Firm Dynamics Depend on Cash and Capital

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Abstract

Models of the firm traditionally have one state variable for tractability. Our innovation is to solve a dynamic model of a firm with two states – cash and capital, which jointly determine a firm’s optimal default, investment, payout, and risk management decisions. Our two-dimensional model allows for diminishing returns to scale, non-linear issuance costs, and fixed debt. The model connects disperse strands of the empirical literature, and we find support in the data for novel non-linearities: (1) equity issuance scaled by capital is declining and convex in capital and (2) payout scaled by capital is concave in capital.

JEL classification: E22, G12, G32, G33, G35

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1. Introduction

A primary question is how costly external financing affects a firm’s investment, payout, and risk management policies. The seminal paper by Bolton, Chen, and Wang (2011) (BCW) highlights the intricacies among these closely intertwined firm decisions. BCW introduced the first elements of a dynamic corporate risk management framework, in which a dimensionality reduction technique is employed for tractability so that a firm’s decision problem only depends on one state variable – the cash-to-capital ratio. Nevertheless, the dimensionality reduction excludes several modelling components which map to realistic firm characteristics: fixed long-term debt, decreasing returns to scale in production, and non-linear issuance costs. In this paper, we introduce a model where the size of a firm’s capital stock and cash reserve are separate state variables for the firm’s problem. This two-dimensional model incorporates the aforementioned firm characteristics and provides novel prescriptions. The model’s predictions unify several aspects of the empirical literature and motivate several new hypotheses on non-linearities in a firm’s optimal strategies, which we find strong support for in the data.

The model firm has a stock of capital and a cash reserve. The firm generates cash flows from production using its stock of capital. The productivity rate is subject to a random sequence of productivity shocks and the production exhibits diminishing returns to scale. The capital depreciates and investment is assumed to be irreversible and is subject to convex adjustment costs. Cash held in the firm carries a liquidity premium and hence earns a low rate of return. The firm is financed by equity and long-term debt, which we model as a consol bond with a constant coupon rate and a constant principal. Equityholders of the firm determine the cash holding, dividend payout, equity issuance, and investment strategies, together with the default time, to maximize the expected value of future payouts net of equity issuance costs.

The model firm faces costly equity issuance and bankruptcy. The equity issuance costs have a fixed component (that does not scale with capital) and a proportional component (which scales with issuance size). This implies that external financing costs are declining
relative to proceeds and firm size.\textsuperscript{1} The firm trades off the costs of holding cash against the flexibility that cash offers in funding investment and meeting liabilities without having to pay the cost of raising new equity. The firm may run out of cash because of negative productivity shocks or because equityholders strategically default, paying out the cash reserve as a lump sum dividend and then defaulting.\textsuperscript{2} The firm continuously compares the value of the firm without issuance with the best value for issuance and only issues the optimal amount of equity when the later value is strictly higher. We show that the firm only optimally issues when its cash reserve is depleted. The firm liquidates the first time it runs out of cash and chooses not to raise equity. At default, equityholders liquidate the capital in a fire sale and apply the proceeds to the secured debt claim, with any remaining proceeds, if any, paid out as a dividend.

We characterize the firm’s optimal value via a dynamic programming equation with two state variables: capital $k$ and cash $c$. We prove that the value function of the firm is the unique solution of this equation. We numerically solve this equation with a reasonable set of parameters (matching BCW for overlapping parameters) and prove that the numerical algorithm converges to the unique solution of this dynamic programming equation (i.e., the numeric solution converges to the firm’s value function) in the appendix. We present the solution in a series of simple yet insightful figures.

Our solution shows that the state space $(k, c)$ is partitioned into several regions with different strategies: a dividend region, an investment region, a continuation region, and an issuance or default region. When the cash reserve is sufficiently high, the firm pays out a dividend. When the cash reserve reaches zero, the firm either issues equity or defaults. Issuance is infrequent and in lump sum. Between the payout and the issuance/default regions,

\textsuperscript{1}For tractability, Bolton et al. (2011) scales fixed issuance costs by capital. They acknowledge, “In practice, external costs of financing scaled by firm size are likely to decrease with firm size.” Prior work finds declining costs relative to proceeds and size (Lee et al., 1996; Altinkilic and Hansen, 2000).

\textsuperscript{2}If equityholders decide to default while the cash reserve of the firm is still positive, they will optimally pay out all remaining cash as dividends the instant before triggering default, because debt holders are senior claimants on the firm and will seize all the cash when the firm defaults. Therefore, default will only happen when the cash level of the firm is at zero.
the firm invests or retains its earnings. Interestingly, the dividend region contains a strategic
default region; whenever the state process reaches its boundary, the equityholders optimally
payout the remaining cash as a lump sum dividend and then default afterward. We prove
that the strategic default region exists when the capital is low and the debt coupon rate is
higher than a threshold, which increases with firm’s expected productivity. The intuition
is that the debt burden is too heavy relative to the continuation value of a firm with low
capital. Even though the marginal productivity of capital is high, it takes time to build up
the capital; hence, equityholders find it sub-optimal to continue and prefer paying out the
remaining cash immediately and defaulting. The existence of the strategic default region,
which has not been observed in various one-dimensional models in the literature, highlights
the flexibility of our two-dimensional model.

We find strong support in the data for the non-linearities in a firm’s issuance and payout
behavior predicted by the model. First, the empirical literature on issuances is predominately
concerned with when firms choose to issue. By contrast, there is a scarcity of work on
how much firms choose to issue conditional on issuance. A novel prediction of the model,
supported by the data, is that the amount a firm optimally issues scaled by capital is declining
and convex in capital. Second, our model and empirical work advance the literature on
payout policy in the presence of costly external financing. Rauh (2006) and Campbell et al.
(2011) show that firms decrease dividends after a negative shock to cash in the presence of
costly financing. A novel prediction of the model, supported by the data, is that for certain
levels of cash, the optimal payout amount scaled by capital is concave in capital.

In the empirical literature, our model connects disperse findings related to costly financing,
liquidity, and investment. Firms with higher costs of external financing tend to save a
higher proportion of cash flows as cash (Almeida et al., 2004; Acharya et al., 2007; Harford...
et al., 2014). Relatedly, several papers find that firms with riskier cash flows hold more cash, especially for distressed firms with less access to capital markets.\(^5\) Consistent with a hedging motive, Opler et al. (1999) and Lins et al. (2010) find that operating losses are the main reason for excess cash changes. Additionally, several papers find cash holdings are associated with investment for financially constrained firms in the presence of costly external financing.\(^6\) Minton and Schrand (1999) show that cash flow volatility is associated with lower investment. Our model predictions are consistent with the aforementioned empirical literature. In our model, firms are effectively risk averse due to costly issuance or default, which increases the marginal value of hoarding cash.\(^7\) Higher cash flow volatility increases the value of cash because the likelihood of issuance or default increases. The higher marginal value of cash competes with the marginal value of investment, dampening investment relative to a frictionless setting.

Additionally, our model predictions are consistent with the empirical facts documented on cash holdings and credit risk. First, Davydenko (2013) finds that approximately 10% of firms in default are solvent (market values of assets exceeding the face value of liabilities), and these firms face high external financing costs. In our model, as external financing costs increase, firms with larger capital stocks find it optimal to default rather than pay the issuance costs to keep running the firm. Second, Davydenko (2013) finds that about 13% of defaulting

\(^5\)See Opler et al. (1999); Han and Qiu (2007); Bates et al. (2009); Duchin (2010); Subramaniam et al. (2011); Tong (2011); Palazzo (2012); Alfaro et al. (2016). Opler et al. (1999) and Duchin (2010) also find that cash flow volatility matters more for distressed firms with less access to capital markets. Bates et al. (2009) report that partially because of riskier cash flows, corporate cash balances in the U.S. increased from 10.5% of book assets in 1980 to over 23% of assets in 2006.

\(^6\)See Faulkender and Wang (2006); Pinkowitz and Williamson (2006); Denis and Sibilkov (2009); Campello et al. (2010); Duchin et al. (2010); Brown and Petersen (2011). Relatedly, payouts are negatively associated with a firm’s growth opportunities (Fama and French, 2001; DeAngelo et al., 2006; Denis and Osobov, 2008). Also, a number of papers consider the intertemporal links between financing constraints and investment (Kaplan and Zingales, 1997; Kim et al., 1998; Boyle and Guthrie, 2003; Stein, 2003; Almeida et al., 2011). Early studies identified firms with higher financing frictions by analyzing the sensitivity of investment to cash flow (Fazzari et al., 1987; Kaplan and Zingales, 1997; Erickson and Whited, 2000; Alti, 2003; Almeida et al., 2004; Levitas and McFadyen, 2009; Kusnadi and Wei, 2011; Officer, 2011).

\(^7\)The marginal cost from a smaller cash holding is higher than the marginal benefit from a larger cash holding because the increase in the likelihood of liquidation or issuance outweighs the benefit from otherwise relaxing the firm’s financing constraints. It is the concavity of the value function that gives rise to the demand for a cash reserve.
firms are insolvent but have more than enough cash to cover their current liabilities. This finding is consistent with the strategic default region that emerges in our model, in which the equityholders of an insolvent firm with large cash balances choose to strategically default because the benefits from investing to grow the capital (which takes time) are less valuable than paying out the cash reserve. Third, our model sheds light on why profitable firms with low expected distress costs may use debt conservatively (Opler et al., 1999; Graham, 2000). In our setting, high leverage firms forgo profitable investment opportunities to save cash and reduce costly issuance.

There are few theoretical studies that examine a firm’s optimal investment and risk management policies facing external financing costs. The closest paper to ours is BCW. Our main methodology contribution is to extend their one-dimensional model based on the cash-to-capital ratio to a two-dimensional model in which cash and capital separately affect firm’s choice. This leads to several new model predictions. First, in BCW, the ratio between the payout boundary and capital is constant. Our model predicts a decreasing and convex ratio with respect to capital. In BCW, the investment over depreciation is also constant, while our model generates a decreasing and convex relation with capital. Second, our model incorporates firm leverage, revealing its impact on a firm’s default, investment, payout, and issuance decisions. Third, our model allows for a size-independent fixed component of issuance costs. For low capital firms, continuation values are small relative to the fixed issuance costs, resulting in default rather than issuance. For high capital firms, the fixed issuance costs are small relative to continuation values; resulting in issuance rather than default. Therefore, our model generates different issuance or default decision for firms with different size.

More generally, our two-state model extends other models in the dynamic liquidity management literature. Most examine a firm’s cash policy assuming that cash-flow shocks are independent and identically distributed through time, which implies that a firm’s solvency - or economic health - is constant through time. In our model, firm cash flows depend on

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8Girgis (1968), Riddick and Whited (2009), Bolton et al. (2011, 2014), Décamps et al. (2011), Hugonnier
the dynamic capital stock. Gomes (2001), Hennessy and Whited (2005), and Hennessy et al. (2007) numerically solve discrete-time dynamic capital structure models with investment for financially constrained firms. They allow for stochastic investment opportunities and have no adjustment costs for investment. However, these studies do not model cash accumulation. In contrast, for our model firm, cash and capital continually change, and the firm may raise equity or default depending on the level of capital and cash in the firm. The capital state variable also gives rise to a rich cash policy.

