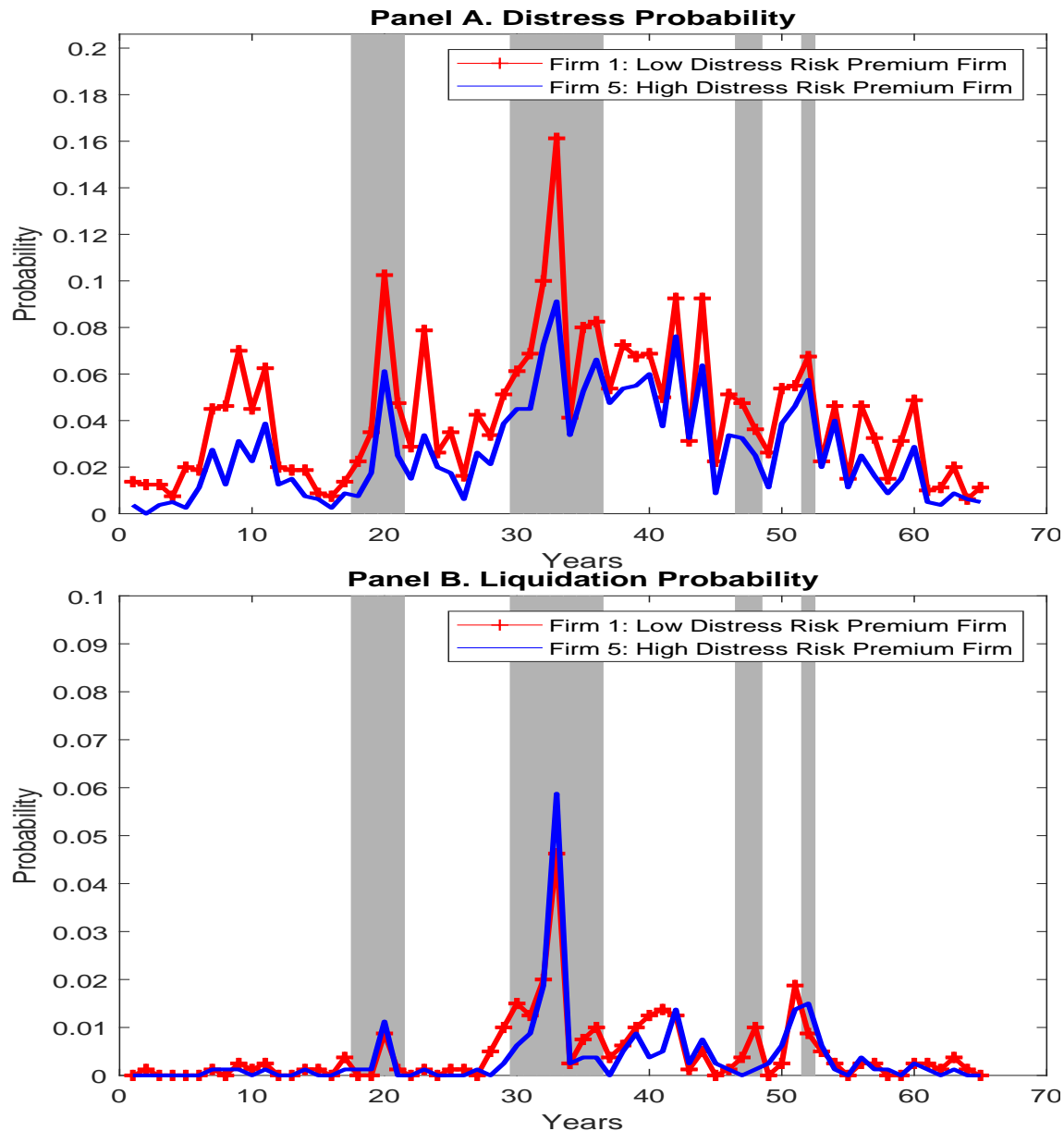


Figure 5. Time Series of Heterogeneous Distress Probabilities

This figure plots over years for a typical economy the time series of distress probability for two types of firms, such as type 1 firms with a low distress risk premium ($\beta_{B,D} = 1.2$) and type 5 firms with a high distress risk premium (i.e., $\beta_{B,D} = 2$). Each type consists of 800 firms. Both probabilities are cumulative over one year. Gray areas show times when the economy is in the bad state, i.e., $s_t = B$.

Table 1. Parameter Values for Calibration

We list the common parameter values in Panel A and heterogeneous loading of the distress risk premium in Panel B. The transition intensities of the Markov chain are chosen to match the average duration of NBER-dated expansions and recessions, i.e., $\hat{\lambda}_G = 0.5$ and $\hat{\lambda}_B = 0.1$, which gives the average durations 10 years for expansions and 2 years for recessions over the business cycle. We set the risk free rate $r_G = r_B = 4\%$ in both aggregate states to abstract away from any term structure effects. The distress cost has a nonsystematic component $\eta_{s_t}^0$ and a systematic component $\eta_{s_t}^s$. For the non-systematic cost of distress, we set $\eta_{s_t}^0$ to 0.015, in both aggregate states. For the systematic cost of distress cost, we assume this systematic cost is proportional to the market risk premium in the bad states. The increase in the cash flow beta $\beta_{B,D}$ suggests an increase in the systematic distress cost η_B^s . That is, $\eta_B = \eta_B^0 + \eta_B^s = 0.015 + 0.380 \times 0.12 (\beta_{B,D} - \beta_{B,H})$.

Panel A. Baseline Values					
	$s_t = B$	$s_t = G$			
Probability of leaving current state s_t , $\hat{\lambda}_{s_t}$	0.500	0.100			
Aggregate state-switching risk premium, κ_{s_t}	1/1.5	1.5			
Nominal interest rate, r_{s_t}	0.040	0.040			
Market price of risk, θ_{s_t}	0.380	0.220			
Systematic volatility, $\sigma_{s_t}^m$	0.120	0.100			
Debt issue cost, ϕ_{s_t}	0.010	0.010			
Asset sale, ϖ_{s_t}	0.038	0.043			
Liquidation cost, α_{s_t}	0.100	0.100			
Effective tax rate, τ_{s_t}	0.150	0.150			
Idiosyncratic volatility, $\sigma_{s_t, w_t}^{i,X}$	0.200	0.150			
Physical growth rate of healthy firms, $\hat{\mu}_{s_t, H}$	-0.010	0.060			
Cash flow Beta of healthy firms, $\beta_{s_t, H}$	1	1			
Cash flow Beta, $\beta_{G, D}$		1			

Panel B. Heterogeneous Distress Costs					
Firm type	1	2	3	4	5
Nonsystematic Distress Cost, $\eta_{s_t}^0$	0.015	0.015	0.015	0.015	0.015
Loadings on Systematic distress risk premium, $\beta_{B, D}$	1.2	1.4	1.6	1.8	2

Table 2. Optimal Policies

Given the parameter values in Table 1, we present the optimal policies for the fully fledged model in Panel A, and quasi-market leverage (QML) at the entry time X_0 in Panel B. The optimal policies in Panel A include coupon $c(s_t)$, the refinance threshold $X_u(s_t)$, and the default threshold $X_d(s_t)$ and the distress threshold $X_s(s_t)$ in the states $s_t = B, G$ when the firm enters at the initial state $s_0 = G$. Panel B show reports the par value of debt $P(s_0; s_0)$, equity $E(s_0; s_0)$ and the quasi-market leverage ratios, i.e., $QML(s_0) = P(s_0; s_0)/(P(s_0; s_0) + E(s_0; s_0))$. The last row of both panels reports the average values.

Panel A. Optimal Policies								
	$c(B)$	$c(G)$	$X_u(B; G)$	$X_u(G; G)$	$X_d(B; G)$	$X_d(G; G)$	$X_s(B; G)$	$X_s(G; G)$
Firm Type 1	0.17	0.42	12.98	1.93	0.17	0.15	0.42	0.42
Type 2	0.17	0.39	7.68	1.93	0.16	0.14	0.39	0.39
Type 3	0.16	0.37	7.72	1.94	0.15	0.14	0.37	0.37
Type 4	0.16	0.36	9.07	1.94	0.14	0.13	0.36	0.36
Firm Type 5	0.16	0.34	6.39	1.94	0.13	0.13	0.34	0.34
Average	0.16	0.38	8.77	1.94	0.15	0.14	0.38	0.38

Panel B. Optimal Leverages at (Re)financing Points						
	$P(B)$	$P(G)$	$E(B)$	$E(G)$	$QML(B)$	$QML(G)$
Firm Type 1	3.89	9.03	23.30	23.57	0.14	0.28
Type 2	3.82	8.52	23.28	23.95	0.14	0.26
Type 3	3.74	8.12	23.29	24.25	0.14	0.25
Type 4	3.68	7.80	23.29	24.50	0.14	0.24
Firm Type 5	3.63	7.53	23.29	24.69	0.13	0.23
Average	3.75	8.20	23.29	24.19	0.14	0.25

Table 3. Moments of Calibrated Model

This table reports the key moments across the 100 simulated data panels. Panel A reports the mean and percentiles of the value-weighted market risk premium (MRP) and Sharpe Ratio (SR), quasi-market leverage (QML), the standard deviation (Std) and interquartile of quarterly QML, and interest coverage (IntCov) across the simulated panels. Panel B reports default probability, distressing probability, rebounding probability and refinancing probability as well as the fraction of distressed firms. We simulate the model and generate 100 artificial panels of data at the quarterly frequency for a period of 150 years. The first 100 years of observations are discarded to reduce the dependence on initial values. In each data panel there are 4000 firms.