We contribute to the broader theoretical corporate finance literature in a few ways. First, most models of risky debt assume that a firm defaults when the market value of its assets falls below the face value of debt or when the option value of equity is not high enough to keep the firm alive (e.g., Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995)). In most such models, there is no cash balance, and if the firm’s cash flow is insufficient for debt service while the asset value is still high enough, equityholders can contribute additional funds at no cost. As a result, these models cannot speak to how financial distress interacts with issuance costs to define a firm’s liquidity management. Second, the contingent claims models of the firm in the literature generally model default as occurring due to severe negative shocks to the market value of a firm’s assets. There are no frictions to issuing equity and so no role for cash in these models.9

2. Model

The firm’s operating revenue depends on its capital stock and productivity. We assume that the firm’s productivity evolves according to:

\[ dZ_t = \mu dt + \sigma dW_t, \]

\[ (1) \]
where $W$ is a 1-dimensional Brownian motion, $\mu$ and $\sigma$ are positive constants. Thus, productivity shocks $dZ_t$ are assumed to be i.i.d. with mean $\mu dt$ and variance $\sigma^2 dt$. The firm’s cumulative cash flows $Y$ follow the dynamics

$$dY_t = k^\alpha_t dZ_t,$$

(2)

where $k$ is the size of capital stock of the firm and $\alpha \in (0, 1)$ is a scale parameter following Bertola and Caballero (1994). Therefore the production is decreasing return to scale. This is the first modelling difference with BCW, who consider a constant returns to scale technology.

The firm can invest in the capital stock. As is standard in capital accumulation models, for an investment process $i$, the dynamics of the capital stock follows

$$dk_t = (i_t - \delta k_t) dt,$$

(3)

where $\delta \geq 0$ is the depreciation rate. We assume that investment is irreversible, i.e., $i \geq 0$. Similar to BCW, investment is subject to a convex adjustment cost

$$g(k, i) = \theta \left( \frac{i}{k} \right)^2 k,$$

(4)

for a positive constant $\theta$ that measures the degree of the adjustment cost.

The firm is financed by equity and long term debt. The firm pays a coupon at a constant rate $b$ for the long term debt. This is the second difference with BCW, who does not model debt, assuming $b$ is zero. Having $b \geq 0$ allows us to study the impact of debt on firm’s payout, investment, and risk management decisions.

The firm determines its own investment and cash management strategies, which include when to pay a dividend and raise equity. The value of cash reserve follows the dynamics

$$dc_t = (r - \lambda_c)c_t dt + dY_t - bdt - i_t dt - g(k_t, i_t) dt - dD_t + dI_t.$$

(5)

\(^{10}\)See the Internet Appendix C.1 for empirical work consistent with diminishing returns to scale. Also, see Caballero (1991), Basu and Fernald (1997) and Grullon and Ikenberry (2021).
Here $r$ is the interest rate, $\lambda_c$ is the cash holding cost (liquidity premium), $D$ is the cumulative dividend payout, and $I$ is the cumulative equity issuance. Both $D$ and $I$ are non-decreasing processes. Cash earns a return equal to the risk free rate ($r$) net of a carry cost of holding cash ($\lambda_c$).\(^{11}\) Even though cash earns a lower rate of return, the firm holds cash for precautionary reasons to lower the expected issuance or default costs if it runs out of liquid funds. The firm therefore manages an optimal cash policy to trade off the risk management benefits of maintaining a cash reserve against the dividend payout delay.

Equity issuance is costly. For a lump sum issuance of size $I$, the cost is

$$\lambda(I) = \lambda_f + \lambda_p I,$$

where $\lambda_f$ and $\lambda_p$ are constants, representing the constant component and the proportional parameter. This is the third modelling difference with BCW, where their fixed issuance cost $\lambda_f$ is proportional to the capital size, resulting in a constant ratio between the fixed issuance cost and capital ($\lambda_c/k$).\(^{12}\) In our model, this ratio is decreasing in capital.

Even if firm does not payout cash nor invest, it can run out the cash reserve due to negative productivity shocks. When this happens, the firm compares the benefit of equity issuance and continuing running with liquidation and paying off the debt. If the latter out weighs the former, the firm default. Therefore, the default time of the firm is

$$\tau = \inf\{t \geq 0 : c_t < 0\}.$$

When the firm defaults, its capital stock $k_\tau$ is fire sold. The recovery rate $\ell$ is assumed to

\(^{11}\)If $\lambda_c = 0$, then the cash in the firm earns the rate of return the same as the discounting rate, and the firm trivially finds it optimal to hold as much cash as it can (indefinitely postponing the dividend) to prevent costly equity issuance. The equity is still valuable because equityholders could always choose to extract the cash via a dividend. The more realistic case is when $\lambda_c > 0$. Cash may earn low returns because: (1) Interest earned on a firm’s cash holdings is taxed at the corporate tax rate, which generally exceeds the personal tax rate (Graham, 2000; Faulkender and Wang, 2006). Agency problems may lower cash returns (Jensen, 1986; Harford, 1999; Dittmar and Shivdasani, 2003; Pinkowitz et al., 2006; Dittmar and Mahrt-Smith, 2007; Harford et al., 2008; Caprio et al., 2011; Gao et al., 2013).

\(^{12}\)This assumption is due to tractability reason for their variable reduction method. Nevertheless they acknowledge that “in practice, external costs of financing scaled by firm size are likely to decrease with firm size.”
be constant. The liquidation value \( \ell k_\tau \) is used to pay off the long term debt with the face value \( b/r_{\text{debt}} \), where \( r_{\text{debt}} \) is the cost of financing for long term debt. If there is any value after paying the long term debtholder, the remaining value, \( (\ell k_\tau - b/r_{\text{debt}})_+ \) is distributed to the equityholders.\(^\text{13}\)

2.1. Firm’s problem

The firm chooses investment, dividend payout, and equity issuance to maximize the expected value of future dividend payout net of equity issuance costs:

\[
\sup_{i \geq 0, D, \{\sigma_j, I_j\}} \mathbb{E} \left[ \int_0^\tau e^{-r(s-t)} dD_s - \sum_{j} e^{-r\sigma_j} (I_j + \lambda_c(I_j)) + 1_{\{\tau < \infty\}} e^{-rt} (\ell k_\tau - b/r_{\text{debt}})_+ \right]
\]

where \( \{\sigma_j\} \) is a sequence of stopping time when the lump sum of equity of size \( I_j \) is issued at each \( \sigma_j \).

The capital stock size and the cash reserve value \( k \) and \( c \) are the two state variables for the firm’s problem. The firm’s value function is:

\[
V(k_t, c_t) = \sup_{i \geq 0, D, \{\sigma_j, I_j\}} \mathbb{E}_t \left[ \int_t^\tau e^{-r(s-t)} dD_s - \sum_{\sigma_j \geq t} e^{-r(\sigma_j-t)} (I_j + \lambda_c(I_j)) \right.
\]

\[
+ 1_{\{\tau < \infty\}} e^{-r(\tau-t)} (\ell k_\tau - b/r_{\text{debt}})_+ \left. \right]\]

It follows from the dynamic programming that the value function \( V \) satisfies the HJB equation:

\[
0 = \min \left\{ rV - \sup_{i \geq 0} \left\{ [i - \delta k] \partial_k V + \left[ (r - \lambda_c)c + k^\alpha \mu - b - i - g(k, i) \right] \partial_c V + \frac{1}{2} k^{2\alpha} \sigma^2 \partial^2_{cc} V \right\}, \right. \]

\[
\partial_c V - 1, V(k, c) - \sup_{I \geq 0} \left[ V(k, c + I) - I - \lambda(I) \right] \right\}
\]

In the equation above, the firm chooses among three alternatives: continuation (the group of terms on the first line of the right-hand side), dividend payout (the first group of terms in the second line), and equity issuance (the second group in the second line).

Regarding the continuation term, \( rV \) represents the required rate of return on equity,

\(^{13}a_+ = \max\{a, 0\} \).
which just equals the risk free rate demanded by risk neutral investors. The term $\partial_k V$ is firm’s marginal benefit of capital, hence $[i - \delta k] \partial_k V$ captures the marginal effect of net investment on equity value. The term $\partial_c V$ is firm’s marginal cost of cash, hence $\left( (r - \lambda_c) c + k^\alpha \mu - b - i - g(k, i) \right) \partial_c V$ is the effect of a firm’s expected savings on equity value. The term $\frac{1}{2} k^{2\alpha} \sigma^2 \partial_{cc}^2 V$ captures the effect of the volatility of cash holdings due to volatility in production on equity value.

Regarding the dividend payout term, the firm postpones dividend payout when the marginal cost of cash outweighs the unit marginal benefits of dividend payout, i.e., $\partial_c V - 1 > 0$. The firm only pays dividend when the previous marginal cost and benefit are equalized.

Regarding the issuance term, at each point $(k, c)$ in the state space, the firm compares no issuance value $V(k, c)$ with the best value for issuance $\sup_{I \geq 0} \left[ V(k, c + I) - I - \lambda(I) \right]$, where $V(k, c + I) - I - \lambda(I)$ is the firm value post issuance net issuance costs. The firm only issues equity when the later value is strictly larger.

The dynamic programming principal implies that all three groups are nonnegative and only one group equals to zero at each point $(k, c)$ in the state space and the corresponding action is optimal for the firm.

The boundary condition at $c = 0$ is determined by comparing the default or issuance value:

$$0 = \min \left\{ V(k, 0) - \left( \ell k - b/r_{\text{debt}} \right)_+, V(k, 0) - \sup_{I \geq 0} \left[ V(k, I) - I - \lambda(I) \right] \right\}.$$  \hspace{1cm} (10)

In the equation above, the boundary value $V(k, 0)$ dominates the default value $\left( \ell k - b/r_{\text{debt}} \right)_+$ and the best issuance value $\sup_{I \geq 0} [V(k, I) - I - \lambda(I)]$, and only equals to one for each value of $k$. If $V(k, 0)$ equals to the former value, it is optimal for the firm to default; otherwise, issuance is optimal with an optimal size. Other more technical boundary conditions are described in the appendix.
3. The Model Solution

In this section, we present and discuss the results of our model for a baseline set of parameters. Then we vary the different parameters to examine their impact on model predicted firm’s behavior.

3.1. Numeric results

The baseline values for the parameters are presented in Table 1 along with rationale.

[Insert Table 1 Here]

Figure 1 depicts firm’s choices for the baseline set of parameters. In Figure 1a, there are several grey regions. The light grey region is the dividend payout region. The medium and darker grey regions are the investment regions. The white region is the continuation region, where the firm retains its earning, does not payout dividend, nor invest in capital. Figure 1b shows a heat map of net investment. We now discuss these regions in more detail.

[Figure 1 Here]

Dividend payout. Let us first focus on the dividend payout boundary ABE first. The firm only pays out a dividend when the marginal cost of reducing the cash reserve matches the marginal benefit of dividend payout. When \( c \) is higher than the boundary ABE, the marginal value of cash equals to one which is the marginal benefit of dividend payout. As a result, when \( c \) is higher than the boundary ABE, a lump sum dividend is paid out so that the state process \((k, c)\) lands on ABE exactly after dividend payout. When the state process \((k, c)\) reaches the boundary ABE from below, minimal dividend is paid out to reflect the state process below so that the state process remains lower than ABE.

The firm’s dividend payout boundary depends on the firm’s capital position \( k \). Figure 1 shows that it first decreases and then increases with \( k \). The intuition is that when \( k \) is low, the firm generates little cash flows. In order to invest in the capital and pay the bond coupon, the firm needs a large cash buffer to avoid costly default and equity issuance. Therefore,
the firm chooses a high dividend payout boundary. As $k$ increases, cash flows generated by production partially substitute for the cash reserve to pay the coupon, fund invest, and mitigate costly issuance or default; hence, the need for the cash reserve decreases in capital, resulting in a lower dividend payout boundary. However, equation (2) indicates that both the expected cash flows and their volatility are proportional to $k^\alpha$. As $k$ increases beyond a certain level, the volatility effect dominates, introducing an increasing likelihood of costly issuance or default. Therefore, a deeper cash buffer is needed hedge against the larger cash flow variations.

**Strategic default.** One novel prediction of our model is the existence of a strategic default region, which is the light grey region to the left of the boundary AP in Figure 1. When the state process $(k, c)$ reaches the boundary AP, rather than continuing the firm, equityholders pay out the remaining cash reserve as a lump sum dividend and default afterward. The existence of the strategic default region is ensured by the following result.

**Proposition 1.** When $b$ is higher than a threshold, then the strategic default region exists when $k$ is sufficiently low.

Intuitively, when the debt coupon is high but capital is low, even though the marginal productivity of capital is high, it takes substantial investment and time to build up capital and increase cash flows to service the debt. Comparing with continuation, equityholders find more attractive to payout the remaining cash immediately and then liquidate the firm. Proof of Proposition 1 in Appendix A shows that the threshold is an increasing function of the expected productivity $\mu$. Therefore, firms with lower expected productivity are more likely to have a strategic default region.

**Investment.** Due to diminishing returns to scale and the risk of issuance costs, the optimal investment depends on the size of capital and cash. In the medium grey region, investment $i$ is larger than the depreciation $\delta k$, so that the capital is built up and the firm grows. In the dark grey region, the firm still invests, i.e., $i > 0$, but it is less than the depreciation $\delta k$. Figure 1b provides a heat map of the net investment $i - \delta k$. Evidently, as capital increases
net investment decreases because the marginal returns to investing are lower. In the white region, the firm does not invest nor payout a dividend. Hence, in the dark grey and white regions, capital declines and the firm shrinks. The interface between the medium and dark grey regions, the boundary BGO in Figure 1a, is where investment exactly offsets depreciation - a stationary region for capital size, because capital grows on the left of the boundary BGO and shrinks on the right. This continuum of stationary points, where investment equals depreciation, arises because the optimal exposure to the productivity shock depends on the cash reserve. Holding capital fixed, the continuation region emerges when cash decreases because the marginal value of cash balloons as the cash reserve approaches zero because the likelihood of paying the cost of issuance or default is increasing. Therefore, preserving cash to mitigate costly issuance or liquidation becomes more important than investment. When the capital is close to the strategic default boundary AP, the firm also scales back investment to preserve cash to prepare for the imminent dividend payout and the strategic default.

**Equity issuance and default.** Even though equation (9) indicates that for each point \((k, c)\) in the state space, the firm compares no issuance value with the best value for issuance. The next result shows that the firm optimally issues equity only when its cash reserve is run out.

**Proposition 2.** *It is without loss of generality that firms only consider issuance when the cash reserve reaches zero.*

Since the firm faces a fixed cost of financing, it chooses to raise equity infrequently. It will optimally raise cash only when it has to. First, cash within the firm earns a below market rate \(r - \lambda_c\). Second, there is time value for external financing costs. Therefore, without any benefit for early issuance, it is always better to defer external financing when \(c > 0\). The above argument highlights the pecking order between cash and external financing in the model.\(^{14}\) When \(c\) reaches zero, the firm compares the best issuance value with the liquidation

\(^{14}\)With stochastic financing cost or stochastic arrival of growth options, the firm may time the market by raising cash in times when financing costs are low. See Bolton, Chen, and Wang (2011).
value to determine issuance or default. In Figure 1a, the issuance boundary is the interval QR on the horizontal axis and the default boundary is the interval PQ. When $c$ reaches zero, if $k$ lands on the interval QR, the firm issues a lump sum equity whose size is specified by the curve FGH. We call this curve issuance target. The lumpiness of issuance is to economize fixed issuance costs. After issuance, the state process jumps upward and lands on FGH before continuing its dynamics. If $k$ lands on the interval PQ when $c$ reaches zero, then the firm chooses not to issue and instead defaults.

3.2. Comparative statics

This section examines how a firm’s choices vary when we adjust the assumed values of specific parameters. We specifically vary the coupon rate, expected productivity, the volatility of the productivity shock, the issuance costs, and the cash liquidity premium.

3.2.1. Impact of leverage

As the cash coupon, $b$, increases, the burden from debt becomes increasingly heavier. Figure 2 shows how the debt burden alters firm’s choices in a few ways.

First, when $b$ increases, the strategic default region expands. Specifically, the light grey stripe in the lower left corner grows taller and thicker. In Figure 2 panel (c), it connects with the dividend payout region on top of the dividend payout boundary. Moreover, increasing burden of debt shift the issuance boundary to the right so that when a low capital firm runs out of cash it is more likely to choose default rather than issuance. (See Internet Appendix C.2 for empirical support.)

Second, as $b$ increases, debt overhang emerges as equityholders of distressed firms are less willing to invest than firms with similar characteristics but with less debt. Specifically, as Figure 2 panel (d) shows, as capital decreases, investment decreases as the dark grey region begins to envelop the mid grey region. Moreover, a continuation (white) region emerges between the strategic default region and investment regions. This continuation
region first emerges as a white stripe in panel (c) and expands dramatically in panel (d). This underinvestment leads to lower capital levels in the short-term decreasing firm cash flows, but it also preserves the cash reserve which equityholders can either seize before strategic default or use to delay costly liquidation or issuance. This underinvestment problem resembles the debt-overhang agency problem in which equityholders choose not to fund positive NPV investments because the required funds come out of the equityholders resources (cash reserves or new issuances) but the benefits are shared with the debtholders in the form of more capital at the time of bankruptcy.

Third, when capital $k$ is low, Figure 2 panels (a) to (c) show that increases in leverage increase sharply the dividend payout boundary and the equity issuance target. By contrast, when $k$ is high, the dividend payout boundary and issuance target are less sensitive to the coupon $b$. The intuition is that when $k$ is low, there is less cash flow generated, so the firm needs a large cash buffer to fund the higher coupon. When $k$ is high, cash flow generated by production can better support the debt coupon payment, reducing the need for a cash buffer.

3.2.2. Impact of expected productivity

As $\mu$ decreases, the expected productivity and cash flows of the firm decrease. Figure 3 shows how lower expected cash flows change firm’s choices (moving from 3c to 3a).

First, because the expected returns to capital are lower, the investment region (combination of medium and dark grey regions) shrinks in size.

Second, the default region (including the strategic default region) grows and the issuance boundary shifts to the right. Intuitively, when capital and profitability are low, the benefits of issuing equity to invest declines. Consequently equityholders find it more attractive to default when cash reaches zero or distribute remaining cash and default strategically.

Third, lower expected cash flows lower the dividend payout boundary. When a firm is less profitable, and the firm is less willing to invest, then equityholders find it more optimal to
extract more cash from the firm, resulting in lower cash balances and earlier dividend payout. Lower cash balances increase the risk of default, because it is harder for the firm to recover from bad productivity shocks, but the lost capital at default is also less valuable.

Fourth, the white continuation region wraps underneath the investment region. Evidently, when profitability is low, firms find it optimal to start investing only when cash balances are sufficiently high. Low profitability firms with low capital require higher cash balances before investing because default risk is higher and equityholders prefer to have cash than capital when default or issuance is imminent.

3.2.3. Impact of changing the fixed component of issuance costs

As $\lambda_f$ increases, the fixed costs of issuing equity increase. Figure 4 (moving from (a) to (d)) shows how higher fixed issuance costs change firm’s choices.

First, Figure 4 shows that fixed issuance costs drive the issuance boundary of the firm. Specifically, when there are no fixed issuance costs, Figure 4a shows that the issuance boundary collapses to the x-axis. Intuitively, firms still value a cash reserve to hedge against costly issuance (there is still a proportional cost), but when the cash reserve reaches zero, firms raise just enough, leading the cash reserve back to the positive domain. In the presence of fixed issuance costs, the firm optimally issues equity in lumps to reduce the fixed issue costs. As the fixed issuance costs increase, the issuance boundary rises because firms have a larger incentive to avoid paying the fixed costs in future issuances.

Second, higher fixed issuance costs incentivize a higher issuance target and a higher dividend payout boundary. Intuitively, if fixed issuance costs are larger, then the firm optimally holds more cash in any given issuance. Similarly, Figure 5 shows that without fixed issuance costs, increases in the proportional issuance costs ($\lambda_p$) increase the dividend boundary because hedging against costly issuance becomes more important.
Third, higher fixed issuance costs move the equity issuance boundary to the right, expanding the default region, because higher issuance costs increase the future costs to run the firm, hence reduce the incentive for equityholders to invest and build up the capital.

3.2.4. Impact of production volatility

Figure 6 shows how changes in the volatility of the productivity shocks, $\sigma$, alters firm’s choices.

[Figure 6 Here]

When productivity shocks are more volatile, the dividend payout boundary and the equity issuance boundaries increase. Equityholders facing more volatility demand higher cash reserves to hedge against the possibility of larger negative cash flow shocks that could lead to costly issuance. Relatedly, conditional on issuance, equityholders raise more capital. This effect is more pronounced when capital $k$ is high because the nominal size of the productivity shock is larger for higher capital levels.

3.2.5. Impact of the liquidity premium $\lambda_c$

Figure 7 shows how firm’s choices vary with the liquidity premium $\lambda_c$.

[Figure 7 Here]

When the liquidity premium is lower (the left panel of Figure 7), the dividend boundary is higher. Intuitively, lower liquidity premium reduces the difference between the net rate of return for cash and the equityholder discounting rate. Hence early dividend payout becomes less attractive and the dividend payout boundary increases.

4. Empirical Support

The analyses thus far has been mostly theoretical. In the preceding sections, we have built a dynamic model of the firm with two states - cash and capital - and related the predicted dynamics to the existing empirical literature. This section provides empirical support for the model’s key predictions about firms’ choices of issuances, payouts, and investment.
4.1. Data

The primary data source is the annual Compustat data file, providing detailed financial statement information on public firms from 1962 through 2017. After filtering the data there are 6,731 firms and 93,822 firm-years. (See Internet Appendix Table C1 for details on the filtering.) For certain tests, the data series is more limited in time frame because of availability of the data.

The two primary state variables in the model are a firm’s cash and capital positions. To proxy for the cash state variable, we use a firm’s cash and cash equivalents (che) from the fiscal year end balance sheet. The capital position includes the tangible capital on the balance sheet, the intangible capital on the balance sheet, and the intangible capital not on the balance sheet per Peters and Taylor (2017).

Other variables of interest include a firm’s issuance, payout, and net investment activity.

To proxy for a firm’s issuance activity, we use a firm’s total sale of common stock listed on the cash flow statement (sstk, available starting 1971).

To proxy for a firm’s payout activity, we primarily use a firm’s annual dividend (dvt) because share repurchase data (cshopq) is only available starting in 2004. However, in one specification we also show our results hold in the more limited sample for which repurchase data is available.