A. Aggregate Moments							
	Data	Mean	5th	25th	50th	75th	95th
Market Risk Premium (MRP)	0.06	0.05	0.02	0.04	0.05	0.06	0.08
Sharpe Ratio (SR)	0.329	0.31	0.11	0.22	0.31	0.38	0.47
QML	0.31	0.30	0.25	0.28	0.30	0.33	0.36
Std of QML	0.12	0.11	0.08	0.10	0.11	0.13	0.15
Interquartile of QML	0.15	0.15	0.10	0.12	0.14	0.17	0.20
IntCov	2.63	2.69	2.36	2.54	2.73	2.83	3.02

B. Probabilities							
	Mean	5th	25th	50th	75th	95th	
Default Prob (%)	0.16	0.01	0.06	0.12	0.22	0.45	
Distress Prob (%)	1.94	0.57	1.18	1.76	2.63	3.68	
Rebounding Prob (%)	1.79	0.64	1.15	1.65	2.31	3.16	
Refinancing Prob (%)	4.74	2.81	3.48	4.67	5.66	7.27	
Fraction of Distressed Firms (%)	5.01	1.44	3.15	4.53	6.85	9.56	

Table 4. Predicting Regressions of Distress and Default Events

This table reports the results of probit regressions for predicting the distress and liquidation over different horizons. We simulate 100 artificial panels of data at the quarterly frequency for a period of 150 years for 4000 firms and discard the first 100 year observations. We use records on annual distress and default events for five types of firms, with each type having 500 firms. We discard the firms with the median risk exposure (i.e., $\beta_{B,D} = 1.6$) and keep the top two and bottom two types of firms. The dependent variable equals 1 if the firm becomes distressed or defaults within the specified horizon. Independent variables are quasi-market leverage (QML) and the indicator variable $\mathbf{1}_{\beta_{B,D}=L} = 1$ if $\beta_{B,D}=1.2, 1.4$. For each data set, a pooled probit regression is run for each of the following three horizons: one, two, and five years. The absolute value of t-statistics is given in parentheses. Log-likelihood is the log-likelihood ratio, and R^2 is the McFadden pseudo- R^2 . The coefficients and t-statistics are averaged across all the panels.

	Panel A. Predictive Regression of Distress								
	Panel A. Horizon = 1 year			Panel B. Horizon = 2 years			Panel C. Horizon = 5 years		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	-2.184	-3.914	-3.931	-2.184	-3.675	-3.696	-2.180	-3.241	-3.270
(t)	(-212.399)	(-153.188)	(-145.148)	(-150.015)	(-112.455)	(-106.092)	(-91.901)	(-72.410)	(-67.800)
$\mathbf{1}_{\beta_{B,D}=L}$	0.143		0.033	0.144		0.042	0.142		0.058
(t)	(10.208)		(1.848)	(7.274)		(1.779)	(4.381)		(1.605)
QML_t		4.350	4.345		3.908	3.902		3.019	3.008
(t)		(89.730)	(89.453)		(60.394)	(60.136)		(31.388)	(31.153)
Log-like	112.406	10060.886	10067.265	57.748	4198.320	4203.722	21.790	1017.227	1021.510
pseudo- R^2	0.003	0.290	0.290	0.004	0.243	0.244	0.004	0.157	0.158

	Panel B. Predictive Regression of Default								
	Panel B. Horizon = 1 year			Panel B. Horizon = 2 years			Panel B. Horizon = 5 years		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	-3.163	-7.616	-7.616	-3.042	-6.066	-6.072	-3.042	-4.864	-4.883
(t)	(-100.089)	(-27.135)	(-26.762)	(-78.090)	(-26.615)	(-26.083)	(-47.737)	(-23.948)	(-22.877)
$\mathbf{1}_{\beta_{B,D}=L}$	0.106		-0.013	0.107		-0.009	0.108		0.009
(t)	(2.410)		(-0.333)	(1.709)		(-0.367)	(1.064)		(-0.022)
QML_t		7.202	7.209		5.364	5.374		3.703	3.714
(t)		(20.978)	(20.975)		(18.068)	(18.063)		(11.981)	(11.954)
Log-like	7.668	2268.215	2270.124	-840.888	1064.302	1065.692	-326.311	248.696	249.863
pseudo- R^2	0.003	0.514	0.515	-1.613	0.432	0.433	-4.074	0.294	0.297

Table 5. Key Characteristics of Distressed Firms

We report the cross-sectional key moments for decile portfolios sorted on the probability of failure (Campbell et al., 2008) in Panel A and on the proxy of distress risk premium in Panel B, using the simulated data panels. The key moments include financial leverage, default probability of Merton (1974) and the fraction of distressed firms within each decile of firms. Following Andrade and Kaplan (1998), a firm is classified as “distressed” if its interest coverage is less than one. We simulate the model and generate 100 artificial panels of data at the quarterly frequency for a period of 150 years. The first 100 years of observations are discarded to reduce the dependence on initial values. In each data panel there are 4000 firms. Then, we sort the simulated firms at the end of each year, based on the failure probability and on the implied default risk premium, and rebalance the portfolios each year. The failure probability is calculated based on equation (29), which is largely based on Table 3 of Campbell et al. (2008), and the proxy of implied risk premium is given by $\log(\pi_t) - \log(\hat{\pi}_t)$, where π_t is the risk-neutral and $\hat{\pi}_t$ the objective default probability of Merton (1974), respectively.

Panel A. Sorted on Failure Probability						
	L(ow)	2	3	4	H(igh)	H-L
QML	0.19	0.23	0.27	0.33	0.49	0.30
STD(QML)	0.02	0.03	0.05	0.07	0.10	0.08
Merton Default Prob (%)	2.26	0.41	0.18	1.02	18.27	16.01
Fraction of Distressed Firms (%)	0.00	0.00	0.06	1.23	24.09	24.09
Fraction of Distressed Firms in bad times	0.00	0.00	0.11	1.44	25.40	25.40
Fraction of Distressed Firms in good times	0.00	0.00	0.05	1.17	23.91	23.91
Deleveraging Frequence(%)	0.00	0.00	0.03	0.31	2.77	2.77
Deleveraging Frequence in bad times	0.00	0.00	0.05	0.57	5.51	5.51
Deleveraging Frequence in good times	0.00	0.00	0.02	0.24	2.30	2.30

Panel B. Sorted on Implied Default Risk Premium						
	L(ow)	2	3	4	H(igh)	H-L
QML	0.37	0.32	0.29	0.27	0.26	-0.11
STD(QML)	0.07	0.06	0.06	0.05	0.05	0.05
Merton Default Prob (%)	7.05	3.55	3.42	3.48	4.65	-2.40
Fraction of Distressed Firms (%)	7.82	3.84	3.89	3.90	5.94	-1.87
Fraction of Distressed Firms in bad times	12.19	6.82	3.91	1.84	2.23	-9.95
Fraction of Distressed Firms in good times	7.07	3.22	3.79	4.32	6.73	-0.34
Deleveraging Frequence(%)	1.58	0.48	0.32	0.25	0.48	-1.10
Deleveraging Frequence in bad times	2.61	1.05	0.79	0.68	1.00	-1.61
Deleveraging Frequence in good times	1.41	0.38	0.22	0.17	0.38	-1.03

Table 6. Equity Returns Sorted on the Failure Probability and Implied Distress Risk Premium

We report the average returns of value-weighted portfolios sorted on the probability of failure (Campbell et al., 2008) in Panel A and on the proxy of distress risk premium in Panel B, using the simulated data panels. We report the value-weighted averages of annualized excess returns in percent $r_{i,t}^{ex}$ and unconditional CAPM alpha α^u for stock portfolios from the 100 simulated data panels. We simulate the model and generate 100 artificial panels of data at the quarterly frequency for a period of 150 years. The first 100 years of observations are discarded to reduce the dependence on initial values. We sort firms at the end of each year, based on the failure probability, and then rebalance the portfolios each year. We calculate the failure probability of the simulated firms based on equation (29), which is largely based on Table 3 of (Campbell et al., 2008).

Panel A. Sorted on Failure Probability						
	L(ow)	2	3	4	H(igh)	H-L
r^{ex} (%)	10.38	4.26	0.42	-3.26	-7.27	-17.66
(t)	(5.71)	(2.08)	(0.21)	(-1.33)	(-2.56)	(-15.62)
α (%)	5.97	-0.73	-4.91	-8.99	-13.97	-19.94
(t)	(34.33)	(-4.70)	(-22.62)	(-29.25)	(-27.62)	(-32.01)
β	0.89	1.01	1.08	1.16	1.37	0.48
(t)	(112.63)	(207.34)	(129.53)	(79.06)	(39.14)	(10.80)
$Adj.R^2$	0.98	1.00	0.99	0.97	0.94	0.54

Panel B. Sorted on Implied Risk Premium						
	L(ow)	2	3	4	H(igh)	H-L
r^{ex} (%)	-4.59	-0.13	3.47	7.08	11.97	16.57
(t)	(-2.14)	(-0.04)	(1.65)	(3.41)	(6.14)	(27.94)
α (%)	-9.61	-5.29	-1.65	2.08	7.32	16.93
(t)	(-24.41)	(-23.62)	(-9.19)	(12.69)	(30.99)	(29.68)
β	1.02	1.04	1.04	1.01	0.94	-0.07
(t)	(59.27)	(103.34)	(141.61)	(157.61)	(84.23)	(-2.80)
$Adj.R^2$	0.95	0.98	0.99	0.99	0.97	0.05