To proxy for a firm’s net investment activity, we use proxies for a firm’s investment in physical and intangible capital. The measure of expenditures on physical capital is a firm’s capital expenditures on property, plant, and equipment from the cash flow statement (capx). The measure of expenditures on intangible capital follows from Hulten and Hao (2008), Eisfeldt and Papanikolaou (2014), and Peters and Taylor (2017). Specifically, a firm’s

---

15 The original data file is from 1950 to 2021. However, our main explanatory variable - total capital - is only available for 1960 to 2017 from WRDS (Peters and Taylor, 2017).
16 The amount repurchased in a year is the sum of the quarterly amounts repurchased in the quarterly Compustat file. CSHOPQ refers to the total shares repurchased in a quarter. We multiply this amount by the average per share purchase price in the quarter. In the few cases where an average purchase price is unavailable, we use the close price for the quarter.
expenditures on intangible capital is a firm’s R&D ($x_{rd}$ plus $rd_{ip}$) plus 30% of a firm’s SG&A ($x_{sga}$) minus R&D expenses ($x_{rd}$) minus in-process R&D ($rd_{ip}$). When $x_{rd}$ exceeds $x_{sga}$ but is less than the cost of goods sold ($cogs$), or when $x_{sga}$ is missing, then we measure SG&A as $x_{sga}$ with no further adjustments or zero if missing. Both R&D and SG&A are set to zero if the corresponding data elements on Compustat are missing. We compare the investment amount to the depreciation on the firm’s physical capital ($dp$) and intangible capital. Like the literature, we assume that the depreciation on intangible capital is 20% of the stock of intangible capital.

4.2. The amount issued as a proportion of capital is declining and convex in capital

Figure 8a shows that the amount issued as a proportion of capital is declining and convex in capital. Contributing factors to this phenomenon are non-linear issuance costs and diminishing returns to scale.

First, as discussed in Section 3.2.3, the fixed issuance costs create incentives for lumpy issuance and thus drive the issuance target. The issuance target scaled by capital is declining because the fixed issuance costs as a proportion of capital are declining in capital. In other words, for low levels of capital, the fixed component of issuance costs represents a larger proportion of firm value. Thus, conditional on issuing, firms with low capital have a stronger precautionary motive to issue more equity to reduce the possibility of paying the fixed issuance costs again in the future.

Second, firms with low capital have strong incentivizes to invest because marginal returns are higher. Additionally, when capital is low, cash reserves play a larger role in supporting investment than cash flows, which puts downward pressure on the cash reserve. As capital increases, cash flows fund a larger proportion of investment. Thus, when capital is low, issuing more equity enables the firm to capitalize on the higher investment returns while managing the risks of future fixed issuance costs. Altogether, the declining issuance costs and declining

\footnote{R&D is substracted from SG&A because Compustat adds R&D expenses to SG&A. Also, the literature interprets the remaining 70% of SG&A as operating costs that support the current period’s profits.}
incentives to invest create a convexity in the amount a firm issues as a proportion of capital.

This reasoning leads to the following hypothesis:

**H1:** The amount issued as a proportion of capital is declining and convex in capital.

To evaluate this hypothesis, we estimate the following empirical specification.

\[
\frac{\text{Amt Raised}_{i,t+1}}{\text{Capital}_{i,t}} = \beta_1 \text{Capital}_{i,t} + \beta_2 \text{Capital}_{i,t}^2 + \beta_3 \text{Cash}_{i,t} + \mu_i + \delta_{j,t} + \epsilon_{i,t}
\]  

(11)

The outcome is the proceeds from common stock sales by firm \(i\) in year \(t+1\) scaled by firm \(i\)'s total capital (tangible plus intangible) at the end of year \(t\) (See Section 4.1 for details). The main explanatory variable is a firm’s total capital at the end of year \(t\) standardized within firm and winsorized at the 1% level. \(\beta_2\) multiplies the quadratic form of capital (standardized). We control for a firm’s cash and equivalents, firm fixed effects \(\mu_i\), and SIC-2–by–year industry trends \((\delta_{j,t})\). \(\epsilon_{i,t}\) is the unexplained variation. Standard errors are double clustered by firm and year. The sample begins in 1971 because of availability of issuance proceeds data. Additionally, the sample is restricted to firm-years in which more than 5% of capital is sold.

Table 2 presents results consistent with **H1**. Column (1) shows the full sample regression. \(\beta_1\) multiplying a firm’s capital position is -4.97 and highly significant. \(\beta_2\) multiplying the quadratic form of a firm’s capital is +2.75 and highly significant. Together, there is strong evidence that the amount raised scaled by capital is declining in capital in a convex manner. Columns (2) and (3) show that the relation holds similarly when splitting the sample by the median year for the issuance sample of 1997. Column (4) shows larger coefficients when the sample is restricted to offerings exceeding 10% of capital, rather than 5% of capital. Columns (5) to (11) show the results hold similarly across SIC-1 industry classifications.
Figure 8b complements the regression results. The bincscatter plot shows a convex pattern between scaled issuance and within firm capital.

4.3. The amount paid out to equityholders is concave in capital

Figure 8c shows that, holding cash fixed, a firm’s optimal payout amount scaled by capital is concave in capital. Intuitively, as capital increases, the marginal product increases at a diminishing rate because of the diminishing returns to capital. Thus, for each additional unit of capital, there are fewer incremental cash flows generated to distribute so that total proceeds scaled by capital is declining. However, in the region of low capital, as capital increases the larger cash flows substitute for the role of the cash reserve in funding investment. Thus, as capital increases, the optimal cash reserve declines allowing for larger payouts. Together, these dynamics predict a concave relation between scaled payouts and capital.

This reasoning leads to the following hypothesis:

**H2:** The total payouts to equityholders scaled by capital is concave in capital.

To evaluate this hypothesis, we estimate the following empirical specification:

\[
\frac{\text{Payout}_{i,t+1}}{\text{Capital}_{i,t}} = \beta_1 \text{Capital}_{i,t} + \beta_2 \text{Capital}_{i,t}^2 + \beta_3 \text{Cash}_{i,t} + \mu_i + \delta_{j,t} + \epsilon_{i,t}
\] (12)

The outcome is the total dividends paid by firm \(i\) in year \(t + 1\) scaled by firm \(i\)’s total capital (tangible plus intangible) at the end of year \(t\) (See Section 4.1 for details). The main explanatory variable is a firm’s total capital at the end of year \(t\) standardized within firm and winsorized at the 1% level. \(\beta_2\) multiplies the quadratic form of capital (standardized). We control for a firm’s cash and equivalents, firm fixed effects \(\mu_i\), and SIC-2–by–year industry trends \(\delta_{j,t}\). \(\epsilon_{i,t}\) is the unexplained variation. Standard errors are double clustered by firm and year.

Table 3 presents the results. Column (1) uses the full sample of firms. \(\beta_1\) multiplying a firm’s capital position is -0.19 and highly significant. \(\beta_2\) multiplying the quadratic form...
of a firm’s capital is -0.09 and also highly significant. Together, there is strong evidence
that payouts scaled by capital is concave in capital. Columns (2) and (3) show that the
concave relation may be stronger in the more recent half of the sample. Column (4) shows
larger coefficients when payouts include dividends and repurchases (sample starts in 2004).
Columns (5) to (11) show the concavity mostly exists across SIC-1 industry classifications,
with the exception of SIC=1 (Mining and Construction).

[Table 3 Here]

Figure 8d complements the regression results. The binscatter plot shows a concave pattern
between payouts scaled by capital and within firm capital.

5. Conclusion

Our two-dimensional model of the firm provides new insights into how cash and capital
jointly determine a firm’s optimal dynamics in the presence of costly financing and bankruptcy.
We present the solution and proofs of convergence in Appendix A and Appendix B. The
model’s predictions connect many existing empirical studies and also provide novel predictions
for future work. We examine and find strong support for two of these novel predictions
related to the non-linearities in issuance and payout policies.
References


Han, Seungjin, and Jiaping Qiu, 2007, Corporate precautionary cash holdings, *Journal of Corporate Finance* 13, 43–57.


Figure 1: Optimal regions

Parameters used are summarized in Table 1.
Figure 2: Impact from $b$

Other parameters used are summarized in Table 1.
Figure 3: **Impact from $\mu$**

Other parameters used are summarized in Table 1.
Figure 4: Impact from fixed issuance cost

Other parameters used are summarized in Table 1.
Figure 5: No fixed issuance cost, $\lambda_f = 0$

*Other parameters used are summarized in Table 1.*
Figure 6: Different $\sigma$

Other parameters used are summarized in Table 1.
Figure 7: Different $\lambda_c$ (Cash liquidity premium)

*Other parameters used are summarized in Table 1.*
Figure 8: Predicted Issuance, Payout and Investment from Figure 1 vs. Actual

(a) Figure 8a is the issuance target in Figure 1 scaled by capital. Y-axis is the amount raised scaled by capital; X-axis is capital. Here $b = 0.02$ and other parameters used are summarized in Table 1.

(b) Offered amount is the value of stock sold in year $t + 1$ scaled by capital at the end of year $t$. The x-axis is a firm’s total capital (tangible and intangible) standardized within firm. Sample is offerings in year $t + 1$ with proceeds greater than or equal to 5% of capital stock at the end of year $t$.

(c) Figure 8c shows the dividend amount in Figure 1 scaled by capital at $c = 0.15$. Here $b = 0.02$ and other parameters used are summarized in Table 1.

(d) The y-axis is the total payout amount of dividends in year $t + 1$ scaled by capital at the end of year $t$. The x-axis is a firm’s total capital (tangible and intangible) standardized within firm.
Table 1: Model Parameters

<table>
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<tr>
<th>Parameter</th>
<th>Name</th>
<th>Values</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>Interest rate</td>
<td>6%</td>
<td>Same as Bolton et al. (2011).</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Cash holding cost, liquidity premium</td>
<td>1%</td>
<td>Same as Bolton et al. (2011).</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Expected productivity shock</td>
<td>18%</td>
<td>Same as Bolton et al. (2011). In line with the estimates of Eberly et al. (2009) for large U.S. firms.</td>
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<td>$\sigma$</td>
<td>Volatility of productivity shock</td>
<td>9%</td>
<td>Same as Bolton et al. (2011). In line with the estimates of Eberly et al. (2009) for large U.S. firms.</td>
</tr>
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<td>$\theta$</td>
<td>Degree of adjustment cost</td>
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<td>Same as Bolton et al. (2011). See Whited (1992).</td>
</tr>
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<td>$\delta$</td>
<td>Depreciation rate</td>
<td>10.07%</td>
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<td>Variable issuance cost</td>
<td>6%</td>
<td>Same as Bolton et al. (2011).</td>
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<tr>
<td>$\alpha$</td>
<td>Curvature of production function. When $\alpha &lt; 1$, then diminishing returns to scale</td>
<td>0.7</td>
<td>$\alpha = 0.75$ in Riddick and Whited (2009) and $\alpha = 0.627$ with std 0.219 for the full sample of firms in Hennessy and Whited (2007).</td>
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<tr>
<td>$\ell$</td>
<td>Recovery rate in liquidation of capital</td>
<td>90%</td>
<td>The choice of $\ell$ is consistent with Hennessy and Whited (2007), where the recovery rate is estimated to be 0.896 for the full sample of firms.</td>
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<tr>
<td>$r_{\text{debt}}$</td>
<td>Cost of financing for long-term debt</td>
<td>9%</td>
<td>$r_{\text{debt}}$ is chosen as 1.5$r$. Results are insensitive to this parameter, because firms are left with no proceed after the bondholder is repaid after liquidation in most of numeric experiments.</td>
</tr>
</tbody>
</table>
Table 2: H1: Amount issued as a proportion of capital is declining and convex in capital
The outcome variable is the amount of common stock sold in year $t + 1$ divided by the total capital of the firm (tangible plus intangible) at the end of year $t$. The sample is all offerings that exceeded 1% of capital. The main explanatory variable is the total capital of the firm (standardized within firm) and its square (standardized). We control for the cash reserve (standardized within firm), industry trends, and year fixed effects. Columns (2) and (3) split the sample after and on or before 1997, respectively. Column (4) restricts the sample to larger offerings—those exceeding 10% of capital. Columns (5) to (11) restrict the sample to industries based on the SIC-1 identifier. Standard errors are double clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

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<th>Yr≤97</th>
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<td>Yes</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
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</tr>
<tr>
<td>% Adjusted $R^2$</td>
<td>30.81</td>
<td>34.52</td>
<td>29.06</td>
<td>31.87</td>
<td>37.26</td>
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<td>30.70</td>
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<td>6.05</td>
<td>6.99</td>
<td>8.32</td>
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<td>7.40</td>
<td>7.11</td>
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<td>Observations</td>
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<td>3731</td>
<td>3351</td>
<td>4079</td>
<td>774</td>
<td>1152</td>
<td>410</td>
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<table>
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<th>Amt Raised (t+1) / Capital (t) × 100</th>
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<td>Capital (t)</td>
<td>-7.17***</td>
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<tr>
<td>Capital (t)^2</td>
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<tr>
<td>Cash (t)</td>
<td>-3.93***</td>
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</table>

| | Specification | All | Yr>97 | Yr≤97 | Ob>10% | SIC1=1 | SIC1=2 | SIC1=3 | SIC1=4 | SIC1=5 | SIC1=7 | SIC1=8 |
|---------------|-----|-------|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| Firm FE       | Yes | Yes   | Yes   | Yes    | Yes    | Yes    | Yes    | Yes    | Yes    | Yes    | Yes    |
| SIC-2x FE     | Yes | Yes   | Yes   | Yes    | Yes    | Yes    | Yes    | Yes    | Yes    | Yes    | Yes    |
| % Adjusted $R^2$ | 30.81 | 34.52 | 29.06 | 31.87 | 37.26 | 23.40 | 24.98 | 32.46 | 30.70 | 39.26 | 34.71 |
| % Within $R^2$ | 7.31 | 6.05 | 6.99 | 8.32 | 15.30 | 7.40 | 7.11 | 8.38 | 7.04 | 9.54 | 2.90 |
| Observations  | 7685 | 3731 | 3351 | 4079 | 774 | 1152 | 410 | 546 | 1465 | 478 |
Table 3: H2: The total payouts to equityholders scaled by capital is declining and concave in capital
The outcome variable is the total dividends paid in year $t+1$ divided by the total capital of the firm (tangible plus intangible) at the end of year $t$. The main explanatory variable is the total capital of the firm (standardized within firm) and its square (standardized). We control for the cash reserve (standardized within firm), industry trends, and year fixed effects. Columns (2) and (3) split the sample after and on or before 1996, respectively. In column (4), the outcome variable is dividends plus share repurchases; however, the repurchase data is only available starting in 2004. Columns (5) to (11) restrict the sample to industries based on the SIC-1 identifier. Standard errors are double clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<td>Yes</td>
<td>Yes</td>
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<tr>
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<td>60.53</td>
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Appendix A. Omitted proofs

We first present a more rigorous formulation of Proposition 1. The following result provides a sufficient condition for a strategic default region to exist. This result shows that a strategic default region exists when the debt coupon rate $b$ is higher than a threshold, which is an increasing function of the expected productivity $\mu$. Therefore, firms with lower expected productivity are more likely to have a strategic default region. This result also provides the existence of a strong type of strategic default region, for which strategic default is optimal regardless of initial cash reserves.

**Proposition 3.** For $b > \mu(1 - \alpha)(\mu \alpha / (r + \delta))^{\alpha/(1 - \alpha)}$, there is a region $[0, \infty) \times [0, k]$ where strategic default is optimal for all $c$, i.e., $V(c, k) = c$ for $k \in [0, k]$.

**Proof.** Consider the following optimization problem

$$V_R(k, c) = c + V_R(k) = c + \sup_{\tau, i \geq 0} \mathbb{E} \left[ \int_0^\tau e^{-rt} \left( k^\alpha \mu - b - i_t - g(k_t, i_t) \right) dt + 1_{\{\tau < \infty\}} e^{-(r - t)(\ell k^\tau - b/r_{\text{debt}})} \right].$$

This is the optimization problem for a firm which is not subject to external financing costs nor cash liquidity premium, and can invest and choose its default time optimally, i.e., a real option. This value function $V_R$ dominates the value function $V$ in (8), i.e., $V(k, c) \leq V_R(k, c)$ for any $(k, c)$. The equation for $V_R(k)$ is given by dynamic programming:

$$0 = \min \left\{ rV_R - \sup_{i \geq 0} (i - \delta k) V'_R + k^\alpha \mu - b - i - g(k, i), V_R - (\ell k - b/r_{\text{debt}}) \right\}. \quad (A.1)$$

We assume that this equation satisfies the comparison principle. This can be proven as in Proposition 4, which proves comparison for a more complicated but related case.

Let

$$v(k) = (k - k)_+$$

for some $k > 0$. We show that this is a viscosity supersolution to (A.1) for some sufficiently small $k$. First, we restrict $k$ so that $k \leq b/r_{\text{debt}}$, then clearly $v - (\ell k - b/r_{\text{debt}})_+ \geq 0$. Let $\phi$ be any $C^1$ function such that $v - \phi$ attains a local minimum at $k$. Then,

$$\phi'(k) \in \begin{cases} \{0\}, & k < k \\ [0, 1], & k = k \\ \{1\}, & k > k \end{cases}$$

For all these cases, because $\phi' \geq 1$, we obtain

$$r(k - k)_+ - k^\alpha \mu + b - \sup_{i \geq 0} (i - \delta k) \phi' - i - g(k, i)) = r(k - k)_+ + \delta k \phi' - k^\alpha \mu + b. \quad (A.2)$$

When $k < k$, the previous expression is

$$b - k^\alpha \mu,$$
which is non-negative for \( k \leq (b/\mu)^{1/\alpha} \).

When \( k > k_* \), the right-hand side of (A.2) is \( r(k - k) + \delta k - k^\alpha \mu + b \). This expression is minimized at \( ((r + \delta)/\mu \alpha)^{(1/(\alpha - 1))} \) with the minimum value

\[
(r + \delta)(1 - 1/\alpha)(\mu \alpha/(r + \delta))^{1/(\alpha - 1)} - r k + b.
\]

If

\[
b > (r + \delta)(1/\alpha - 1)(\mu \alpha/(r + \delta))^{1/(\alpha - 1)},
\]

we can choose a sufficiently small \( k > 0 \) so that the previous minimum value is still nonnegative.

Finally, when \( k = k_* \),

\[
\delta k \phi' - (k)^\alpha \mu + b \geq -(k)^\alpha \mu + b \geq 0
\]

is satisfied if we choose \( k \leq (b/\mu)^{1/\alpha} \). Combining the previous three cases, we confirm that \( \nu \) is a viscosity supersolution to (A.1).

By the comparison principle, \( V_R \leq \nu \), so we have

\[
c \leq V(c, k) \leq c + V_R(k) \leq c + \nu(k) = c,
\]

for \( k \leq k_* \). The first inequality above holds because for problem (8), the firm can always pay out the remaining case and default. Hence, \( V(c, k) = c \) for all \( k \leq k_* \), which is attained by the immediate payout of all cash as dividends. \( \square \)

Next we prove Proposition 2.

**Proof of Proposition 2.** Because of the term \( (r - \lambda_c)c \) appearing in the dynamics for \( c \), the proof of Akyildirim et al. (2014) does not extend to this case. Instead, we define an equivalent control problem by letting

\[
\tilde{c}_t = e^{-(r-\lambda_c)t}c_t, \quad \tilde{D}_t = \int_0^t e^{-(r-\lambda_c)s}dD_s, \quad \tilde{I}_t = \int_0^t e^{-(r-\lambda_c)s}dI_s.
\]

Then,

\[
d\tilde{c}_t = e^{-(r-\lambda_c)t}dY_t - e^{-(r-\lambda_c)t}(b + i_t + g(k_t, i_t))dt - d\tilde{D}_t + d\tilde{I}_t,
\]

and the optimization problem is

\[
V(k, c) = \sup_{i \geq 0, \tilde{D}, \{\sigma, \tilde{I}_j\}} \mathbb{E} \left[ \int_0^\tau e^{-\lambda c t}d\tilde{D}_t - \sum_{\sigma_j \geq 0} e^{-\lambda c \sigma_j}(\tilde{I}_j + \lambda_p \tilde{I}_j + e^{-(r-\lambda_c)\sigma_j \lambda_f}) + 1_{\{\tau < \infty\}} e^{-r(\tau - t)}(\ell k - b/r_{\text{debt}})_{+} \right],
\]

where we note that \( \tau \) can be defined equivalently for \( c \) or \( \tilde{c} \).

For this alternative formulation of the control problem, we note that issuance can be delayed at a discount. To make things clear, we fix an investment strategy \( i \) and a dividend strategy \( D \), index \( \tilde{c}' \) by an issuance strategy \( \nu = \{\sigma_j, \tilde{I}_j\}_{j \in \mathbb{N}} \) with at least one issuance time say \( \sigma_i \) at which \( \tilde{c}'_{\sigma_i} > 0 \). Let \( \tilde{I}_i \) be the corresponding issuance amount. Consider another
issuance strategy \( \nu^- \) that omits this issuance, but keep the rest issuance strategy as \( \nu \). Finally, construct a third strategy \( \nu' \) like \( \nu^- \) but with an additional issuance of size \( \tilde{I} \) at a time \( \sigma' = \inf\{t > \sigma_i : \tilde{c}_t^- < 0\} \).\(^{18}\) Note that for the same \((i, D)\), the increment of \( \tilde{c}_t^\nu \) and \( \tilde{c}_t^\nu' \) are the same between \( \sigma_i \) and \( \sigma' \). With the same issuance size, we have \( \tilde{c}_t^\nu = \tilde{c}_t^\nu' \), so the continuation values must coincide, because the strategies are identical after \( \sigma'_i \). Moreover, dividends and issuance until \( \sigma' \) have been identical, with one exception, for which \( \nu' \) has resulted in a larger discounting factor and a smaller discounted fixed cost.

We therefore conclude that the original strategy is dominated by the one issuing equity only at \( c = 0 \). As this is true for any strategy, we may consider only strategies that issue equity when \( c = 0 \). \( \square \)

Appendix B. Comparison result and numeric algorithm

Proposition 2 lets us simplify the HJB equation where \( c \neq 0 \). We further restrict ourselves to the case where there is a maximal permitted investment rate \( i_{\text{max}} < \infty \). The proof here only relies on the boundedness of \( i_{\text{max}} \), but not on its size. We are now ready to state the resulting HJB equation. Let \( \mathcal{O} = (0, \infty) \times (0, k_{\text{max}}) \).

\[
0 = \min\left\{ rV - \sup_{i \in [0,i_{\text{max}}]} \left( [i - \delta k] \partial_k V + \left( (r - \lambda c) c + c^\alpha \mu - b - i - g(k,i) \right) \partial_c V + \frac{1}{2} k^{2\alpha} \sigma^2 \partial^2_{cc} V \right), \partial_c V - 1 \right\} \quad \text{in } \mathcal{O}. \tag{B.1}
\]

At \( k = 0 \), \( g(k,i) = \infty \) for any \( i > 0 \), i.e., investment is infinitely costly, so \( k \) will remain zero forever. Hence, at the boundary, the value function satisfies

\[
0 = \min\{rV - (r - \lambda c) c + b, \partial_c V - 1\},
\]

which has the solution \( V = c \). Consider therefore the following boundary condition (with precedence to the first one in the corner)

\[
c = V \quad \text{at } k = 0, \quad 0 = \min\{V - (\ell k - b/r_{\text{debt}}), V - \mathcal{I}V\} \quad \text{at } c = 0. \tag{B.2}
\]

Here \( \mathcal{I}(V) = V(0, k) - \sup_{I \geq 0} \left[ V(I, k) - I - \lambda(I) \right] \).