Table 7. Procyclical Leverage vs. Expected Distress Risk Premium in the Data

This table reports the results of panel regression of quarterly financial leverage on HP filtered GDP (Panel A), expected market risk premium (Panel B), expected distressed risk premium (Panel C), and alternative distressed risk premium (Panel D) at the firm-level as follows:

$$\frac{\Delta D_{i,t}}{A_{i,t-1}} = \alpha_{0,i} + \sum_{j=1}^J I_{i,t}(j) \{ \alpha_{j,t} + \alpha_{j,Y} Y_t + \alpha_{j,CF} \left(\frac{CF_{i,t-1}}{A_{i,t-2}} - \frac{CF_{j,t-1}}{A_{j,t-2}} \right) + \alpha_{j,Q} (Q_{i,t-1} - \overline{Q_{j,t-1}}) + \alpha_{j,S} (\overline{Size_{i,t-1}} - \overline{Size_{j,t-1}}) \} + u_{i,t},$$

where $\Delta D_{i,t}$ is calculated as the change in the long term debt (compustat item DLTTQ) scaled by total assets (compustat item ATQ) of last quarter. We follow Covas and Den Haan (2011) and classify firms into seven categories based on the failure probability at the end of last quarter. The seven groups include [0, 25], [25, 50], [50, 75], [75, 90], [90, 95], [95, 99], [99, 100]. The quarterly long term market risk premium are obtained from Haddad et al. (2017). To decompose the expected market risk premium $E_t(R_{M,t+1}^e)$, we regress it on the *HY* Spread, which is a composite of the Merrill Lynch high-yield bond indices less the yield on the three-month T-bill (Axelson et al., 2013). We use the fitted value to proxy for the aggregate distress risk premium (DRP) and the residuals for the non-distress risk premium (NDRP). Alternatively, we regress $E_t(R_{M,t+1}^e)$ on the *HY* Spread, *EBITDA* Spread (Kaplan and Stromberg, 2009) and *GZ* Spread (Gilchrist and Zakrajšek, 2012) to obtain the distress risk premium and non-distress risk premium components.

Panel A. Debt Financing versus GDP							
	j = [0,25]	j = [25,50]	j = [50,75]	j = [75,90]	j = [90,95]	j = [95,99]	j = [99,100]
$\mathbf{I}(j)GDP_t$	0.02	0.07	0.10	0.08	0.05	0.08	0.13
(t)	(1.53)	(5.39)	(7.54)	(5.12)	(1.79)	(2.47)	(2.17)

Panel B. Debt Financing v.s. Aggregate Discount Rate							
	j = [0,25]	j = [25,50]	j = [50,75]	j = [75,90]	j = [90,95]	j = [95,99]	j = [99,100]
$\mathbf{I}(j)MRP_t$	-0.04	-0.05	-0.05	-0.07	-0.14	-0.14	-0.15
(t)	(-5.26)	(-6.93)	(-6.18)	(-7.97)	(-11.15)	(-10.05)	(-6.12)

Panel C. Debt Financing v.s. Expected Distress Risk Premium							
	j = [0,25]	j = [25,50]	j = [50,75]	j = [75,90]	j = [90,95]	j = [95,99]	j = [99,100]
$\mathbf{I}(j)DRP_t$	-0.09	-0.10	-0.09	-0.10	-0.19	-0.21	-0.24
(t)	(-10.49)	(-12.53)	(-10.78)	(-10.56)	(-12.39)	(-12.32)	(-7.61)
$\mathbf{I}(j)NDRP_t$	0.01	0.01	-0.00	-0.01	-0.02	-0.00	0.03
(t)	(2.58)	(2.65)	(-0.14)	(-1.30)	(-1.70)	(-0.09)	(1.49)

Panel D. Debt Financing v.s. Alternative Expected Distress Risk Premium							
	j = [0,25]	j = [25,50]	j = [50,75]	j = [75,90]	j = [90,95]	j = [95,99]	j = [99,100]
$\mathbf{I}(j)DRP_t$	-0.04	-0.05	-0.05	-0.07	-0.14	-0.14	-0.16
(t)	(-5.40)	(-7.08)	(-6.15)	(-7.98)	(-11.14)	(-10.09)	(-6.26)
$\mathbf{I}(j)NDRP_t$	0.00	0.00	-0.01	-0.01	-0.01	-0.01	0.03
(t)	(0.92)	(0.97)	(-2.11)	(-1.86)	(-1.31)	(-0.63)	(1.24)

Table 8. Distress Risk Premium of the Failure Probability in the Data

This table reports the results from unconditional CAPM regressions in Panel A and annualized excess stock returns $r_{i,t}^{ex}$ and unconditional alphas α^u implied by the conditional CAPM in Panel B, for portfolios sorted on the failure probability. In Panel A, we report annualized excess stock returns $r_{i,t}^{ex}$ and unconditional alphas α^u and β^u from the unconditional time series regressions for each portfolio. We calculate the probability of failure according to the third column of Table 4 of Campbell et al. (2008): $F - Prob = -9.164 - 20.264NIMTAAVG + 1.416TLMTA - 7.129EXRETAVG + 1.411SIGMA - 0.045RSIZE + 0.075MB - 0.058PRICE - 2.132CASHMTA$

where $NIMTAAVG$ is the moving average of the net income, $EXRETAVG$ is the moving average of the relative excess returns; $TLMTA$ is the ratio of total liabilities divided by the sum of market equity and total liabilities; $SIGMA$ is the volatility of daily stock returns over the past three months; $RSIZE$ is the relative size measured as the log ratio of the firm's market equity to that of the total market; $CASHMTA$ is the ratio of cash and short-term investments divided by the sum of market equity and total liabilities; MB is the market-to-book equity; $PRICE$ is the log price per share. In Panel B, we report the value-weighted average of the time-varying CAPM beta $\beta_{i,t}^E$ from the data in the first row. They are obtained from the regression of daily returns on the daily excess market returns, month by month, and are adjusted by the procedure of Lewellen and Nagel (2006). Then, we report the two main components, $\mathbf{E}[\beta_{i,t}^E]\mathbf{E}[r_t^m]$ and $\mathbf{cov}(\beta_{i,t}^E, r_t^m)$ in the next two rows. Finally, we follow Lewellen and Nagel (2006) and calculate the unconditional expected excess return as $r_{i,t}^{ex} = \mathbf{E}[\beta_{i,t}^E]\mathbf{E}[r_t^m] + \mathbf{cov}(\beta_{i,t}^E, r_t^m)$ and the unconditional CAPM alpha $\alpha^u \approx \mathbf{cov}(\beta_{i,t}^E, r_t^m) - \frac{\mathbf{E}[r_t^m]}{(\mathbf{E}[\sigma_t^m])^2 \mathbf{cov}(\beta_t^E, (\sigma_t^m)^2)}$.

Panel A. Excess Return and Unconditional CAPM Results from the Data											
	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H-L
$r_{i,t}^{ex}$	6.14	6.00	6.70	5.04	6.45	5.71	2.96	1.05	0.20	-5.61	-11.75
(t)	(2.04)	(2.19)	(2.43)	(1.62)	(1.93)	(1.47)	(0.67)	(0.22)	(0.04)	(-0.84)	(-2.14)
α^u	1.39	1.23	1.41	-0.52	0.25	-0.65	-4.17	-6.53	-7.91	-14.29	-15.68
(t)	(0.96)	(1.25)	(1.44)	(-0.53)	(0.22)	(-0.44)	(-2.12)	(-2.76)	(-2.77)	(-3.53)	(-3.41)
β^u	0.92	0.92	1.02	1.07	1.19	1.23	1.37	1.46	1.56	1.67	0.76
(t)	(22.25)	(34.44)	(40.94)	(60.55)	(42.94)	(29.52)	(23.00)	(19.07)	(20.77)	(14.15)	(5.20)
$Adj.R^2$	0.98	1.00	1.03	1.08	1.08	1.10	1.09	1.16	1.12	1.11	0.12

Panel B. Excess Return and Unconditional CAPM α^u Implied from the Conditional CAPM											
	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H-L
$\beta_{i,t}^E$	0.97	0.98	1.01	1.10	1.13	1.14	1.17	1.25	1.30	1.32	0.34
$\mathbf{E}[\beta_{i,t}^E]\mathbf{E}[r_t^m]$	5.10	5.17	5.32	5.63	5.59	5.70	5.66	6.01	5.82	5.75	0.64
$\mathbf{cov}(\beta_{i,t}^E, r_t^m)$	0.27	0.52	-0.26	-0.37	-0.09	-1.70	-1.94	-3.87	-4.66	-5.33	-5.61
$r_{i,t}^{ex}$	5.37	5.69	5.06	5.26	5.50	4.00	3.72	2.14	1.16	0.41	-4.96
α^u	0.79	0.96	-0.02	-0.39	-0.32	-2.16	-2.85	-4.96	-6.01	-7.06	-7.84

Online Appendixes of A Unified Model of Distress Risk Puzzles

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- Appendix A: Baseline Model
- Appendix B: Full Model
- Appendix C: Theoretical Justification for the Proxy of Distress Risk Premium

Appendix A

A Baseline Model

For the baseline model, we list boundary conditions, present the value functions for equity and debt, and then finally the closed form solution of the expected stock return.