Theorem 4. Let \( u \) and \( v \) be, respectively, possibly discontinuous viscosity sub- and supersolutions to (B.1) with the above boundary conditions. Assume further that \( u \) and \( v \) are both of linear growth in \( c \) and polynomial growth in \( k \), i.e., they take values in \([c, c + M + p(k)]\) for some constant \( M > 0 \) and polynomial \( p \). Then \( u \leq v \) everywhere in \( \mathcal{O} \).

\(^{18}\) If \( \tilde{c}_t^- \) fell below zero due to a lump sum dividend payout, we can balance out the dividend payout and the issuance to obtain the same result in the next step. If there are multiple issuance in \( \nu \) between \( \sigma_i \) and \( \sigma' \), we omit all of them in \( \nu^- \) and issue the sum of all missed size at \( \sigma' \) in \( \nu' \).
Proof. Suppose there exists a point for which \( u > v \). Fix some \( \eta > 0 \) and consider a maximizing sequence \((c_n, k_n)_{n \geq 1}\) to \( \sup_{\mathcal{O}} e^{-\eta k}(u - v) > 0 \). By the growth condition, \( k_n \) is bounded by some \( k^* \), where \( k^* \) depends only on \( \eta \). Now, for any \( \zeta > 0 \), there exists a point \((\bar{c}, \bar{k})\) such that \( e^{-\eta k}(u - v)(\bar{c}, \bar{k}) = \delta \zeta \geq \sup_{\mathcal{O}} e^{-\eta k}(u - v) - \zeta > 0 \). We emphasize that \( \bar{k} \) remains bounded, irrespective of \( \zeta \). In particular, for any \( \eta, \delta \zeta/(\zeta + \sqrt{\zeta}) \) can be chosen arbitrarily large.

We begin by showing that if such a point lies on the boundary \( c = 0 \), then there is another with the same property on the interior. Consider points \((\bar{c}, \bar{k})\) such that \( \bar{c} = 0 \). Then depending on whether \( u(0, \bar{k}) \leq \bar{k} - \ell/r_{\text{debt}} \) or \( u(0, \bar{k}) \leq \mathcal{I}u \), we have

\[
(u - v)(0, \bar{k}) \leq \ell \bar{k} - b/r_{\text{debt}} - \max\{\ell \bar{k} - b/r_{\text{debt}}, \mathcal{I}v\} \leq 0
\]
or

\[
(u - v)(0, \bar{k}) \leq \mathcal{I}u - \max\{\ell \bar{k} - b/r_{\text{debt}}, \mathcal{I}v\} \leq \sup_{I > 0} [u(I, \bar{k}) - v(I, \bar{k})].
\]

The first case contradicts \( \delta \zeta > 0 \) and the second shows that another a point with the same properties exists in the interior. Similarly, for \( \bar{k} = 0 \), we also get \( (u - v)(\bar{c}, 0) \leq \bar{c} - \bar{c} \). Hence, without loss of generality, we may assume \((\bar{c}, \bar{k})\) lies away from \( c = 0 \) and \( k = 0 \).

Define

\[
\Phi^{c,\gamma}(c, k, d, \ell) = (1 - \gamma)e^{-\eta k}u(c, k) - e^{-\eta \ell}v(d, \ell)
\]

\[
- \beta(c - \bar{c})^4 - \frac{1}{2\epsilon}((c - d)^2 + (k - \ell)^2)
\]

in \( \mathcal{O} \times \mathcal{O} \).

Clearly,

\[
\sup_{\mathcal{O} \times \mathcal{O}} \Phi^{c,\gamma} \geq \Phi^{c,\gamma}(\bar{c}, \bar{k}, \bar{c}, \bar{k}) = e^{-\eta \bar{k}}((1 - \gamma)u(\bar{c}, \bar{k}) - v(\bar{c}, \bar{k})) > \delta \zeta,
\]

for \( \gamma > 0 \) small enough. In particular, for any \( \gamma > 0 \) and \( \eta > 0 \), \( \Phi^{c,\gamma} \) has a maximizer \((c_{\epsilon,\gamma}, k_{\epsilon,\gamma}, d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma})\), because of the growth conditions on \( u \) and \( v \). Moreover, the growth conditions give an upper bound for this maximizer, depending only on \( \gamma \) and \( \eta \). Therefore \((c_{\epsilon,\gamma}, k_{\epsilon,\gamma}, d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma})\) converges along a subsequence as \( \epsilon \to 0 \). From here on, let us only consider \( \epsilon \) along this subsequence. Because the lower bound at the maximum above is independent of \( \epsilon \),

\[
0 < \delta \zeta < \liminf_{\epsilon \to 0} \Phi^{c,\gamma}(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}, d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma})
\]

which implies

\[
\limsup_{\epsilon \to 0} \frac{1}{2\epsilon}((c_{\epsilon,\gamma} - d_{\epsilon,\gamma})^2 + (k_{\epsilon,\gamma} - \ell_{\epsilon,\gamma})^2) < \infty,
\]

so \((c_{\epsilon,\gamma}, k_{\epsilon,\gamma}), (d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma}) \to (c_{\gamma}, k_{\gamma})\). Note that \( k_{\gamma} \leq k^* \), again because of the growth condition.
Rearranging terms and letting $\epsilon \to 0$,

$$
\lim_{\epsilon \to 0} \beta(c_{\epsilon,\gamma} - \bar{c})^4 + \lim_{\epsilon \to 0} \frac{1}{2\epsilon} \left( (c_{\epsilon,\gamma} - d_{\epsilon,\gamma})^2 + (k_{\epsilon,\gamma} - \ell_{\epsilon,\gamma})^2 \right) \\
\leq \limsup_{\epsilon \to 0} e^{-\eta k_{\epsilon,\gamma}}(1 - \gamma)u(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}) - e^{-\eta k_{\epsilon,\gamma}}v(d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma}) - \delta \zeta \\
\leq e^{-\eta k_{\epsilon}}(1 - \gamma)u(c_{\gamma}, k_{\gamma}) - v(c_{\gamma}, k_{\gamma}) - \delta \zeta \\
\leq \zeta.
$$

That is,

$$
\lim_{\epsilon \to 0} \beta(c_{\epsilon,\gamma} - \bar{c})^4 + \lim_{\epsilon \to 0} \frac{1}{2\epsilon} \left( (c_{\epsilon,\gamma} - d_{\epsilon,\gamma})^2 + (k_{\epsilon,\gamma} - \ell_{\epsilon,\gamma})^2 \right) \leq \zeta. \tag{B.3}
$$

As $\beta$ may be taken arbitrarily large, we ensure that $\zeta < \beta \bar{c}^4$, so that $c_\gamma > 0$.

If $k_\gamma = 0$, we directly obtain $u(c_\gamma, 0) \leq c_\gamma \leq v(c_\gamma, 0)$, which is a contradiction. Hence, $(c_\gamma, k_\gamma)$ must lie in the interior, and so will $(c_{\epsilon,\gamma}, k_{\epsilon,\gamma})$ and $(d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma})$ for sufficiently small $\epsilon$.

Because the maxima are attained in interior points, we proceed to use Ishii’s lemma, from which we obtain $(p^u_\gamma, X) \in \mathcal{J}^{2,+}(e^{-\eta k_{\epsilon,\gamma}}(1 - \gamma)u(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}))$ and $(p^v_\gamma, Y) \in \mathcal{J}^{2,-}(e^{-\eta k_{\epsilon,\gamma}}v(d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma}))$ (Crandall et al., 1992, Theorem 3.2), satisfying

$$
p^u_\gamma = (p^u_c, p^u_k) = (p^v_c + 4\beta(c_{\epsilon,\gamma} - \bar{c})^3, p^v_k), \quad p^v_\gamma = (p^v_c, p^v_k) = \left( \frac{c_{\epsilon,\gamma} - d_{\epsilon,\gamma}}{\epsilon}, \frac{k_{\epsilon,\gamma} - \ell_{\epsilon,\gamma}}{\epsilon} \right)
$$

and

$$
k^{2\alpha}_{\epsilon,\gamma}X - \ell^{2\alpha}_{\epsilon,\gamma}Y \leq k^{2\alpha}_{\epsilon,\gamma}12\beta(c_{\epsilon,\gamma} - \bar{c})^2 + \frac{(k^{\alpha}_{\epsilon,\gamma} - \ell^{\alpha}_{\epsilon,\gamma})^2}{\epsilon} + o(1)
$$

where $o(1)$ denotes a term that converges to 0 as $\epsilon \to 0$.

Because $u$ is a subsolution, $\bar{u} = (1 - \gamma)e^{-\eta k}u$ satisfies

$$
0 \geq \min \left\{ r\bar{u} - \sup_{i \in [0, i_{max}]} \left[ \left( i - \delta \xi \right)(\eta\bar{u} + \partial_i \bar{u}) \right. \\
\left. + \left( (r - \lambda_c)\sigma + k^\alpha \mu - b - i - g(k, i) \right) \partial_i \bar{u} \right. \\
\left. + \frac{1}{2}k^{2\alpha} \sigma^2 \partial_{\xi e}^2 \bar{u} \right) \right. \\
\partial_i \bar{u} - (1 - \gamma)e^{-\eta k} \left. \right\}. \tag{B.4}
$$

Similarly, $\tilde{v} = e^{-\eta k}v$ satisfies

$$
0 \leq \min \left\{ r\tilde{v} - \sup_{i \in [0, i_{max}]} \left[ \left( i - \delta \xi \right)(\eta\tilde{v} + \partial_i \tilde{v}) \right. \\
\left. + \left( (r - \lambda_c)\sigma + k^\alpha \mu - b - i - g(k, i) \right) \partial_i \tilde{v} \right. \\
\left. + \frac{1}{2}k^{2\alpha} \sigma^2 \partial_{\xi e}^2 \tilde{v} \right) \right. \\
\partial_i \tilde{v} - e^{-\eta k} \left. \right\}. \tag{B.5}
$$
We split into two cases, depending on which expression is smallest in (B.4). We begin with the simplest case of

\[ p_c^u \leq (1 - \gamma)e^{-\eta k, \gamma}. \]

Subtracting the two equations (B.4), (B.5) thus gives

\[ 4\beta(c_{\epsilon, \gamma} - \bar{c})^3 = p_c^u - p_c^v \leq (e^{-\eta k, \gamma} - e^{\eta k, \gamma}) - \gamma e^{-\eta k, \gamma} \]

Letting \( \epsilon \to 0 \) in the last inequality

\[ 4\beta(c_{\gamma} - \bar{c})^3 \leq -\gamma e^{-\eta k, \gamma}, \]

which contradicts with (B.3), because \( \zeta \) can be chosen arbitrarily small, independently of \( k^\ast \).