A.1 Boundary Conditions

Equity Boundary Conditions

The boundary conditions for equity are as follows:

$$\lim_{X_t \uparrow X_d} E(X_t, D) = 0; \quad (\text{A1})$$

$$\lim_{X_t \uparrow X_u} E(X_t, H) = \lim_{X_t \downarrow X_u} \frac{X_t}{X_0} (E(X_0, H) + P(1 - \phi)) - P; \quad (\text{A2})$$

$$\lim_{X_t \uparrow X_s} E(X_t, H) = \lim_{X_t \downarrow X_s} E(X_t, D); \quad (\text{A3})$$

$$\lim_{X_t \uparrow X_s} E(X_t, H)' = \lim_{X_t \downarrow X_s} E(X_t, D)'. \quad (\text{A4})$$

Equation (A1) states that equity holders receive nothing at bankruptcy, if increasing idiosyncratic volatility to the high level, i.e., $w_t = H$, does not save the firm. Equation (A2) states that the equity value $E(X_u, H)$ increases by a scaling factor $\frac{X_u}{X_0}$ at the refinancing threshold X_u , after retiring the existing debt-in-place at par P and issuing more debt $\frac{X_u}{X_0}P(1 - \phi)$. Equations (A3) and (A4) are the value-matching condition and smoothing pasting conditions for the distress threshold X_s .

Debt Boundary Conditions

The boundary conditions for debt are as follows:

$$\lim_{X_t \uparrow X_d} D(X_t, D) = \lim_{X_t \downarrow X_d} (1 - \alpha)A(X_t, D)(1 - \tau); \quad (\text{A5})$$

$$\lim_{X_t \uparrow X_u} D(X_t, H) = P; \quad (\text{A6})$$

$$\lim_{X_t \uparrow X_s} D(X_t, H) = \lim_{X_t \downarrow X_s} D(X_t, D); \quad (\text{A7})$$

$$\lim_{X_t \uparrow X_s} D(X_t, H)' = \lim_{X_t \downarrow X_s} D(X_t, D)'. \quad (\text{A8})$$

Equation (A5) shows that debt holders take over the assets and receive the residual value of assets $V(X_d, D)(1 - \tau)$ after the liquidation cost α . Equation (A6) states that debt holders receive the par value of debt P when equity holders retire the existing debt if they never increase risk before. Equations (A3) and (A4) are the value-matching condition and smoothing pasting conditions for the distress threshold X_s .

A.2 Asset Valuations

Under the risk-neutral measure, the Bellman equation describes the valuation of any claim $J(X_t, w_t)$ on operating cash flows X_t in state, s , as follows:

$$J(X_t, w_t) = CF_t dt + e^{-rdt} \mathbb{E}^Q(J(X_t + dX_t, w_t)), \quad (\text{A9})$$

where CF_t denotes the cash flows accruing to claim holders. Standard dynamic programming suggests that $J(X_t, w_t) \equiv J_{t,w_t}$ must satisfy the ordinary differential equation

$$\mu_{w_t} X J'_{t,w_t} + \frac{\sigma_{w_t}^2}{2} X^2 J''_{t,w_t} - r J_{t,w_t} + CF_t = 0, \quad (\text{A10})$$

where $J(X_t, w_t)$, J'_{t,w_t} and J''_{t,w_t} denote the first and second-order derivatives of J_{t,w_t} with respect to X_t , respectively.

The firm is operating under two financial status of w_t . That is, $w_t = H$ for $X_t \geq X_s$ and $w_t = D$ for $X_t < X_s$. Under the risk-neutral measure, the value of assets-in-place, A_{t,w_t} , is

$$A_{t,w_t} \equiv A(X_t, w_t) = \begin{cases} \frac{X_t}{r - \mu_{w_t}}, & w_t = H; \\ \frac{X_t(1 - \varpi)}{r - \mu_{w_t}}, & w_t = D. \end{cases} \quad (\text{A11})$$

When the firm is healthy, the dividend d_t accruing to equity holders is $(X_t - c)(1 - \tau)$. The dividend becomes $(X_t(1 - \varpi) - c(1 - k))(1 - \tau)$ when the firm is distressed. Hence, the value function of equity is

$$E(X_t, w_t) = \begin{cases} (1 - \tau) \left(\frac{X_t}{r - \mu_{w_t}} - \frac{c}{r} \right) + e_{w_t,1} X_t^{\omega_{w_t,1}} + e_{w_t,2} X_t^{\omega_{w_t,2}}, & w_t = H; \\ (1 - \tau) \left(\frac{X_t(1 - \varpi)}{r - \mu_{w_t}} - \frac{c(1 - k)}{r} \right) + e_{w_t,1} X_t^{\omega_{w_t,1}} + e_{w_t,2} X_t^{\omega_{w_t,2}}, & w_t = D. \end{cases} \quad (\text{A12})$$

where $\omega_{w_t,1} < 0$ and $\omega_{w_t,2} > 1$ are the two roots of the characteristic equation

$$\frac{1}{2} \sigma_{w_t}^2 \omega_{w_t} (\omega_{w_t} - 1) + \mu_{w_t} \omega_{w_t} - r = 0. \quad (\text{A13})$$

The two roots are:

$$\omega_{w_t,1} = \frac{1}{2} - \frac{\mu_{w_t}}{\sigma_{w_t}^2} - \sqrt{\left(\frac{\mu_{w_t}}{\sigma_{w_t}^2} - \frac{1}{2} \right)^2 + 2 \frac{r}{\sigma_{w_t}^2}} < 0, \quad (\text{A14})$$

and

$$\omega_{w_t,2} = \frac{1}{2} - \frac{\mu_{w_t}}{\sigma_{w_t}^2} + \sqrt{\left(\frac{\mu_{w_t}}{\sigma_{w_t}^2} - \frac{1}{2} \right)^2 + 2 \frac{r}{\sigma_{w_t}^2}} > 1. \quad (\text{A15})$$

To solve for $e_{L,1}$, $e_{L,2}$, $e_{H,1}$ and $e_{H,2}$, we use the boundary conditions from equation (A1) to (A4).

The cash flow accruing to debt holders is the coupon c before the debt reduction and $c(1 - k)$

after the debt reduction, respectively. Hence, the value function of debt is

$$D(X_t, w_t) = \begin{cases} \frac{c}{r} + d_{w_t,1} X_t^{\omega_{w_t,1}} + d_{w_t,2} X_t^{\omega_{w_t,2}}, & w_t = H; \\ \frac{c(1-k)}{r} + d_{w_t,1} X_t^{\omega_{w_t,1}} + d_{w_t,2} X_t^{\omega_{w_t,2}}, & w_t = D. \end{cases} \quad (\text{A16})$$

To solve for four coefficients, $d_{L,1}$, $d_{L,2}$, $d_{H,1}$ and $d_{H,2}$, we use the boundary conditions in equations (A5) to (A8).

A.3 Equity Returns

In this section, we provide the proofs for equity returns in proposition 1. We obtain Proposition 1 for the simplified baseline model where we assume the firm does not have the option to refinance its debt upward after it is out of distress.

To prove the Proposition 1, we start with the general formula for the equity return and then use the boundary conditions. Ito's lemma implies that the equity value $E(X_t, w_t) \equiv E_{t,w_t}$ satisfies

$$\frac{dE_{t,w_t}}{E_{t,w_t}} = \frac{1}{E_{t,w_t}} \left(\frac{\partial E_{t,w_t}}{\partial t} + \hat{\mu}_{w_t} X_t \frac{\partial E_{t,w_t}}{\partial X_t} + \frac{\sigma_{w_t}^2}{2} X_t^2 \frac{\partial^2 E_{t,w_t}}{\partial X_t^2} \right) dt + (\sigma^m d\hat{W}_t^m + \sigma_{w_t}^{i,X} d\hat{W}_t^i) \frac{X_t}{E_{t,w_t}} \frac{\partial E_{t,w_t}}{\partial X_t}. \quad (\text{A17})$$

The standard asset pricing argument gives

$$\mathbb{E} \left[\frac{dE_{t,w_t} + d_t dt}{E_{t,w_t}} \right] - r dt = -\mathbb{E} \left(\frac{dE_{t,w_t}}{E_{t,w_t}}, \frac{dm_t}{m_t} \right) = \frac{X_t}{E_{t,w_t}} \frac{\partial E_{t,w_t}}{\partial X_t} \beta_{w_t} (\sigma^m \theta) dt. \quad (\text{A18})$$

Denoting $(dE_{t,w_t} + d_t dt)/E_{t,w_t}$ by r_{t,w_t}^E and $(X_t \partial E_{t,w_t})/(E_{t,w_t} \partial X_t)$ by γ_{t,w_t} , we have the excess equity return

$$r_{t,w_t}^e(X_t) = \mathbb{E}_t[r_{t,w_t}^E] - r dt = (\gamma_{t,w_t} \beta_{w_t}) \sigma^m \theta dt = \beta_{w_t}^E \zeta^m dt. \quad (\text{A19})$$

The sensitivity of the equity to the underlying cash flows $\gamma_{t,w_t} = \frac{X_t \partial E_{t,w_t}}{E_{t,w_t} \partial X_t}$ can be obtained by differentiating (A12), and is as follows:

$$\gamma_{t,w_t} = \begin{cases} 1 + \frac{c(1-\tau)}{r E_{t,w_t}} + \frac{(\omega_{w_t,1}-1)}{E_{t,w_t}} e_{w_t,1} X_t^{\omega_{w_t,1}} + \frac{(\omega_{w_t,2}-1)}{E_{t,w_t}} e_{w_t,2} X_t^{\omega_{w_t,2}}, & w_t = H \\ 1 + \frac{c(1-k)(1-\tau)}{r E_{t,w_t}} + \frac{(\omega_{w_t,1}-1)}{E_{t,w_t}} e_{w_t,1} X_t^{\omega_{w_t,1}} + \frac{(\omega_{w_t,2}-1)}{E_{t,w_t}} e_{w_t,2} X_t^{\omega_{w_t,2}}, & w_t = D \end{cases} \quad (\text{A20})$$