In the other case, we subtract the equations and get that

\[ \Delta(\tilde{u} - \tilde{v}) \leq \sup_{i \in [0, i_{\text{max}}]} \left\{ i - \delta c \eta \epsilon, \gamma \right\} \left[ (\eta \epsilon, \gamma, k_{\epsilon, \gamma}) + p_k^u \right] \]

\[ + \left[ (r - \lambda \epsilon, \gamma) c + k_{\epsilon, \gamma}^\alpha \mu - b - i - g(k_{\epsilon, \gamma}, i) \right] (p_c^v + 4\beta(c_{\epsilon, \gamma} - \bar{c})^3) + \frac{1}{2} k_{\epsilon, \gamma}^2 \sigma^2 X \]

\[ - \left[ i - \delta c \epsilon, \gamma \right] (\eta \epsilon, \gamma, \epsilon, \gamma) + p_k^v \]

\[ - \left[ (r - \lambda \epsilon, \gamma) c + \ell_{\epsilon, \gamma}^\alpha \mu - b - i - g(\epsilon, \gamma, i) \right] p_c^v - \frac{1}{2} \ell_{\epsilon, \gamma}^2 \sigma^2 Y \]

\[ \leq \sup_{i \in [0, i_{\text{max}}} \left\{ i(\epsilon, \gamma, k_{\epsilon, \gamma}, \epsilon, \gamma) - \tilde{v}(d_{\epsilon, \gamma}, \epsilon, \gamma) \right\} \]

\[ + \left[ (r - \lambda \epsilon, \gamma) c + k_{\epsilon, \gamma}^\alpha \mu - b - i - g(k_{\epsilon, \gamma}, i) \right] 4\beta(c_{\epsilon, \gamma} - \bar{c})^3 \]

\[ - \delta c \epsilon, \gamma - k_{\epsilon, \gamma} \right\} (k_{\epsilon, \gamma}^\alpha - \ell_{\epsilon, \gamma}^\alpha) \mu - \epsilon \left( g(k_{\epsilon, \gamma}, i) - g(\epsilon, \gamma, i) \right) \right| p_c^v \]

\[ + 6k_{\epsilon, \gamma}^2 \sigma^2 \beta(c_{\epsilon, \gamma} - \bar{c})^2 + \frac{(k_{\epsilon, \gamma}^\alpha - \ell_{\epsilon, \gamma}^\alpha)^2}{\epsilon} + o(1) \}

Let \( \eta < (r - \Delta)/i_{\text{max}} \) for \( \Delta \in (0, r) \). Then, taking \( \limsup \) as \( \epsilon \to 0 \), and using that \( g(\epsilon, i) \) and \( k \mapsto k^\alpha \) are Lipschitz in a neighborhood of \( (c_{\epsilon, \gamma}, \epsilon, \gamma) \), i.e.,

\[ |g(k_{\epsilon, \gamma}, i) - g(\epsilon, \gamma, i)| + \mu |k_{\epsilon, \gamma}^\alpha - \ell_{\epsilon, \gamma}^\alpha| \leq R |k_{\epsilon, \gamma} - \epsilon, \gamma| \]

we get

\[ \limsup_{\epsilon \to 0} \Delta(\tilde{u}(c_{\epsilon, \gamma}, k_{\epsilon, \gamma}) - \tilde{v}(d_{\epsilon, \gamma}, \epsilon, \gamma)) \]

\[ \leq \lim_{\epsilon \to 0} \left[ \delta(\epsilon, \gamma) + R^3 \frac{(k_{\epsilon, \gamma} - \ell_{\epsilon, \gamma})^2}{\epsilon} + R \frac{(c_{\epsilon, \gamma} - d_{\epsilon, \gamma}) (k_{\epsilon, \gamma} - \ell_{\epsilon, \gamma})}{\sqrt{\epsilon}} \right] \]

\[ + R'(|c_{\epsilon, \gamma} - \bar{c}|^2 + |c_{\epsilon, \gamma} - \bar{c}|^3) + o(1), \]

for some constant \( R' \) depending on \( k^\ast \) (i.e., \( \eta, i_{\text{max}}, \beta \), and the model parameters. In other words, \( R' \) is independent of \( \zeta \). By (B.3), the right hand side is bounded by \( R''(\zeta + \sqrt{\zeta}) \), for
some constant $R'' > 0$ that is also independent of $\zeta$. Finally, because $\Delta > 0$,

$$\delta_{\zeta} \leq e^{-\eta k_{\zeta}}((1 - \gamma)u - v)(c_{\zeta}, k_{\zeta}) \leq \lim_{\epsilon \to 0} \sup \left( \bar{u}(c_{\epsilon, \gamma}, k_{\epsilon, \gamma}) - \bar{v}(d_{\epsilon, \gamma}, \ell_{\epsilon, \gamma}) \right) \leq \frac{R''}{\Delta} (\zeta + \sqrt{\zeta}),$$

which is a contradiction, because $\delta_{\zeta}/(\zeta + \sqrt{\zeta})$ can be chosen arbitrarily large. Hence, there cannot exist a point $(c, k)$ such that $(u - v)(c, k) > 0$.

Following the steps of the proof of Proposition 3, it can be easily proven that $V(c, k) \leq M + c + k$ for some $M$. As a consequence, $V$ satisfies the assumptions of Proposition 4. The following results are standard consequences of comparison of viscosity solutions.

**Corollary 5.** The value function $V$ is the unique solution to (B.1) with its boundary conditions.

Now we present our numerical algorithm to solve the model. The equation (B.1) is solved in a square domain $[0, c_{\text{max}}] \times [0, k_{\text{max}}]$ via policy iteration, which produces the value function $V(k, c)$ and investment policy function $i(k, c)$ in addition to the regions of dividend and equity issuance. The singular structure of the dividends is approximated as in (Reppen et al., 2020, Section 4), which also describes the policy iteration algorithm, and the impulse control issuance as in (Reppen et al., 2020, Section 6.1.2).

In addition to (B.2), the boundary conditions where $c = c_{\text{max}}$ and $k_{\text{max}}$ are given by

$$0 = \partial_c V - 1 \quad \text{at } c = c_{\text{max}}$$

$$0 = \min \left\{ r V + \delta_k \partial_k V - \left[ r c + k^\alpha \mu - b \right] \partial_c V - \frac{1}{2} k^{2\alpha} \sigma^2 \partial^2_{cc} V, \partial_c V - 1, \mathcal{I} V \right\} \quad \text{at } k = k_{\text{max}}$$

At the corners, the $c$-conditions are used.

Another consequence of the comparison result in Proposition 4 is the convergence of the numerical scheme, see Barles and Souganidis (1991).

**Corollary 6.** Numerical solutions converge to the value function as the discretization gets finer.
Internet Appendix to
Firm Dynamics Depend on Cash and Capital

This Internet Appendix contains supplementary empirical work. These include the following:

1. Appendix C provides additional empirical work
   (a) Table C1 shows the sample selection criteria
   (b) Section Appendix C.1 provides support in the data for the model’s predicted relation between investment-to-depreciation and capital
   (c) Section Appendix C.2 provides support in the data for the model’s prediction that leveraged firms with low capital are less likely to offer and more likely to default
Appendix C. Empirical Appendix

Table C1: Sample Selection

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<th>Obs. Remaining</th>
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<tr>
<td>Less:</td>
<td></td>
<td></td>
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<td>Firms missing Intangible Capital Data (1960-2017)</td>
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<tr>
<td>Pre-IPO Data</td>
<td>(28,778)</td>
<td>455,740</td>
<td></td>
</tr>
<tr>
<td>Firms headquartered outside of USA</td>
<td>(77,440)</td>
<td>378,300</td>
<td></td>
</tr>
<tr>
<td>Firms incorporated outside of USA</td>
<td>(4,117)</td>
<td>374,183</td>
<td></td>
</tr>
<tr>
<td>Financials (SIC-1==6)</td>
<td>(94,916)</td>
<td>277,267</td>
<td></td>
</tr>
<tr>
<td>Utilities (SIC-2==49)</td>
<td>(18,997)</td>
<td>258,270</td>
<td></td>
</tr>
<tr>
<td>Public Administration (SIC-1==9)</td>
<td>(4,943)</td>
<td>253,327</td>
<td></td>
</tr>
<tr>
<td>Listed on non-major U.S. exchange</td>
<td>(107,798)</td>
<td>145,529</td>
<td></td>
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<tr>
<td>Firms with missing or zero assets</td>
<td>(2,321)</td>
<td>143,208</td>
<td></td>
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<tr>
<td>Firms with missing common stock price at close of fiscal year</td>
<td>(11,926)</td>
<td>131,282</td>
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<td>Firms with missing shares outstanding</td>
<td>(375)</td>
<td>130,907</td>
<td></td>
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<tr>
<td>Firms with missing or zero book value of equity</td>
<td>(1,286)</td>
<td>129,621</td>
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<tr>
<td>Firms with cash and cash equivalents</td>
<td>(14)</td>
<td>129,607</td>
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<tr>
<td>Firms with missing or zero total liabilities</td>
<td>(253)</td>
<td>129,354</td>
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<tr>
<td>Firms with missing net income</td>
<td>(151)</td>
<td>129,203</td>
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<td>Firms with missing operating income</td>
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<td>129,202</td>
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<td>Firms with missing retained earnings</td>
<td>(1,573)</td>
<td>127,629</td>
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<td>Firms with missing working capital</td>
<td>(2,740)</td>
<td>124,889</td>
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<td>Firms with PP&amp;E less than $5M or missing PP&amp;E</td>
<td>(26,685)</td>
<td>98,204</td>
<td></td>
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<td>Firms with negative cash and cash equivalents</td>
<td>(2)</td>
<td>98,202</td>
<td></td>
</tr>
<tr>
<td>Firms with less than $1M in sales</td>
<td>(375)</td>
<td>97,827</td>
<td></td>
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<tr>
<td>Firms with zero market equity</td>
<td>(3)</td>
<td>97,824</td>
<td></td>
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<td>Singleton Firms</td>
<td>(566)</td>
<td>97,258</td>
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<td>SIC-4 industries with one firm</td>
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<td>93,822</td>
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<td>Final sample (6,731 firms)</td>
<td></td>
<td></td>
<td>93,822</td>
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Appendix C.1. Investment-to-depreciation is declining and convex in capital

In our model, the assumption that capital exhibits diminishing returns creates convexity in the investment-to-depreciation ratio. Figure C.1a shows that investment to depreciation is declining and convex in capital. Intuitively, as capital increases the marginal returns to investing decrease, discouraging investment. As capital increases, depreciation increases linearly in capital at the rate $\delta$. Together, the ratio of investment to depreciation should be declining and convex in capital.

This reasoning leads to the following hypothesis:

**H**: The ratio of investment to depreciation is declining and convex in capital.

To evaluate this hypothesis, we estimate the following empirical specification:

$$\frac{\text{Investment}_{i,t+1}}{\text{Depreciation}_{i,t+1}} = \beta_1 \text{Capital}_{i,t} + \beta_2 \text{Capital}^2_{i,t} + \beta_3 \text{Cash}_{i,t} + \mu_i + \delta_{j,t} + \epsilon_{i,t} \quad (C.1)$$

The outcome is firm $i$’s investment in physical and intangible capital in year $t + 1$ scaled by the depreciation of physical and intangible capital in year $t$ (See Section 4.1 for details). The main explanatory variable is a firm’s total capital at the end of year $t$ standardized within firm and winsorized at the 1% level. $\beta_2$ multiplies the quadratic form of capital (standardized). We control for a firm’s cash and equivalents, firm fixed effects $\mu_i$, and SIC-2–by–year industry trends ($\delta_{j,t}$). $\epsilon_{i,t}$ is the unexplained variation. Standard errors are double clustered by firm and year.

Table C2 presents the results. Column (1) uses the full sample of firms. $\beta_1$ multiplying a firm’s capital position is -42.50 and highly significant. $\beta_2$ multiplying the quadratic form of a firm’s capital is +17.39 and also highly significant. Together, there is strong evidence that payouts are declining in capital in a convex pattern. Columns (2) and (3) show the relation is similar splitting the sample for the investment analyses at the median year of 1995.
Columns (4) to (10) show the convexity exists across SIC-1 industry classifications.

Figure C.1b complements the regression results. The binscatter plot shows a convex relation between the investment-to-depreciation ratio and within firm capital.

Table C2: The ratio of investment to depreciation is convex and declining in capital

The outcome variable is the ratio of investment to depreciation. Investments include spending on physical capital as well as intangible capital on and off the balance sheet. We also determine the depreciation of physical and intangible capital. The main explanatory variable is the total capital of the firm (standardized within firm) and its square (standardized). We control for the cash reserve (standardized within firm), industry trends, and year fixed effects. Columns (2) and (3) split the sample after and on or before 1995, respectively. Columns (4) to (10) restrict the sample to industries based on the SIC-1 identifier. Standard errors are double clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

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<th>Specification</th>
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<th>SIC1=8</th>
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<tr>
<td>Investment (t+1) / Depreciation (t+1) × 100</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
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<td>Capital (t)</td>
<td>-42.50***</td>
<td>-41.38***</td>
<td>-39.96***</td>
<td>-56.50***</td>
<td>-29.30***</td>
<td>-39.60***</td>
<td>-46.71***</td>
<td>-48.26***</td>
<td>-53.47***</td>
</tr>
<tr>
<td>(1.96)</td>
<td>(3.70)</td>
<td>(2.25)</td>
<td>(6.31)</td>
<td>(2.21)</td>
<td>(2.02)</td>
<td>(5.56)</td>
<td>(3.65)</td>
<td>(3.83)</td>
<td>(4.73)</td>
</tr>
<tr>
<td>Capital (t)²</td>
<td>17.39***</td>
<td>14.96***</td>
<td>16.28***</td>
<td>18.55***</td>
<td>10.18***</td>
<td>18.09***</td>
<td>16.41***</td>
<td>21.97***</td>
<td>23.27***</td>
</tr>
<tr>
<td>(1.21)</td>
<td>(2.05)</td>
<td>(1.34)</td>
<td>(3.32)</td>
<td>(1.27)</td>
<td>(1.38)</td>
<td>(3.39)</td>
<td>(2.16)</td>
<td>(2.51)</td>
<td>(3.29)</td>
</tr>
<tr>
<td>Cash (t)</td>
<td>8.28***</td>
<td>7.93***</td>
<td>10.54***</td>
<td>19.51***</td>
<td>6.54***</td>
<td>7.55***</td>
<td>12.63***</td>
<td>7.31***</td>
<td>5.50**</td>
</tr>
<tr>
<td>(0.66)</td>
<td>(0.86)</td>
<td>(0.79)</td>
<td>(3.59)</td>
<td>(0.95)</td>
<td>(0.90)</td>
<td>(2.62)</td>
<td>(1.71)</td>
<td>(2.13)</td>
<td>(2.73)</td>
</tr>
<tr>
<td>Constant</td>
<td>153.38***</td>
<td>136.22***</td>
<td>170.29***</td>
<td>242.16***</td>
<td>133.24***</td>
<td>142.78***</td>
<td>165.23***</td>
<td>167.18***</td>
<td>152.94***</td>
</tr>
<tr>
<td>(0.06)</td>
<td>(0.86)</td>
<td>(0.72)</td>
<td>(0.34)</td>
<td>(0.15)</td>
<td>(0.10)</td>
<td>(0.29)</td>
<td>(0.24)</td>
<td>(0.23)</td>
<td>(0.36)</td>
</tr>
</tbody>
</table>

Firm FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
SIC-2× Year FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
% Adjusted R² | 53.41 | 59.10 | 52.04 | 46.13 | 46.12 | 48.81 | 52.48 | 56.18 | 55.01 | 61.11 |
% Within R² | 8.92 | 10.10 | 5.94 | 5.35 | 5.87 | 11.82 | 7.82 | 13.47 | 12.41 | 8.07 |
Observations | 84511 | 41290 | 42812 | 5762 | 17936 | 30620 | 6121 | 10421 | 9710 | 3635 |
Figure C.1a is the amount a firm invests scaled by depreciation at $c = 0.15$ from Figure 1. Here $b = 0.02$ and other parameters used are summarized in Table 1.

(b) The y-axis is a firm’s investment in physical and intangible capital in year $t + 1$. Depreciation is the depreciation of tangible and intangible capital in year $t$. The x-axis is a firm’s total capital (tangible and intangible) standardized within firm.

Figure C.1: Predicted Investment from Figure 1 vs. Actual
We provide evidence of diminishing returns to scale in our sample by examining how profitability varies with capital stock. Table C3 shows that a firm’s return on assets and return on equity decline with capital. Note that the specification includes firm and industry-by-year fixed effects.

Table C3: Decreasing Returns to Scale
This table provides evidence of decreasing returns to scale. The outcome variable in column (1) is the Return on Assets (EBITDA/Book Value of Assets), measured in percentage points. The outcome variable in column (2) is the Return on Equity (Net Income/Book Value of Equity). All explanatory variables are standardized to facilitate comparisons. Each specification includes firm fixed effects and industry trends. Standard errors are double clustered by stock and quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

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<th></th>
<th>Return on Assets (%)</th>
<th>Return on Equity (%)</th>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
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<tr>
<td>Capital (t)</td>
<td>-6.46***</td>
<td>-1.93***</td>
</tr>
<tr>
<td></td>
<td>(0.66)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Cash (t)</td>
<td>5.32***</td>
<td>1.33***</td>
</tr>
<tr>
<td></td>
<td>(0.38)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>Constant</td>
<td>6.04***</td>
<td>13.16***</td>
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<tr>
<td></td>
<td>(0.01)</td>
<td>(0.00)</td>
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<td>Firm FE</td>
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<td>Yes</td>
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<tr>
<td>SIC-4 x Year FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>% Adjusted R$^2$</td>
<td>15.25</td>
<td>61.46</td>
</tr>
<tr>
<td>% Within R$^2$</td>
<td>0.48</td>
<td>0.88</td>
</tr>
<tr>
<td>Observations</td>
<td>93916</td>
<td>93840</td>
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</table>
Appendix C.2. Leverage, Offerings, and Restructurings

Our model predicts that, in the presence of costly financing, distressed firms with low capital are less likely to issue equity to continue and more likely to default. Intuitively, when capital is low, the cash flows are low. Thus, equityholders have to decide whether to default or pay the issuance costs (which are larger for small firms because of the fixed component) to raise cash to fund the coupon payments and to invest over time (because of convex adjustment costs) to rebuild the capital. For more leveraged firms, the coupon rates are higher, which increases the coupon burden on the firms, especially when capital stock is low.

This reasoning leads to the following hypothesis:

H: Leveraged firms are less likely to conduct offerings and more likely to restructure when capital is low.

To evaluate this hypothesis, we estimate the following two empirical specifications:

\[
\text{Offering}_{i,t+1} = \beta_1 \text{Capital}_{i,t} + \beta_2 \text{Leverage}_{i,t} + \beta_3 \text{Capital}_{i,t} \times \text{Leverage}_{i,t} + \beta_4 \text{Cash}_{i,t} + \mu_i + \delta_{j,t} + \epsilon_{i,t} \tag{C.2}
\]

\[
\text{Restructuring}_{i,t+1} = \beta_1 \text{Capital}_{i,t} + \beta_2 \text{Leverage}_{i,t} + \beta_3 \text{Capital}_{i,t} \times \text{Leverage}_{i,t} + \beta_4 \text{Cash}_{i,t} + \mu_i + \delta_{j,t} + \epsilon_{i,t} \tag{C.3}
\]

The outcome in Eq. C.2 is an indicator that equals one if firm \( i \) conducts an offering in year \( t + 1 \) amounting to more than 5% of firm \( i \)'s capital at the end of year \( t \). The outcome in Eq. C.3 is an indicator that equals one if firm \( i \) has restructuring expenses in year \( t + 1 \) and no restructuring expenses in year \( t \). The main explanatory variable is a firm’s total capital at the end of year \( t \) standardized within firm and winsorized at the 1% level. \( \beta_2 \) multiplies the quadratic form of capital (standardized). We control for a firm’s cash and equivalents, firm fixed effects \( \mu_i \), and SIC-2–by–year industry trends (\( \delta_{j,t} \)). \( \epsilon_{i,t} \) is the unexplained variation.
Standard errors are double clustered by firm and year.

Table C2 presents the results. Panel A Column (1) shows that firms with high leverage and low capital are less likely to conduct an offering. Figure C.2a complements the results, showing that for firms with above average leverage, the propensity to conduct an offering is lower for small firms but largely similar for larger firms. Columns (2) and (3) show the result holds before and after 1997. Column (4) shows the result holds for when the outcome is whether the firm conducts an offering in year \( t + 1 \) that is greater than 10% of capital as of the end of year \( t \). Columns (5)-(11) show that the relation is fairly common across industries.

By contrast, Table C2 Panel B Column (1) shows that leveraged firms with low capital are more likely to restructure. Figure C.2b complements the results, showing that the propensity to restructure is higher for high-leverage firms with low capital than for low-leverage firms with low capital. There is no difference in the restructuring propensity for high and low leverage firms with capital is high. Columns (2) and (3) show the relation holds before and after the median year of 2005 for the sample with restructuring data. Columns (4) to (10) show directionally similar results across industries.
Table C4: Firms with high leverage and low capital are less likely to issue stock and more likely to restructure

In Panel A, the outcome variable is an indicator that equals 100 if the a firm receives proceeds from stock sales of at least 5% of capital in year \( t + 1 \). In Panel B, the outcome variable is an indicator that equals 100 if the firm has restructuring expenses in year \( t + 1 \). The sample in Panel B is limited to years without any restructuring expenses in year \( t \). The main explanatory variable in both panels is the total capital of the firm (standardized within firm) and its square (standardized). We control for the cash reserve (standardized within firm), industry trends, and year fixed effects. In Panel A, Columns (2) and (3) split the sample after and on or before 1997, respectively. Column (4) restricts the outcome to offerings of at least 10% of capital. Columns (5) to (11) restrict the sample to industries based on the SIC-1 identifier. In Panel B, Columns (2) and (3) split the sample after and on or before 2005 (the median year in the sample with restructuring data), respectively. Columns (4) to (10) restrict the sample to industries based on the SIC-1 identifier. Standard errors are double clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

### Panel A: Levered firms less likely to issue when capital is low

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<th>( \leq '97 )</th>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>% Adjusted R²</td>
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<td>21.12</td>
<td>18.50</td>
<td>16.88</td>
<td>24.57</td>
<td>20.61</td>
<td>19.82</td>
<td>15.69</td>
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<td>Capital (t) × Leverage (t)</td>
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<td>0.88***</td>
<td>0.76***</td>
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<td>-0.31</td>
<td>1.25***</td>
<td>0.51</td>
<td>1.30***</td>
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<td>Cash (t)</td>
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<td>-1.91***</td>
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### Panel B: Levered firms more likely to restructure when capital is low

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<td>% Adjusted R²</td>
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<td>% Within R²</td>
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<td>0.27</td>
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<td>2.18**</td>
<td>0.84*</td>
<td>0.74*</td>
<td>0.54*</td>
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<td>Capital (t) × Leverage (t)</td>
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<td>-0.91***</td>
<td>-0.30*</td>
<td>-1.46</td>
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<td>-0.44*</td>
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(a) The y-axis is the percentage of firms issuing shares worth at least 1% of their capital stock in year $t + 1$. The x-axis is a firm’s total capital (tangible and intangible) standardized within firm.

(b) The y-axis is the percentage of firms restructuring in year $t + 1$. The x-axis is a firm’s total capital (tangible and intangible) standardized within firm.

Figure C.2: Leverage, Offerings, and Restructuring