Because we assume the firm does not have the refinancing option in the further simplified model, the no-bubble condition implies $e_{H,2} = 0$. The condition in equation (A3) implies:

$$e_{H,1} = \left(\frac{E(X_s, D)}{(1-\tau)} - \left(A(X_s, H) - \frac{c}{r} \right) \right) (1-\tau) \left(\frac{1}{X_s} \right)^{\omega_{H,1}}. \quad (\text{A21})$$

When the firm is distressed, $w_t = D$, the value-matching conditions in equations (A1) and (A3)

jointly determine²⁰

$$e_{D,1} = \left(\frac{c(1-k)}{r} - A(X_d, D) \right) (1-\tau) \mathcal{L}'_t, \quad (\text{A22})$$

and

$$e_{D,2} = \left(\frac{E(X_s, H)}{1-\tau} - \left(A(X_s, D) - \frac{c(1-k)}{r} \right) \right) (1-\tau) \mathcal{H}'_t, \quad (\text{A23})$$

where

$$\mathcal{L}'_t = \frac{X_t^{\omega_{D,1}} (X_s)^{\omega_{D,2}} - X_t^{\omega_{D,2}} (X_s)^{\omega_{D,1}}}{(X_s)^{\omega_{D,2}} (X_d)^{\omega_{D,1}} - (X_s)^{\omega_{D,1}} (X_d)^{\omega_{D,2}}} \quad (\text{A24})$$

and

$$\mathcal{H}'_t = \frac{X_t^{\omega_{D,2}} (X_d)^{\omega_{D,1}} - X_t^{\omega_{D,1}} (X_d)^{\omega_{D,2}}}{(X_s)^{\omega_{D,2}} (X_d)^{\omega_{D,1}} - (X_s)^{\omega_{D,1}} (X_d)^{\omega_{D,2}}}. \quad (\text{A25})$$

Substituting $e_{H,1}$, $e_{D,1}$ and $e_{D,2}$ into equations (A12), we obtain the equity value of a healthy firm

$$E_{t,H} = \left[\underbrace{\left(A(t, H) - \frac{c}{r} \right)}_{\text{Equity-in-Place}} + \underbrace{\left(\frac{E(X_s, D)}{(1-\tau)} - \left(A(X_s, H) - \frac{c}{r} \right) \right) \left(\frac{1}{X_s} \right)_{H,1}^{\omega}}_{\text{Put Option of deleveraging}} \right] (1-\tau), \quad (\text{A26})$$

and the equity value of a distressed firm

$$E_{t,D} = \left[\underbrace{\left(A(t, D) - \frac{c(1-k)}{r} \right)}_{\text{Equity-in-Place}} + \underbrace{\left(\frac{c(1-k)}{r} - A(X_d, D) \right) \mathcal{L}'_t}_{\text{Put Option of Going Bankruptcy}} + \underbrace{\left(\frac{E(X_s, H)}{1-\tau} - \left(A(X_s, D) - \frac{c(1-k)}{r} \right) \right) \mathcal{H}'_t}_{\text{Call Option of Rebounding}} \right] (1-\tau) \quad (\text{A27})$$

Finally, we insert $e_{H,1}$, $e_{D,1}$ and $e_{D,2}$ into (A20) and obtain the equity-cash flow elasticity as follows:

$$\gamma_{t,H} = \frac{\partial E_{t,H} / E_{t,H}}{\partial X_t / X_t}, \quad (\text{A28})$$

$$= 1 + \underbrace{\frac{\frac{c}{r}(1-\tau)}{E_{t,H}}}_{\text{Leverage}} + \underbrace{(1 - \omega_{H,1}) \frac{\left(A(X_s, H) - \frac{c}{r} - \frac{E(X_s, D)}{1-\tau} \right) \left(\frac{X_t}{X_s} \right)^{\omega_{H,1}} (1-\tau)}{E_{t,H}}}_{\text{Put Option of Deleveraging (-)}}, \quad (\text{A29})$$

²⁰ Although we have not used equation (A4) to express the value function of equity, we need this condition it when we solve the model numerically because we have three unknowns and need three boundary conditions.

and

$$\begin{aligned}
\gamma_{t,D} &= \frac{\partial E_{t,D}/E_{t,D}}{\partial X_t/X_t}, \tag{A30} \\
&= 1 + \underbrace{\frac{\frac{c(1-k)}{r}(1-\tau)}{E_{t,D}}}_{\text{Leverage}} + \underbrace{\frac{(\frac{c(1-k)}{r} - A(X_d, D))\mathcal{L}_t}{E_{t,D}}(1-\tau)}_{\text{Put Option of Going Bankruptcy (-)}} + \underbrace{\frac{\left(\frac{E(X_s, H)}{1-\tau} - \left(A(X_s, D) - \frac{c(1-k)}{r}\right)\right)\mathcal{H}_t}{E_{t,D}}(1-\tau)}_{\text{Call Option of Rebounding (+)}}. \tag{A31}
\end{aligned}$$

B Fully fledged Model

We list the boundary conditions, present the value functions of equity and debt for the firm over the healthy and distress status over the business cycle.

B.1 Boundary conditions

Given the setup of the model, we list the boundary conditions to solve for the model. The closed-form solutions are presented in the appendix.

B.1.1 Boundary Conditions of Equity Value Functions

When the firm is distressed, $w_t = D$, we have the following conditions:

$$\lim_{X_t \downarrow X_d(B; s_0)} E(X_t, B, D; s_0) = 0, \tag{B1}$$

$$\lim_{X_t \downarrow X_d(G; s_0)} E(X_t, G, D; s_0) = 0, \tag{B2}$$

$$\lim_{X_t \uparrow X_d(B; s_0)} E(X_t, G, D; s_0) = \lim_{X_t \downarrow X_d(B; s_0)} E(X_t, G, D; s_0), \tag{B3}$$

$$\lim_{X_t \uparrow X_d(B; s_0)} E'(X_t, G, D; s_0) = \lim_{X_t \downarrow X_d(B; s_0)} E'(X_t, G, D; s_0). \tag{B4}$$

Equations (B1) and (B2) state equity holders receive nothing at bankruptcy at both aggregate state, $s_t = B, G$. Equations (B3) and (B4) are to ensure that the equity value function $E(X_t, G, D; s_0)$ be continuous and smooth at $X_d(B; s_0)$.

Before the firm goes bankrupt or restructure its debt, it switches between the healthy and distress financial status in both aggregate states. We impose the following conditions for equity

value functions :

$$\lim_{X_t \uparrow X_s(B; s_0)} E(X_t, B, D; s_0) = \lim_{X_t \downarrow X_s(B; s_0)} E(X_t, B, H; s_0), \quad (\text{B5})$$

$$\lim_{X_t \uparrow X_s(G; s_0)} E(X_t, G, D; s_0) = \lim_{X_t \downarrow X_s(G; s_0)} E(X_t, G, H; s_0). \quad (\text{B6})$$

$$\lim_{X_t \uparrow X_s(B; s_0)} E'(X_t, B, D; s_0) = \lim_{X_t \downarrow X_s(B; s_0)} E'(X_t, B, H; s_0), \quad (\text{B7})$$

$$\lim_{X_t \uparrow X_s(G; s_0)} E'(X_t, G, D; s_0) = \lim_{X_t \downarrow X_s(G; s_0)} E'(X_t, G, H; s_0). \quad (\text{B8})$$

Equations (B5) and (B6) are value matching conditions, which state the equity value are identical at the distress threshold $X_s(s_t; s_0)$ for the same state, s_t . Equations (B7) and (B8) are smooth pasting conditions, respectively.

When the firm is currently in a healthy status, i.e., $w_t = H$ in both aggregate states, it restructures its debt upward. We impose the boundary conditions as follows:

$$\lim_{X_t \uparrow X_u(B; s_0)} E(X_t, B, H; s_0) = \lim_{X_t \downarrow X_u(B; s_0)} \frac{X_t}{X_0} [(1 - \phi)D(X_0, B, H; B) + E(X_0, B, H; B)] - D(X_0, s_0, H; s_0), \quad (\text{B9})$$

$$\lim_{X_t \uparrow X_u(G; s_0)} E(X_t, G, H; s_0) = \lim_{X_t \downarrow X_u(G; s_0)} \frac{X_t}{X_0} [(1 - \phi)D(X_0, G, H; G) + E(X_0, G, H; G)] - D(X_0, s_0, H; s_0), \quad (\text{B10})$$

$$\lim_{X_t \uparrow X_u(G; s_0)} E(X_t, B, H; s_0) = \lim_{X_t \downarrow X_u(G; s_0)} E(X_t, B, H; s_0), \quad (\text{B11})$$

$$\lim_{X_t \uparrow X_u(G; s_0)} E'(X_t, B, H; s_0) = \lim_{X_t \downarrow X_u(G; s_0)} E'(X_t, B, H; s_0). \quad (\text{B12})$$

Equations (B9) and (B10) are value matching conditions at the restructuring threshold, $X_u(s_t; s_0)$, which states that equity holders retire debt at par $D(X_0, s_0, H; s_0)$, which was issued at the initial state s_0 , and issue more debt $D(X_t, s_t, H; s_t)$ at the current aggregate state $s_t = B, G$. *The scaling property applies only within the same aggregate state, s_t .* That is, if the firm starts at an initial state $s_0 = B$ but refinances at $s_t = G$, we scale up the firm value to $X_t/X_0[(1 - \phi)D(X_0, G, H; G) + E(X_0, G, H; G)]$ as if it starts at $s_0 = G$. Equations (B11) and (B12) are to ensure that equity value function $E(X_t, B, H; s_0)$ is continuous and smooth at $X_u(G; s_0)$.

B.1.2 Boundary Conditions of Debt Value Functions

When the firm is in distress, $w_t = D$, we have the following conditions for debt value functions:

$$\lim_{X_t \downarrow X_d(B; s_0)} D(X_t, B, D; s_0) = (1 - \alpha)A(X_b, B, D; s_0), \quad (\text{B13})$$

$$\lim_{X_t \downarrow X_d(G; s_0)} D(X_t, G, D; s_0) = (1 - \alpha)A(X_b, G, D; s_0), \quad (\text{B14})$$

$$\lim_{X_t \uparrow X_d(B; s_0)} D(X_t, G, D; s_0) = \lim_{X_t \downarrow X_d(B; s_0)} D(X_t, G, D; s_0), \quad (\text{B15})$$

$$\lim_{X_t \uparrow X_d(B; s_0)} D'(X_t, G, D; s_0) = \lim_{X_t \downarrow X_d(B; s_0)} D'(X_t, G, D; s_0). \quad (\text{B16})$$

Equations (B13) to (B14) states that debt holders receive the asset value after liquidation cost α in both states $s_t = B, G$. Equations (B15) and (B16) are to ensure that the debt value function $D(X_t, G, D; s_0)$ be continuous and smooth at $X_d(B; s_0)$.

We impose the following conditions for debt value functions before the firm goes bankrupt or restructure its debt:

$$\lim_{X_t \uparrow X_s(B; s_0)} D(X_t, B, D; s_0) = \lim_{X_t \downarrow X_s(B; s_0)} D(X_t, B, H; s_0), \quad (\text{B17})$$

$$\lim_{X_t \uparrow X_s(G; s_0)} D(X_t, G, D; s_0) = \lim_{X_t \downarrow X_s(G; s_0)} D(X_t, G, H; s_0). \quad (\text{B18})$$

$$\lim_{X_t \uparrow X_s(B; s_0)} D'(X_t, B, D; s_0) = \lim_{X_t \downarrow X_s(B; s_0)} D'(X_t, B, H; s_0), \quad (\text{B19})$$

$$\lim_{X_t \uparrow X_s(G; s_0)} D'(X_t, G, D; s_0) = \lim_{X_t \downarrow X_s(G; s_0)} D'(X_t, G, H; s_0). \quad (\text{B20})$$

$$(\text{B21})$$

The interpretations for equations (B17) to (B20) are similar to those for equations (B5) to (B8).

When the firm restructures its debt upward, we have the following conditions:

$$\lim_{X_t \uparrow X_u(B; s_0)} D(X_t, B, H; s_0) = P(X_0; s_0), \quad (\text{B22})$$

$$\lim_{X_t \uparrow X_u(G; s_0)} D(X_t, G, H; s_0) = P(X_0; s_0), \quad (\text{B23})$$

$$\lim_{X_t \uparrow X_u(G; s_0)} D(X_t, B, H; s_0) = \lim_{X_t \downarrow X_u(G; s_0)} D(X_t, B, H; s_0), \quad (\text{B24})$$

$$\lim_{X_t \uparrow X_u(G; s_0)} D'(X_t, B, H; s_0) = \lim_{X_t \downarrow X_u(G; s_0)} D'(X_t, B, H; s_0). \quad (\text{B25})$$

Equations (B22) and (B23) are value matching condition which indicate that debt holder receive par value at the debt refinancing threshold $X_u(s_t; s_0)$ at both states, respectively. Regardless of which the current aggregate state s_t is, debt holders receive the par value $P(X_0; s_0)$ determined at the initial state s_0 where debt is issued. Equations (B24) and (B25) are to ensure that debt value function $D(X_t, B, H; s_0)$ is continuous and smooth at $X_u(G; s_0)$.

B.2 Asset Valuations

For each initial state, s_0 , there are a total of four cash flow regions for both equity and debt, as shown in Figure 3. The cash flow regions are divided as follows:

$$\mathbb{R}_1 : X_d(G; s_0) \leq X_t < X_d(B; s_0); \quad (\text{B26})$$

$$\mathbb{R}_2 : X_d(B; s_0) \leq X_t < c(s_0); \quad (\text{B27})$$

$$\mathbb{R}_3 : c(s_0) \leq X_t < X_u(G; s_0); \quad (\text{B28})$$

$$\mathbb{R}_4 : X_u(G; s_0) \leq X_t < X_u(B; s_0). \quad (\text{B29})$$

For regions of \mathbb{R}_1 and \mathbb{R}_2 , firms are distressed, $w_t = D$ in both states $s_t = G, B$. For regions of \mathbb{R}_3 and \mathbb{R}_4 , firms are healthy, $w_t = H$ in both states. We will successively characterize the values of equity and debt for each region. We assume the distress threshold is the same in both aggregate states and is dependent on the initial coupon $c(s_0)$, i.e., $X_s(G; s_0) = X_s(B; s_0) = c(s_0)$.

The standard no-arbitrage condition implies that equity E_{s_t, w_t} pays the dividend D_{s_t, w_t} , satisfies

$$(r + p_B)E_{B, w_t} = D_{B, w_t} + \mu_{B, w_t} X_t E'_{B, w_t} + \frac{1}{2} \sigma_{B, w_t}^2 X_t E''_{B, w_t} + p_B E_{G, w_t}, \quad (\text{B30})$$

$$(r + p_G)E_{G, w_t} = D_{G, w_t} + \mu_{G, w_t} X_t E'_{G, w_t} + \frac{1}{2} \sigma_{G, w_t}^2 X_t E''_{G, w_t} + p_G E_{B, w_t}, \quad (\text{B31})$$

In the matrix form,

$$\left[\begin{pmatrix} r_{B, w_t} + p_B & -p_B \\ -p_G & r_{G, w_t} + p_G \end{pmatrix} - \begin{pmatrix} \mu_{B, w_t} & 0 \\ 0 & \mu_{G, w_t} \end{pmatrix} X_t \frac{\partial}{\partial X_t} - \frac{1}{2} \begin{pmatrix} \sigma_{B, w_t}^2 & 0 \\ 0 & \sigma_{G, w_t}^2 \end{pmatrix} X_t^2 \frac{\partial^2}{\partial X_t^2} \right] \begin{pmatrix} E_{B, w_t} \\ E_{G, w_t} \end{pmatrix} = \begin{pmatrix} D_{B, w_t} \\ D_{G, w_t} \end{pmatrix} \quad (\text{B32})$$

B.2.1 Equity Value Functions

$$\mathbb{R}_1 = \mathbf{X}_d(\mathbf{G}; \mathbf{s}_0) \leq \mathbf{X}_t < \mathbf{X}_d(\mathbf{B}; \mathbf{s}_0)$$

After becoming distressed, $w_t = D$, the firm has already gone bankrupt in the bad state, but not yet in the good state. Because equity holders receive nothing at bankruptcy, $E(X_t, B, D; s_0) = 0$, in the bad state. They still receive the residuals after interest and taxes before bankruptcy in the good state. In addition, a sudden switch of the economy from the good state to the bad state will cause the firm to go bankrupt immediately.

The value function of $E(X_t, G, D; s_0)$ satisfies the following ODE:

$$(r_G + \lambda_G)E(X_t, G, D; s_0) = (1 - \tau)(X_t(1 - \varpi(G)) - c(s_0)(1 - k(G; s_0))) + \mu_{G, D} X_t E'(X_t, G, D; s_0) + \frac{1}{2} \sigma_{G, D}^2 X_t^2 E''(X_t, G, D; s_0) \quad (\text{B33})$$

Assume that the function form of the equity value is

$$E(X_t, G, D; s_0) = (A(X_t, G, D) - C_G(s_0))(1 - \tau) + \sum_{i=1}^2 a_{G, D, i}^E(s_0) X_t^{\psi_{H, i}}, \quad (\text{B34})$$

where $\psi_{H,i}$ is the roots of

$$\frac{1}{2}\sigma_{G,D}^2\psi_H(\psi_H - 1) + \mu_{G,D}\psi_H - r_G - \lambda_G = 0. \quad (\text{B35})$$

We can easily verify that the particular parts of the above function form are, respectively,

$$A(X_t, G, D) = \frac{X_t(1 - \varpi(G))}{r_G + \lambda_G - \mu_{G,D}}, \quad (\text{B36})$$

and

$$C_G(s_0) = \frac{c(s_0)(1 - k(G; s_0))}{r_G + \lambda_G}. \quad (\text{B37})$$

It is evident that the unleveled asset value $A(X_t, G, D)$ is decreasing with the probability of leaving the good state for the bad state, λ_G , in line with our intuition. While $A(X_t, G, D)$ is independent of initial state s_0 , $C_G(s_0)$ is dependent on the initial state where the firm enters market and issue debt.

$$\mathbb{R}_2 = \mathbf{X}_d(\mathbf{B}; \mathbf{s}_0) \leq \mathbf{X}_t < \mathbf{c}(\mathbf{s}_0)$$

In this region, the firm has become distressed, i.e., $w_t = D$, but has not gone bankrupt in both states. Equity holders receive $(1 - \tau)(X_t - c(s_0))$ in both states so that $E(X_t, B, D; s_0)$ and $E(X_t, G, D; s_0)$ satisfy the following system of ODEs:

$$(r_B + \lambda_B)E(X_t, B, D; s_0) = (1 - \tau)(X_t(1 - \varpi(B) - c(s_0)(1 - k(B; s_0))) + \mu_{B,D}X_tE'(X_t, B, D; s_0) + \frac{1}{2}\sigma_{B,D}^2X_t^2E''(X_t, B, D; s_0) + \lambda_BE(X_t, G, D; s_0) \quad (\text{B38})$$

$$(r_G + \lambda_G)E(X_t, G, D; s_0) = (1 - \tau)(X_t(1 - \varpi(G) - c(s_0)(1 - k(G; s_0))) + \mu_{G,D}X_tE'(X_t, G, D; s_0) + \frac{1}{2}\sigma_{G,D}^2X_t^2E''(X_t, G, D; s_0) + \lambda_GE(X_t, B, D; s_0). \quad (\text{B39})$$

Assume the functional form of the solution in state s_t is

$$E(X_t, s_t, H; s_0) = (A(X_t, s_t, H) - C_{s_t}(s_0))(1 - \tau) + \sum_{i=1}^4 e_{s_t, H, i}^E X_t^{\omega_{H, i}}. \quad (\text{B40})$$

Plugging (B40) into the ODEs (B38) and (B39), we obtain the solutions to the particular parts as follows:

$$\begin{bmatrix} A(X_t, B, D) \\ A(X_t, G, D) \end{bmatrix} = \begin{bmatrix} r_B - \mu_{B,D} + \lambda_B & -\lambda_B \\ -\lambda_G & r_G - \mu_{G,D} + \lambda_G \end{bmatrix}^{-1} \begin{bmatrix} X_t(1 - \varpi(B)) \\ X_t(1 - \varpi(G)) \end{bmatrix} \quad (\text{B41})$$

and

$$\begin{bmatrix} C_B(s_0) \\ C_G(s_0) \end{bmatrix} = \begin{bmatrix} r_B + \lambda_B & -\lambda_B \\ -\lambda_G & r_G + \lambda_G \end{bmatrix}^{-1} \begin{bmatrix} c(s_0)(1 - k(B; s_0)) \\ c(s_0)(1 - k(G; s_0)) \end{bmatrix} \quad (\text{B42})$$

For the homogenous part of the solution, we verify that, for each pair of $e_{B,D,i}^E(s_0)X_t^{\omega_{H,i}}$ and

$e_{G,D,i}^E(s_0)X_t^{\omega_{H,i}}$, we have

$$\left[\begin{pmatrix} r_{B,D} + \lambda_B & -\lambda_B \\ -\lambda_G & r_{G,D} + \lambda_G \end{pmatrix} - \begin{pmatrix} \mu_{B,D} & 0 \\ 0 & \mu_{G,D} \end{pmatrix} \omega_{H,i} - \frac{1}{2} \begin{pmatrix} \sigma_{B,D}^2 & 0 \\ 0 & \sigma_{G,D}^2 \end{pmatrix} \omega_{H,i} (\omega_{H,i} - 1) \right] \begin{pmatrix} e_{B,D,i}^E(s_0) \\ e_{G,D,i}^E(s_0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (\text{B43})$$

Moreover, $e_{B,D,i}^E(s_0) = g_{H,i} e_{G,D,i}^E(s_0)$, where

$$g_{H,i} = \frac{1}{\lambda_G} \left(\frac{1}{2} \sigma_{G,D}^2 \omega_{H,i} (\omega_{H,i} - 1) + \mu_{G,D} \omega_{H,i} - r_G - \lambda_G \right), \quad (\text{B44})$$

and $\omega_{H,i}$ is one of two positive roots and two negative roots of the following function

$$\left(\frac{1}{2} \sigma_{B,D}^2 \omega_H (\omega_H - 1) + \mu_{B,D} \omega_H - r_B - \lambda_B \right) \left(\frac{1}{2} \sigma_{G,D}^2 \omega_H (\omega_H - 1) + \mu_{G,D} \omega_H - r_G - \lambda_G \right) = \lambda_B \lambda_G. \quad (\text{B45})$$

$\mathbb{R}_3 = \mathbf{c}(s_0) \leq \mathbf{X}_t < \mathbf{X}_u(\mathbf{G}; s_0)$

The firm is healthy in both states in this region. Hence, equity value functions $E(X_t, G, H; s_0)$ and $E(X_t, B, H; s_0)$ satisfy the following system of ODEs

$$\begin{aligned} (r_G + \lambda_G)E(X_t, G, H; s_0) &= (1 - \tau)(X_t - c(s_0)) + \mu_{G,H} X_t E'(X_t, G, H; s_0) \\ &\quad + \frac{1}{2} \sigma_{G,H}^2 X_t^2 E''(X_t, G, H; s_0) + \lambda_G E(X_t, B, H; s_0), \end{aligned} \quad (\text{B46})$$

$$\begin{aligned} (r_G + \lambda_B)E(X_t, B, H; s_0) &= (1 - \tau)(X_t - c(s_0)) + \mu_{B,H} X_t E'(X_t, B, H; s_0) \\ &\quad + \frac{1}{2} \sigma_{B,H}^2 X_t^2 E''(X_t, B, H; s_0) + \lambda_B E(X_t, G, H; s_0). \end{aligned} \quad (\text{B47})$$

Assume the functional form of the value function is

$$E(X_t, s_t, L; s_0) = (A(X_t, s_t, L) - C_{s_t}(s_0))(1 - \tau) + \sum_{i=1}^4 e_{s_t,L,i}^E(s_0) X_t^{\omega_{L,i}}. \quad (\text{B48})$$

Plugging (B48) into ODEs (B46) and (B47), we obtain its particular solutions $A(X_t, s_t, L)$ and $C_{s_t}(s_0)$ in the matrix form are as follows:

$$\begin{bmatrix} A(X_t, B, H) \\ A(X_t, G, H) \end{bmatrix} = \begin{bmatrix} r_B - \mu_{B,H} + \lambda_B & -\lambda_B \\ -\lambda_G & r_G - \mu_{G,H} + \lambda_G \end{bmatrix}^{-1} \begin{bmatrix} X_t \\ X_t \end{bmatrix}, \quad (\text{B49})$$

and

$$\begin{bmatrix} C_B(s_0) \\ C_G(s_0) \end{bmatrix} = \begin{bmatrix} r_B + \lambda_B & -\lambda_B \\ -\lambda_G & r_G + \lambda_G \end{bmatrix}^{-1} \begin{bmatrix} c(s_0) \\ c(s_0) \end{bmatrix}. \quad (\text{B50})$$

We can verify for each item $e_{B,H,i}^E(s_0)X_t^{\omega_{L,i}}$ and $e_{G,H,i}^E(s_0)X_t^{\omega_{L,i}}$ of the homogenous solution is

$$\left[\begin{pmatrix} r_{B,H} + \lambda_B & -\lambda_B \\ -\lambda_G & r_{G,H} + \lambda_G \end{pmatrix} - \begin{pmatrix} \mu_{B,H} & 0 \\ 0 & \mu_{G,H} \end{pmatrix} \omega_{L,i} - \frac{1}{2} \begin{pmatrix} \sigma_{B,H}^2 & 0 \\ 0 & \sigma_{G,H}^2 \end{pmatrix} \omega_{L,i} (\omega_{L,i} - 1) \right] \begin{pmatrix} e_{B,H,i}^E(s_0) \\ e_{G,H,i}^E(s_0) \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (\text{B51})$$

Additionally, $e_{B,H,i}^E(s_0) = g_{L,i} e_{G,H,i}^E(s_0)$, where

$$g_{L,i} = \frac{1}{\lambda_G} \left(\frac{1}{2} \sigma_{G,H}^2 \omega_{L,i} (\omega_{L,i} - 1) + \mu_{G,H} \omega_{L,i} - r_G - \lambda_G \right), \quad (\text{B52})$$

and ω_L is two positive roots and two negative roots of the following function

$$\left(\frac{1}{2} \sigma_{B,H}^2 \omega_L (\omega_L - 1) + \mu_{B,H} \omega_L - r_B - \lambda_B \right) \left(\frac{1}{2} \sigma_{G,H}^2 \omega_L (\omega_L - 1) + \mu_{G,H} \omega_L - r_G - \lambda_G \right) = \lambda_B \lambda_G. \quad (\text{B53})$$

$$\mathbb{R}_4 = \mathbf{X}_u(\mathbf{G}; \mathbf{s}_0) \leq \mathbf{X}_t < \mathbf{X}_u(\mathbf{B}; \mathbf{s}_0)$$

In this region, the firm in the good state has already refinanced their debt upward, but not yet in the bad state. By retiring the existing debt at par $D(X_0, s_0, H; s_0)$ and issuing new debt $D(X_u, G, H; G)$ at a fraction cost ϕ , equity holders increase their own wealth to $E(X_t, G, H; s_0) = (1 - \phi) D(X_t, G, H; G) + E(X_t, G, H; G) - D(X_0, s_0, H; s_0)$. By scaling property, we have the following equity value at the refinancing threshold $X_u(G; s_0)$:

$$E(X_u, G, H; s_0) = \frac{X_u(G; s_0)}{X_0} \left((1 - \phi) D(X_0, G, H; G) + E(X_0, G, H; G) \right) - D(X_0, s_0, H; s_0).$$

In contrast, equity holders in the bad state have not refinanced their debt yet. However, an exogenous switch from the bad state to the good state induces equity holders to refinance their debt immediately. Hence, the equity value function in the bad state, $E(X_t, B, H; s_0)$, satisfies the following ODE

$$\begin{aligned} (r_B + \lambda_B) E(X_t, B, H; s_0) &= (1 - \tau)(X_t - c(s_0)) + \mu_{B,H} X_t E'_{B,H} + \frac{1}{2} \sigma_{B,H}^2 X_t E''_{B,H} \\ &+ \lambda_B \left(\frac{X_t}{X_0} \left((1 - \phi) D(X_0, G, H; G) + E(X_0, G, H; G) \right) - D(X_0, s_0, H; s_0) \right). \end{aligned} \quad (\text{B54})$$

Its solution is

$$E(X_t, B, H; s_0) = A(X_t, B, H) - C_B(s_0) + \sum_{i=1}^2 a_{B,H,i}^E(s_0) X_t^{\psi_{L,i}} \quad (\text{B55})$$

where $\psi_{L,i}$ is the negative and positive roots of

$$\frac{1}{2} \sigma_{B,H}^2 \psi_L (\psi_L - 1) + \mu_{B,H} \psi_L - r_B - \lambda_B = 0. \quad (\text{B56})$$

We can verify the particular parts of the value function are as follows:

$$A(X_t, B, H) = \frac{X_t(1 - \tau) + \lambda_B \frac{X_t}{X_0} ((1 - \phi) D(X_0, G, H; G) + E(X_0, G, H; G))}{r_B + \lambda_B - \mu_{B,,L}}, \quad (\text{B57})$$

and

$$C_B(s_0) = \frac{c(s_0)(1 - \tau) + \lambda_B D(X_0, s_0, H; s_0)}{r_B + \lambda_B}. \quad (\text{B58})$$

In total, we have 12 unknown coefficients for equity value function for an initial state s_0 .

B.2.2 Debt Value Functions

$$\mathbb{R}_1 = \mathbf{X}_d(\mathbf{G}; \mathbf{s}_0) \leq \mathbf{X}_t < \mathbf{X}_d(\mathbf{B}; \mathbf{s}_0)$$

In this region, the firm has gone bankrupt in the bad state. Debt holders take over the assets and receive the residual value after the liquidation cost, i.e., $D(X_t, B, D; s_0) = (1 - \alpha_B)A(X_{B,d}, B, D)(1 - \tau)$. In the good state, debt holders still receive the fixed coupon $c(s_0)$ before bankruptcy. Hence, its value function $D(X_t, G, D; s_0)$ satisfies the following ODE:

$$\begin{aligned} (r_G + \lambda_G)D(X_t, G, D; s_0) &= c(s_0)(1 - k(G; s_0) + \mu_{G,D}X_t D'(X_t, G, D; s_0)) \\ &\quad + \frac{1}{2}\sigma_{G,D}^2 X_t^2 D''(X_t, G, D; s_0) + \lambda_G(1 - \alpha_B)A(X_{B,d}, B, D)(1 - \tau) \end{aligned} \quad (\text{B59})$$

The solution of the debt value function is

$$D(X_t, G, D; s_0) = C_G(s_0) + \sum_{i=1}^2 a_{G,D,i}^D(s_0) X_t^{\psi_{H,i}} + a_d \lambda_G (1 - \alpha_B) A(X_{B,d}, B, D)(1 - \tau), \quad (\text{B60})$$

where $C_G(s_0)$ is defined in equation (B37), $\psi_{H,i}$ in (B35),

$$a_d = \frac{1}{r_G + \lambda_G - \mu_{G,D}}, \quad (\text{B61})$$

and

$$A(X_t, B, D) = \frac{X_t(1 - \varpi(B))}{r_B + \lambda_B - \mu_{B,D}}. \quad (\text{B62})$$

$$\mathbb{R}_2 = \mathbf{X}_d(\mathbf{B}; \mathbf{s}_0) \leq \mathbf{X}_t < \mathbf{c}(\mathbf{s}_0)$$

In this region, the firm is distressed and its debt holders receive a stream of fixed coupon $c(s_0)$ in both states. $D(X_t, B, D; s_0)$ and $D(X_t, G, D; s_0)$ satisfy the following system of ODEs:

$$\begin{aligned} (r_B + \lambda_B)D(X_t, B, D; s_0) &= c(s_0)(1 - k(B; s_0)) + \mu_{B,D}X_t D'(X_t, B, D; s_0) \\ &\quad + \frac{1}{2}\sigma_{B,D}^2 X_t^2 D''(X_t, B, D; s_0) + \lambda_B D(X_t, G, D; s_0) \end{aligned} \quad (\text{B63})$$

$$\begin{aligned} (r_G + \lambda_G)D(X_t, G, D; s_0) &= c(s_0)(1 - k(G; s_0)) + \mu_{G,D}X_t D'(X_t, G, D; s_0) \\ &\quad + \frac{1}{2}\sigma_{G,D}^2 X_t^2 D''(X_t, G, D; s_0) + \lambda_G D(X_t, B, D; s_0), \end{aligned} \quad (\text{B64})$$

The debt value function in state s_t is

$$D(X_t, s_t, H; s_0) = C_{s_t}(s_0) + \sum_{i=1}^4 e_{s_t, H, i}^D X_t^{\omega_{H, i}} \quad (\text{B65})$$

where $C_{s_t}(s_0)$ is shown in (B42) and $\omega_{H, i}$ in (B45). Similar to the equity value function in the same region, $e_{B, D, i}^D(s_0) = g_{H, i} e_{G, D, i}^D(s_0)$, where $g_{H, i}$ is in equation (B44).

$$\mathbb{R}_3 = \mathbf{c}(\mathbf{s}_0) \leq \mathbf{X}_t < \mathbf{X}_u(\mathbf{G}; \mathbf{s}_0)$$

The firm is healthy in both states. Debt value functions $D(X_t, G, H; s_0)$ and $D(X_t, B, H; s_0)$ satisfy the following system of ODEs:

$$(r_G + \lambda_G)D(X_t, G, H; s_0) = c(s_0) + \mu_{G, H} X_t D'(X_t, G, H; s_0) + \frac{1}{2} \sigma_{G, H}^2 X_t^2 D''(X_t, G, H; s_0) + \lambda_G D(X_t, B, H; s_0), \quad (\text{B66})$$

$$(r_G + \lambda_B)D(X_t, B, H; s_0) = c(s_0) + \mu_{B, H} X_t D'(X_t, B, H; s_0) + \frac{1}{2} \sigma_{B, H}^2 X_t^2 D''(X_t, B, H; s_0) + \lambda_B D(X_t, G, H; s_0). \quad (\text{B67})$$

And the solution function in both states is

$$D(X_t, s_t, H; s_0) = C_{s_t}(s_0) + \sum_{i=1}^4 e_{s_t, L, i}^D(s_0) X_t^{\omega_{L, i}}. \quad (\text{B68})$$

where $C_{s_t}(s_0)$ is shown in (B50) and $\omega_{L, i}$ in (B53). Similar to the equity value function in the same region, $e_{B, H, i}^D(s_0) = g_{L, i} e_{G, H, i}^D(s_0)$, where $g_{L, i}$ is in equation (B52).

$$\mathbb{R}_4 = \mathbf{X}_u(\mathbf{G}; \mathbf{s}_0) \leq \mathbf{X}_t < \mathbf{X}_u(\mathbf{B}; \mathbf{s}_0)$$

Because the firm refinances earlier in the good state than in the bad state, debt holders have already redeemed the par value, $D(X_t, G, H; s_0) = D(X_0, s_0, H; s_0)$, in the good state. Because debt holders have not received the payment at par in the bad state, the debt value function, $D(X_t, B, H; s_0)$, satisfies the following ODE:

$$(r_B + \lambda_B)D(X_t, B, H; s_0) = c(s_0) + \mu_{B, H} X_t D'(X_t, B, H; s_0) + \frac{1}{2} \sigma_{B, H}^2 X_t^2 D''(X_t, B, H; s_0) + \lambda_B D(X_0, s_0, H; s_0) \quad (\text{B69})$$

Its solution is

$$D(X_t, B, H; s_0) = C_B(s_0) + \sum_{i=1}^2 a_{B, H, i}^D(s_0) X_t^{\psi_{L, i}} \quad (\text{B70})$$

where $\psi_{L, i}$ is in (B56) and

$$C_B(s_0) = \frac{c(s_0) + \lambda_B D(X_0, s_0, H; s_0)}{r_B + \lambda_B}. \quad (\text{B71})$$

In total, we have 12 unknown coefficients for debt value function for an initial state s_0 .

Appendix C

By directly applying the property of hitting time distribution of a geometric Brownian motion according to equation (11) of p.14 on Harrison (1985), we obtain the cumulative physical default probability $\hat{\pi}$ for the firm issuing a bond with the time-to-maturity T :

$$\hat{\pi}_{it} = \mathbb{N}(h(T)) + \frac{X_{it}}{X_{iB}}^{-2\xi_i/\sigma_i^2} \mathbb{N}\left(h(T) + \frac{2\xi_i T}{\sigma_i \sqrt{T}}\right), \quad (\text{C72})$$

where $\xi_i = \hat{\mu}_i - 0.5\sigma_i^2 > 0$ and $h(T) = \frac{\log(X_{iB}/X_{it}) - \xi_i T}{\sigma_i \sqrt{T}}$.

For the perpetual bond in our model, $T \rightarrow \infty$. Therefore,

$$\hat{\pi}_{it} = \left(\frac{X_{it}}{X_{iB}}\right)^{-2(\hat{\mu}_i - 0.5\sigma_i^2)/\sigma_i^2}. \quad (\text{C73})$$

When $\mu_i \rightarrow r$, we obtain $\omega_i \rightarrow -2r/\sigma_i^2$ from equation (A14). Therefore,

$$\pi_{it} = \left(\frac{X_{it}}{X_{iB}}\right)^{\omega_i} \rightarrow \left(\frac{X_{it}}{X_{iB}}\right)^{-2r/\sigma_i^2}. \quad (\text{C74})$$

By taking logarithm of $\hat{\pi}_{it}$ and π_{it} , we can easily obtain:

$$\lambda_i = \hat{\mu}_i - \mu_i \rightarrow \hat{\mu}_i - r = \left(\frac{\log(\pi_{it}) - \log(\hat{\pi}_{it})}{\log(X_{it}) - \log(X_{iB})} + 1\right) \frac{\sigma_i^2}{2}. \quad (\text{C75})$$